Particle-Tracking Analysis of Contributing Areas of Public-Supply Wells in Simple and Complex Flow Systems, Cape Cod, Massachusetts

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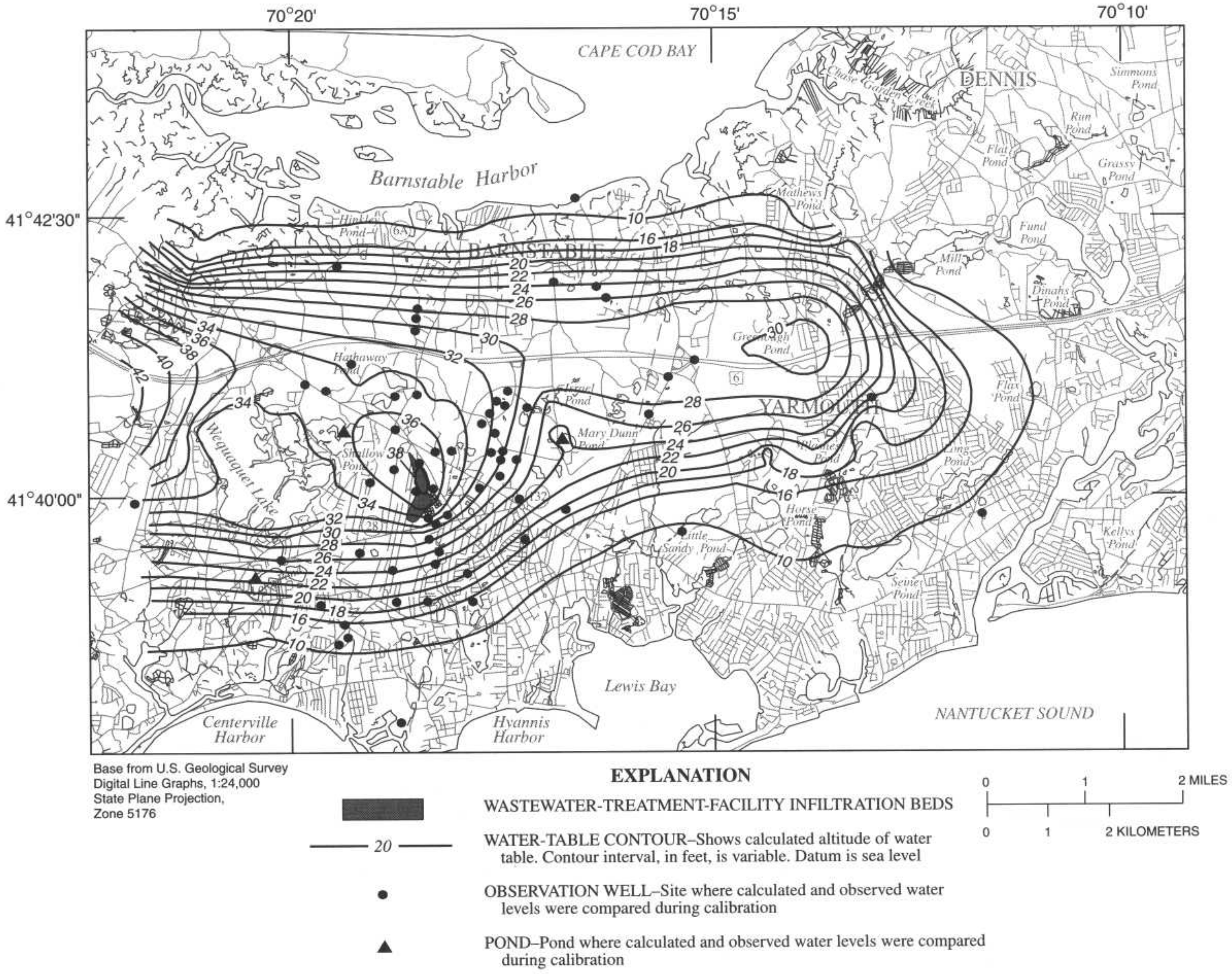


Figure 11. Calculated water-table configuration for the two-dimensional model of the complex flow system.

