Particle-Tracking Analysis of Contributing Areas of Public-Supply Wells in Simple and Complex Flow Systems, Cape Cod, Massachusetts

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Prepared in cooperation with the Massachusetts Departments of Environmental Management and Environmental Protection, and the Cape Cod Commission

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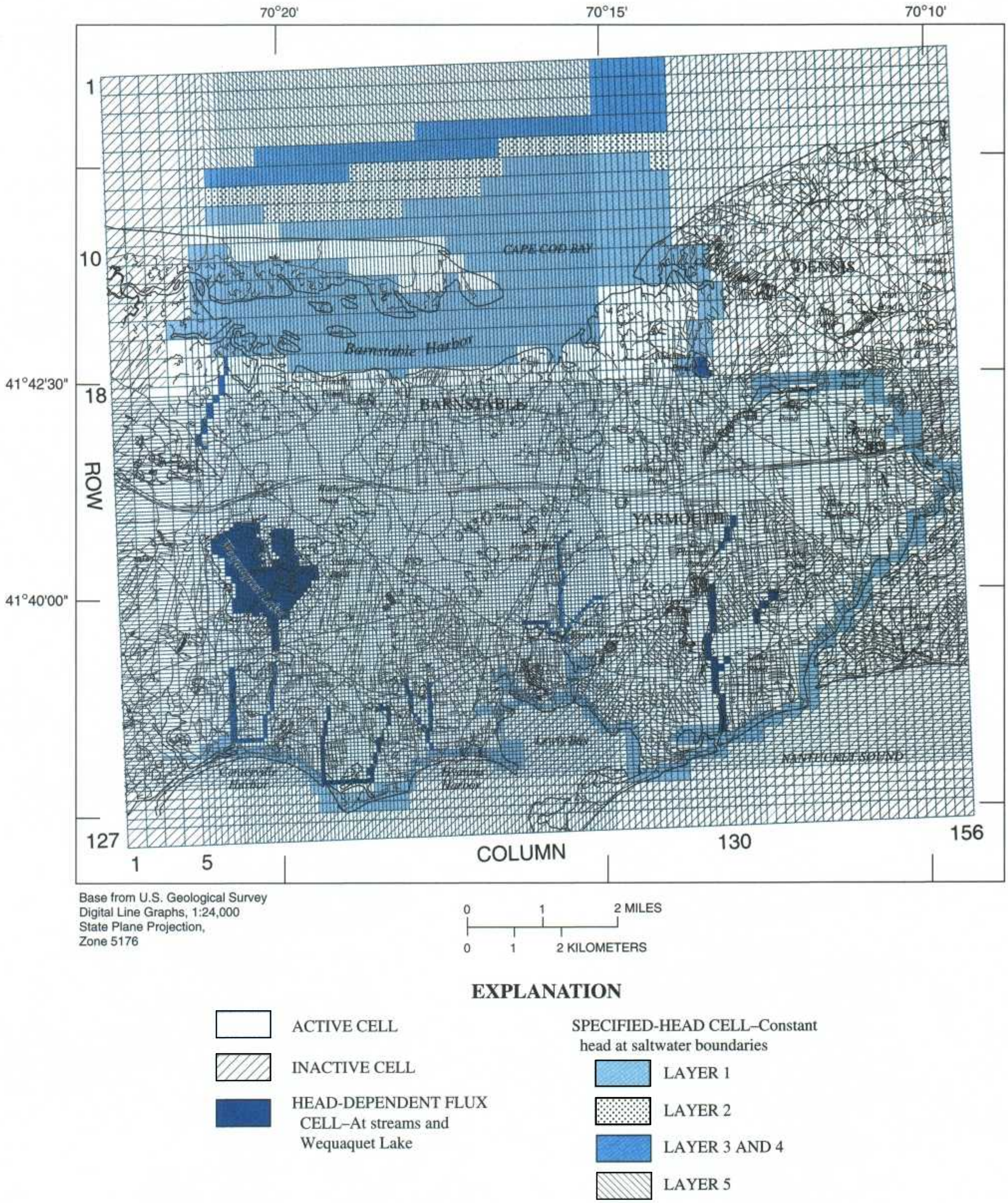


Figure 8. Model grid and lateral boundary conditions for the three-and two-dimensional models of the complex flow system. (Boundary conditions for layers 2 through 5 of the three-dimensional model are simulated in layer 1 of the two-dimensional model.)

Three-Dimensional Model

Hydrogeologic data, well-design characteristics, and the conceptual model of ground-water flow were used to develop the three-dimensional model of the complex flow system. The model was calibrated to heads measured in the flow system during average water-level conditions.

Grid

The three-dimensional model consists of eight layers (table 6) that were chosen on the basis of the lithology of the flow system and the location of the screened-interval of public-supply wells (many of which are screened at altitudes of 0–50 ft below sea level). The layers represent as closely as possible the sloping contact between the upper sand and gravel aquifer and middle confining unit of Barnstable. The horizontal grid consists of 127 rows and 156 columns (fig. 8). Grid cells range in size from 264 ft by 264 ft to 1,320 ft by 1,320 ft. The smallest grid size is near public-supply wells in Barnstable; the largest is in Yarmouth where head data is limited.

Boundary Conditions

Specified-head boundaries were used to simulate saltwater discharge areas at Cape Cod Bay, the Bass River, and Nantucket Sound (fig. 8). Beneath the Bass River and Nantucket Sound, specified-head boundaries were used only in the top layer of the model. Beneath Cape Cod Bay, however, specified-head boundaries were used in layers 1 through 5 to correspond to the location of offshore discharge (determined from bathymetric maps of the bay). Active cells underlie all specified-head boundaries and extend to the eighth layer of the model. Inactive cells were specified for cells that lie above specified-head boundaries in Cape Cod Bay. Equivalent freshwater heads were determined for specified-head boundaries according to equation 1.

Table 6. Vertical layering, horizontal hydraulic conductivity, and vertical conductance of the calibrated three- and two-dimensional flow models of the complex flow system

Model layer	Altitude of layer bottom, in feet below sea level	Horizontal hydraulic conductivity, in feet per day	Vertical conductance, in day^{-1}
Three-dimensional model:			
1	10	[1]50–250	0.0001–1.0
2	20	0.001–200	0.0001–1.0
3	30	0.001–200	0.0001–1.0
4	40	0.001–200	0.0001–1.0
5	60	0.001–200	0.0001–1.0
6	110	0.001–200	0.0001–1.0
7	200	10–30	0.00001–0.03
8	250–500	0.01–30	--
Two-dimensional model:			
1	200–500	10–100	--

[1]Grid cells underlying ponds were assigned a horizontal hydraulic conductivity of 50,000 feet per day.

A ground-water-flow line along the western edge of the model was used to separate the modeled area from the remainder of the West Cape flow cell. Inactive (no-flow) cells were specified to the west of the flow line in all eight layers. Water-table altitudes are sparse where the western flow line is drawn, and some error is likely in the position of the flow line and in the location of the western no-flow boundary. Inactive cells also were specified for all eight layers along the northeast section of the model where a natural ground-water divide separates the flow system from the adjoining flow cell.

The top of the model is bounded by active cells that receive ground-water recharge. Recharge from precipitation has been estimated to average 18.9 in/yr in Barnstable and Yarmouth (LeBlanc and others, 1986, pl. 2). This recharge rate was used as the base value to the model. Recharge was reduced to 15.7 in/yr for cells that underlie ponds to account for evaporation losses from pond surfaces. No recharge was specified for areas of the model coincident with marshes and swamps because they are assumed to be areas of ground-water discharge. Residential and commercial areas of Barnstable and Yarmouth receive water from the Barnstable Water Company, the Barnstable Fire District, the Centerville and Osterville Water Company, and the Yarmouth Water Company. Most of Barnstable and Yarmouth is unsewered, and water that is supplied to the residences, businesses, and industries in these towns is returned to the aquifer through septic systems. The amount of return flow reaching the aquifer in unsewered areas was estimated to be the 1987 average daily volume of water supplied to these areas (less that amount removed by sewers) divided by the total area receiving the supply. The unsewered area within each water district was assumed to be the urban areas shown on USGS 7.5-minute topographic maps. Estimates of return flow ranged from 3 to 6 in/yr and were added to the base value of precipitation recharge.

Water discharged to sewers is returned to the aquifer through infiltration beds at a wastewater-treatment facility in Barnstable. In 1987, average daily discharge from the facility was 1.3 Mgal; all wastewater was assumed to infiltrate to the water table. The yearly recharge of wastewater, 293 in., was determined by division of the 1987 discharge of wastewater by the area of the model cells underlying the infiltration beds.

Ground-water pumping was simulated at 12 public-supply wells (table 7) at rates equal to those measured on October 14, 1987.

The bottom of the model is a no-flow bedrock boundary. The altitudes of the base of the cells in the lowest layer of the model are equal to the altitude of the bedrock surface as determined from seismic-refraction data (Oldale, 1974a, 1974b) and from borehole logs available in the Massachusetts office of the USGS.