Total inflow to the model is 79.7 ft^3/s (table 9), which is 0.3 ft^3/s more than total inflow to the three-dimensional model. Results of the calibration indicate that approximately 0.8 ft^3/s more water discharges to streams in the two-dimensional model than in the three-dimensional model, and net outflow to Wequaquet Lake in the two-dimensional model is greater than the three-dimensional model by approximately 1.6 ft^3/s. The sensitivity analysis indicates that calculated heads are sensitive to both recharge and hydraulic conductivity (fig.10*B*) and are most sensitive to decreases in horizontal hydraulic conductivity.

PARTICLE-TRACKING ANALYSIS OF CONTRIBUTING AREAS

Tracking of fluid particle pathlines within a ground-water-flow model from areas of recharge to a supply well is the final step in the delineation of the contributing area of a supply well. The simulations reported here were completed to demonstrate the use of particle tracking for delineation of contributing areas and the source of water to public-supply wells, and to assess qualitatively and quantitatively how the location of that area and the source of water are affected by such factors as well location and pumping rate, aquifer heterogeneity, spatial variability of recharge rates, parameter uncertainty, and grid discretization.

Procedure for Delineation of Contributing Areas

The program MODPATH uses the heads and intercell flow rates (the flow rate at the face of each cell in the model) calculated by the flow model of McDonald and Harbaugh (1988) to determine particle pathlines and velocities. To determine particle velocities, MODPATH requires that the porosity of the material represented by each cell be specified. Uniform porosities of 0.39 for sand and gravel and 0.68 for silt and clay were specified for each cell of the models and are based on porosities reported for Cape Cod sediments presented earlier. MODPATH also requires specification of the altitude of the upper and lower boundaries of each cell (except the upper boundary of cells that simulate the water table). Because the bottom altitude of the fifth layer of the three-dimensional model of the simple flow system was not explicitly specified (because a transmissivity was used for that layer), a bottom altitude of 446 ft below sea level was assigned to it in MODPATH. This altitude corresponds to the contact between glacial sediments and bedrock at test hole Eastham 45.

Starting locations of particles must be specified to initiate a particle-tracking analysis. Particles may be tracked either forward (from the water table to a pumped well) or backward (from a pumped well to the water table). Although both forward and backward particle tracking were tried during this investigation, forward tracking proved to be more useful for two reasons. First, complicated, discontinuous contributing areas were better defined by forward tracking than by backward tracking because it was often unclear whether areas located between particles tracked to the water table in backward-tracking analyses should be included in the contributing area to a well. In forward-tracking analyses, however, the contributing area of the well is defined by that area of the water table from which particles captured by the well originate. Second, forward tracking of particles proved useful in ensuring that each well captured a quantity of water sufficient to satisfy its specified pumping rate and, therefore, that the contributing area for each well was correct in size. Correct sizing was accomplished by first calculating the volume flow rate of each particle tracked to a well by multiplying the area of the water table represented by a particle (equal to the area of the cell in which the particle originates divided by the number of particles specified for each cell) by the recharge rate to the cell in which the particle originates; that is,

$$Qp_{(x)} = [Ap\ R]_{(x)}, \tag{5}$$

where

$Qp_{(x)}$ is the volume flow rate of particle x (L^3/T);

Ap is the area of the water table represented by particle x, which is equal to the area of cell i, j, k in which the particle originates divided by the number of particles specified for the cell (L^2); and

R is the total recharge rate to the top face of cell i, j, k in which particle x originates (L/T).

The volume flow rates of all particles captured by a well can then be summed to determine the total quantity of water captured by a well (referred to as the "calculated pumping rate" of the well); that is,

$$Q'w = \sum_{i=1}^{n} Qp_{(x)} , \qquad (6)$$

where

$Q'w$ is the calculated pumping rate of the well (L^3/T); and

n is the total number of particles captured by the well.

Although equations 5 and 6 provide a useful means of ensuring that a well captures a quantity of water equal to its specified pumping rate, calculated and specified pumping rates may not be equal, for three reasons. First, water captured by the well may have originated from sources other than recharge at the water table, such as from specified-head or head-dependent flux boundaries that are not accounted for by equations 5 and 6. Second, an insufficient number of particles may have been used in the delineation of the contributing area of the well. Finally, specified and calculated pumping rates may differ because of the presence of one or more weak internal boundary sinks within the contributing area of the well.