Several small streams in the modeled area receive ground-water discharge. The rate of discharge to the streams depends on the difference between heads in the stream and the underlying aquifer, the altitude of the streambed, and the conductance of streambed sediments. No determination of the characteristics of the streams was made during this investigation; however, visual inspections and conversations with water suppliers and town personnel indicate that local streams are generally less than 5 ft wide, less than 1 ft deep, have sandy streambeds, and flow intermittently throughout the year.

Streams were modeled as head-dependent flux boundaries that can only receive ground-water discharge. Streambed conductance was determined for each stream cell by the following equation:

$$C = \frac{KLW}{M}, \qquad (4)$$

where

C is streambed conductance, in square feet per day;
K is vertical hydraulic conductivity of the streambed, in feet per day;
L is total length of stream in the cell, in feet;
W is stream width, in feet; and
M is streambed thickness, in feet.

Streambed sediments were assumed to consist of the same material as the underlying aquifer. Because the aquifer consists of sand and gravel in the area of the streams, a ratio of vertical to horizontal hydraulic conductivity of 1:5 was used, which is consistent with the generalized values of the ratio of the vertical to horizontal hydraulic conductivity for medium sand to gravel (table 3). Streambed thickness and width were assumed to be 1 and 5 ft, respectively, and streambed altitudes were estimated from topographic maps.

Wequaquet Lake was modeled as a head-dependent flux boundary. The altitude of the lake was set at 33.7 ft above sea level, which is the altitude at which it is maintained by the town of Barnstable (Charles Millen, Barnstable Natural Resources, oral commun., 1988). The conductance between the bottom sediments of the lake and the underlying aquifer was determined for each lake cell to be the product of the area of the cell multiplied by a leakance term. The leakance term was calculated as the quotient of the vertical hydraulic conductivity of the sediments beneath the lake divided by an assumed distance over which head losses between the aquifer and overlying lake take place. Vertical hydraulic conductivity of lake sediments was assumed to be the same as that of the underlying aquifer, or about one-fifth of the horizontal hydraulic conductivity of the underlying aquifer. Head losses between the aquifer and overlying lake were assumed to take place over a distance of 15 ft, which is about one-half the distance from the mean bottom

Table 7. Pumping rates measured on October 14, 1987, for 12 public-supply wells in Barnstable and Yarmouth, Massachusetts

[All well screens were simulated in the single layer of the two-dimensional model; USGS, U.S. Geological Survey; B, Barnstable; Y, Yarmouth]

Model cell			Local well name	USGS well identifier	Pumping rate (cubic feet per day)
Layer	Row	Column			
1	61	77	BWC MD1	B 387	25,400
2	60	79	BWC MD2	B 383	65,500
1	55	77	BWC MD4	B 402	65,500
5	110	48	BWC HY	B 229	60,200
5	108	49	BWC SI	B 384	89,600
4	48	38	BFD 3	B 416	31,300
1	52	53	S and G	B 497	25,400
4	48	146	YWC 6–8	Y 53	25,300
2	62	129	YWC 11	Y 63	55,800
3	65	117	YWC 13	Y 58	89,400
5	35	142	YWC 15	Y 126	96,300
5	74	113	YWC 17	Y 195	58,800

altitude of the lake—23 ft above sea level (McCann, 1969)—to the altitude of the bottom of layer 1 (10 ft below sea level).

Hydraulic Properties

Estimates of horizontal hydraulic conductivity and vertical conductance were made by comparison of lithologic logs of more than 370 test holes in the modeled area to generalized values of hydraulic conductivity (table 3). Each log was divided into intervals that correspond to the eight layers of the model. A value of horizontal hydraulic conductivity and vertical conductance for each interval of the log was then determined from equations 2 and 3. Cells in the top layer of the model that underlie ponds were assigned a horizontal hydraulic conductivity of 50,000 ft/d. Small ponds, covering only a fraction of a grid cell, were not modeled.