A weak internal boundary sink is a specified-flux or head-dependent flux boundary sink—such as a simulated pumped well, gaining stream, or drain—that does not capture all flow crossing the six faces of the model cell in which the sink is located. In contrast, a strong internal boundary sink is one that captures all flow crossing the six faces of the model cell in which the sink is located. An example of a weak internal boundary sink is a simulated well that pumps only 67 percent of the water that flows into the cell in which the well is located. MODPATH does not remove particles from internal boundary sinks on the basis of the percentage of flow discharged by the weak sink. Consequently, there is no explicit way to determine whether a particle that enters a cell in which a weak sink is located should be discharged by the sink or pass through the cell (Pollock, 1989, p. 18).

There are two ways to address the problem of weak internal boundary sinks for the delineation of contributing areas. First, weak internal boundary sinks can be eliminated from a flow model by decreasing the vertical and (or) horizontal size of the model cells until all flow entering cells with internal boundary sinks is discharged by the sinks. It is often not practical, however, to reduce the size of grid cells to the point at which all weak internal boundary sinks are eliminated (Pollock, 1989, p. 20). Second, contributing areas can be delineated in two particle-tracking analyses that result in a maximum and minimum contributing area being defined for each well. In the first analysis, particles are stopped at weak internal boundary sinks, and, in the second, particles are allowed to pass through the weak sinks. Unless the well itself is a weak sink, the first analysis provides an estimate of the minimum size of the contributing area of each well and the second analysis provides an estimate of the maximum size of the contributing area of each well for the particular set of pumping, recharge, and hydrogeologic conditions simulated by the flow model. If the well itself is a weak sink, then the first analysis will provide the maximum contributing area to the well, and no contributing area will be defined for the well in the second analysis because all particles will pass through the weak sink cell in which the well is located. As is evident from the preceding discussion, the presence of weak internal boundary sinks, if not eliminated, can lead to ambiguities in the exact delineation of the contributing area of a well.

Of the two options for addressing the problem of weak internal boundary sinks, refining the model grid or completing two particle-tracking analyses, the latter option was chosen for this investigation. This was done because weak internal boundary sinks were not present in either the two- or three-dimensional models of the simple flow system and did not significantly affect contributing areas delineated for wells simulated by the three-dimensional model of the complex flow system. Though weak internal boundary sinks did affect the delineation of contributing areas delineated for wells simulated by the two-dimensional model of the complex flow system, the results of those particle-tracking analyses were used primarily to demonstrate the limitations of a two-dimensional model for delineation of contributing areas.

Simple Flow System—Analysis of Contributing Areas to Two Hypothetical Wells

Two hypothetical, large-capacity supply wells were simulated in the flow models developed for the simple flow system. Well A (which is located at test hole E45, fig. 2) is near the crest of a natural recharge mound where hydraulic gradients are low; well B is between this mound and discharge areas along the Atlantic coast, where gradients are steeper than those at the mound. These hypothetical wells were chosen for investigation because of their differing locations in the flow system and because they have been cited as possible locations for supply wells for the town of Eastham. Contributing areas were delineated by use of nine particles, evenly spaced at the water table in each grid cell of the top layer of the model, and tracked in the forward direction to the wells. The distribution of nine particles per cell adequately defined the contributing area of each well for all simulations, as indicated by the fact that calculated pumping rates for the wells (from eqs. 5 and 6) were within 1 percent of specified pumping rates in the flow model. Conditions used in the simulations are summarized in table 10.

Delineation of Contributing Areas for a Pumping Rate of 0.5 Million Gallons Per Day Per Well

Contributing areas can be difficult to delineate for conditions in which multiple wells are pumped simultaneously from an aquifer, because pumping at any one well can affect the flow of water to other wells. It is necessary, therefore, to simulate all pumped wells simultaneously in order to delineate the contributing area to any particular well accurately. When all wells are pumped simultaneously, contributing areas to individual wells do not overlap because it is impossible for a particle of water that originates at the water table to flow to more than one discharge point. When wells are pumped individually in separate simulations, however, contributing areas can overlap, and the total area delineated can be smaller than the area that would have resulted from wells being pumped simultaneously.

In the first simulation (the base simulation), each well was pumped at a rate of 0.5 Mgal/d. The sizes of the contributing areas to wells A and B can be determined explicitly because recharge from precipitation is uniform over the contributing area of each well and because it is the only source of water to the wells. The contributing area for each well is 0.57 mi^2, which is equal to the discharge rate of each well, 0.5 Mgal/d, divided by the recharge rate to the aquifer within the contributing area of each well, 18.3 in/yr.

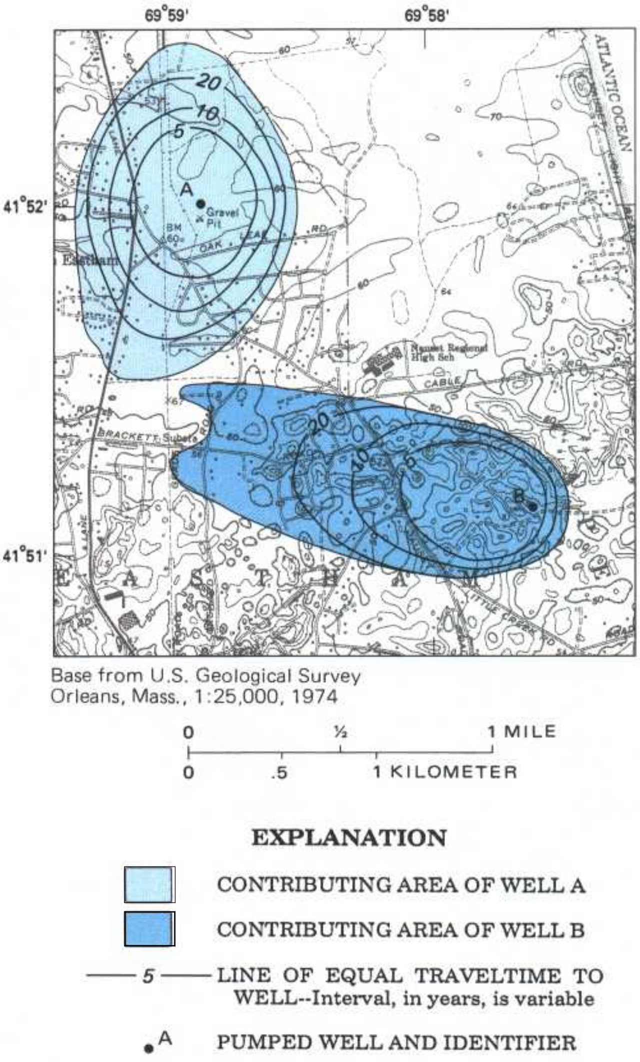
Contributing areas delineated for the wells indicate that the shape of the contributing area of each well depends on the location of the supply well in the flow system. The contributing area for well A (fig.12), located near the crest of the recharge mound where head gradients are nearly flat, is oval; water is captured about equally from all areas around the well. The contributing area for well B, however, located between the crest of the recharge mound and the coastal discharge boundary of the aquifer, is elongate; water is captured primarily from areas that lie upgradient from the well. The bifurcation on the upgradient end of the contributing area for well B results from the fact that particles of water originating at the water table between its tails follow three-dimensional flow paths that end at the coastal boundary.

Pathlines of particles initiated at the water table along the row of the model in which well B is located (row 55) show how particles move from the water table to discharge points at the well or specified-head cells downgradient from the well (fig.13). Because hydraulic gradients produced by the well are steeper near the well than in other parts of the aquifer, ground-water and particle velocities are also highest near the well, as indicated by particle pathlines after 5 and 10 years of travel (fig.13).