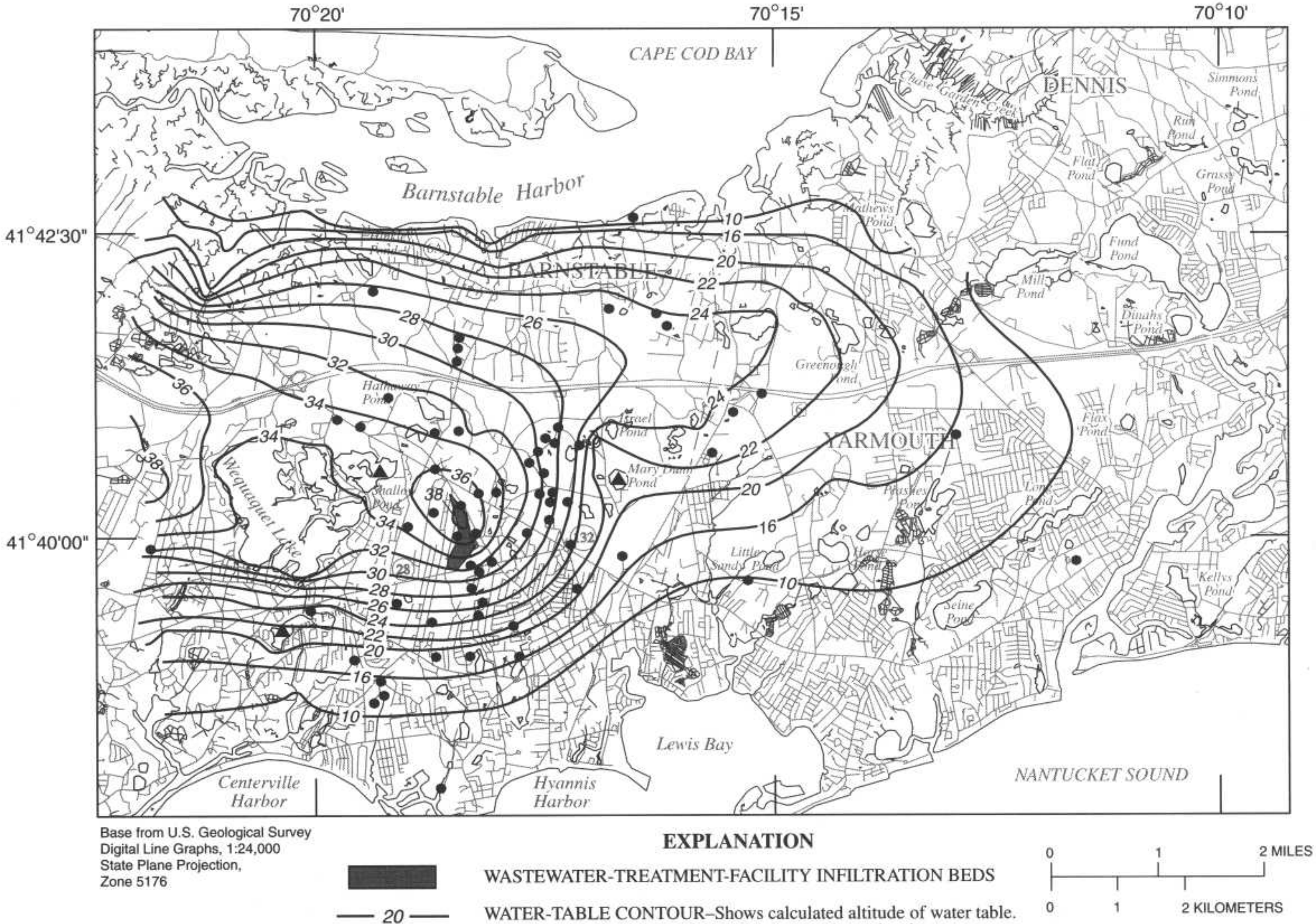
Sediments of the fine-grained unit that underlies stratified drift in eastern Barnstable were assigned a horizontal and vertical hydraulic conductivity of 0.01 ft/d. Lacustrine deposits near and beneath Cape Cod Bay are thought to be of the same origin as those of Eastham and were assigned horizontal and vertical hydraulic conductivities of 0.001 ft/d. Larger hydraulic conductivities were assigned to sediments of the fine-grained unit in eastern Barnstable than to those underlying Cape Cod Bay because they are less compact and are generally coarser than those of the simple flow system for which hydraulic conductivity has been estimated by means of permeameter tests.

Calibration and Sensitivity

The model was calibrated by comparison of calculated heads to heads measured at 63 observation wells and 3 ponds on October 14, 1987. Heads at this time were used for model calibration because they are considered representative of average (steady-state) conditions. Initial estimates of hydraulic conductivity for cells in layers 3 through 6 of the model and for cells that underlie moraine were reduced during model calibration; however, recharge was not adjusted. Generally, horizontal hydraulic conductivity and vertical conductance decrease as the depth of each model layer increases (table 6), which is consistent with the lithology of the flow system.