Table 10. Summary of hydrogeologic and model conditions for delineation of contributing areas of hypothetical wells A and B in the simple flow system

[No., number; --, not applicable to two-dimensional model]

Simulation No.	1	2a	2b	2c	3a	3b	3c	4a	4b	4c	5	6a	6b	7a	7b	7c	7d	7e	7f	7g	7h	8a	8b	9
Hydrogeologic and Model Conditions																								
Model:																								
Three dimensional	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	
Two dimensional																								X
Layer of well screens:																								
1		X			X			X																X
2			X			X			X															
3	X			X			X			X	X	X	X	X	X	X	X	X	X	X	X	X	X	
Well-discharge rates, in million gallons per day:																								
0.10		X	X	X																				
0.25					X	X	X																	
0.40								X	X	X														
0.50	X											X	X	X	X	X	X	X	X	X	X	X	X	X
1.00											X													
Ratio of horizontal to vertical hydraulic conductivity, layers 1–3:																								
Calibrated model	X	X	X	X	X	X	X	X	X	X	X			X	X	X	X	X	X	X	X	X	X	--
0.1 × calibrated model												X												
0.01 × calibrated model													X											
Aquifer recharge:																								
Calibrated model	X	X	X	X	X	X	X	X	X	X	X	X	X					X	X	X	X	X	X	X
0.8 × calibrated model														X		X								
1.2 × calibrated model															X		X							
Aquifer horizontal hydraulic conductivity:																								
Calibrated model	X	X	X	X	X	X	X	X	X	X	X	X	X			X	X			X	X	X	X	X
0.7 × calibrated model														X				X						
1.3 × calibrated model															X				X					
Porosity of coarse-grained stratified drift:																								
Calibrated model (0.39)	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X			X	X	X
0.29																				X				
0.49																					X			
Sediment layering:																								
Calibrated model	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X			X
Continuous zone of low hydraulic conductivity over well B																						X		
Discontinuous zones of low hydraulic conductivity over well B																							X	



Base from U.S. Geological Survey
Orleans, Mass., 1:25,000, 1974

EXPLANATION

🟦	CONTRIBUTING AREA OF WELL A
🟦	CONTRIBUTING AREA OF WELL B
—— 5 ——	LINE OF EQUAL TRAVELTIME TO WELL--Interval, in years, is variable
•A	PUMPED WELL AND IDENTIFIER

Figure 12. Contributing areas of and traveltimes to wells A and B for a pumping rate of 0.5 million gallons per day per well, determined by use of the three-dimensional model of the simple flow system.

Lines of equal particle traveltime to each well for simulation 1 are shown in figure 12. Traveltime estimates commonly are used to define protection zones around supply wells and can be used, for example, to estimate the traveltime to supply wells of contaminants released to an aquifer by an accidental spill. They also provide a means of computing the percentage of the discharge from each well that has traveled to the well in a specified period of time. For example, table 11 indicates that, for simulation 1, approximately 8.4 percent of the particles captured by well A and 6.8 percent of those captured by well B take less than 2 years to travel from their point of entry at the water table to the pumped wells. Table 11 also indicates that more than 25 percent of the water discharged from each well takes longer than 20 years to travel to the well from the water table.

Sensitivity of Contributing Areas to Selected Factors

Hydrogeologic and well-design factors and vertical discretization of the model were varied in several simulations to evaluate the effect of these factors on the delineation of contributing areas of the two wells.

Penetration of Well Screens, Pumping Rates of Wells, and Ratio of Horizontal to Vertical Hydraulic Conductivity

Public-supply wells in the glacial deposits of Cape Cod commonly are screened through only a small part of the vertical thickness of the aquifer; that is, the wells partially penetrate the aquifer. Partial penetration of well screens can affect the area from which water is captured by a supply well because the distribution of head near the well is, in part, a function of the depth from which water is withdrawn and the ratio of horizontal to vertical hydraulic conductivity of aquifer sediments near the well, in addition to the rate at which the well is pumped and location of the well in the flow system. Several simulations (simulations 2–6, table 10) were completed to assess the effect of partial penetration of well screens, pumping rate of the well, and ratio of horizontal to vertical hydraulic conductivity of aquifer sediments to the delineation of contributing areas of the hypothetical wells. The results of the simulations are discussed simultaneously because of the interrelations among the three factors to the delineation of contributing areas.

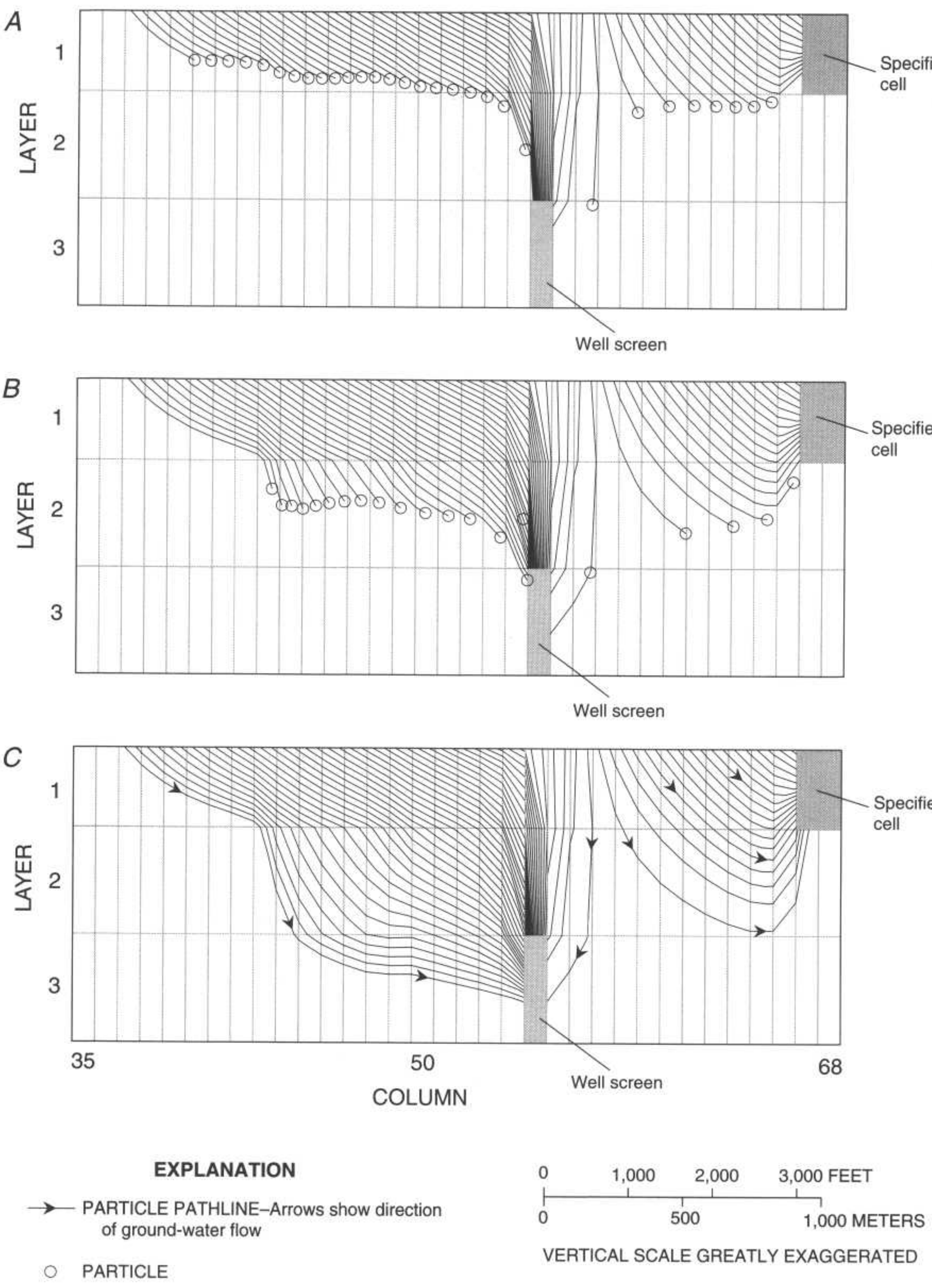


Figure 13. Particle pathlines in row 55 of the three-dimensional model of the simple flow system: (*A*) after 5 years of travel, (*B*) after 10 years of travel, and (*C*) for steady-state distribution of pathlines.