A map of calculated water-table altitudes for the top layer of the model is shown in figure 9. Generally, agreement between observed and calculated heads is close at each of the observation points (table 8). The mean error of the absolute value of observed heads minus calculated heads is 1.7 ft, which corresponds to approximately 5 percent of the total relief of the water table in the flow system. The errors are distributed around a mean value of 0.53 ft, indicating that calculated heads are generally lower than observed heads. This positive bias is most likely the result of high values specified for the hydraulic conductivity of the fine-grained unit consisting of silt and clay in eastern Barnstable. In a second simulation, the horizontal hydraulic conductivity and vertical conductance of these silt and clay sediments were reduced by an order of magnitude to the values used for fine-grained sediments beneath Cape Cod Bay; the resulting mean error of the absolute value of observed heads minus calculated heads was 1.8 ft. Although this mean error was not substantially different from that determined for the first simulation (1.7 ft), the error between observed heads and calculated heads for the second simulation was -0.43 ft, indicating that calculated heads were now generally higher than the observed heads. Results of these two simulations indicate that the true value of hydraulic conductivity of these silt and clay sediments lies somewhere between the two values simulated.

Total steady-state inflow to the model is 79.4 ft^3/s (table 9), 79 percent of which discharges to specified-head, coastal boundaries. Less than 2 percent of the inflow discharges through lacustrine deposits that underlie Cape Cod Bay to specified-head cells located deeper than layer 1. Fifty-two percent of the total inflow discharges to specified-head cells along Nantucket Sound, from the southwestern edge of the model to the Bass River. Nantucket Sound receives much of the discharge from the system because the most transmissive parts of the aquifer extend south from the moraine to Nantucket Sound. Nearly 10 percent of the total inflow reaches the bottom layer of the model, which extends from 200 ft below sea level downward to the top of bedrock.



Base from U.S. Geological Survey
Digital Line Graphs, 1:24,000
State Plane Projection,
Zone 5176

EXPLANATION

WASTEWATER-TREATMENT-FACILITY INFILTRATION BEDS

— 20 — WATER-TABLE CONTOUR–Shows calculated altitude of water table. Contour interval, in feet, is variable. Datum is sea level

● OBSERVATION WELL–Site where calculated and observed water levels were compared during calibration

▲ POND–Pond where calculated and observed water levels were compared during calibration

Figure 9. Calculated water-table configuration for the top layer of the three-dimensional model of the complex flow system.

Table 8. Observed heads and heads calculated by the three- and two-dimensional models of the complex flow system

[B, Barnstable; Y, Yarmouth; 3-D, three-dimensional; 2-D, two-dimensional]

Model node			Well No. or pond name	Observed water level, in feet above sea level	Calculated water level, in feet above sea level		Difference between observed and calculated head, in feet	
Layer	Row	Column			3-D model	2-D model	3-D model	2-D model
1	71	63	B 434	33.48	30.33	30.22	3.15	3.26
1	50	27	B 435	34.25	34.39	34.96	-.14	-.71
1	70	40	B 437	34.67	34.25	35.52	.42	-.85
1	88	44	B 439	26.76	24.66	23.96	2.10	2.80
1	80	44	B 440	31.88	30.15	29.97	1.73	1.91
1	86	21	B 441	26.73	22.68	25.33	4.05	1.40
1	96	31	B 442	16.96	17.47	17.63	-.51	-.67
1	71	53	B 444	37.38	36.51	36.92	.87	.46
1	60	47	B 445	36.29	36.17	36.86	.12	-.57
1	64	69	B 447	29.72	27.79	29.20	1.93	.52
1	74	71	B 448	26.59	24.69	24.25	1.90	2.34
1	83	72	B 449	20.65	20.32	17.82	.33	2.83
1	58	64	B 450	32.43	30.96	32.42	1.47	.01
1	64	54	B 451	36.20	36.09	36.29	.11	-.09
1	78	55	B 452	32.06	31.29	30.32	.77	1.74
1	87	53	B 455	24.91	25.09	23.33	-.18	1.58
1	90	60	B 456	21.23	21.10	18.70	.13	2.53
1	66	51	B 458	37.28	37.70	37.64	-.42	-.36
1	73	50	B 459	33.79	36.34	37.70	-2.55	-3.91
1	68	45	B 470	33.95	36.27	37.09	-2.32	-3.14
1	64	57	B 471	35.14	34.83	35.12	.31	.02
1	78	52	B 472	31.46	32.22	32.06	-.76	-.60
1	82	53	B 460	29.74	28.90	27.43	.84	2.31
1	95	51	B 461	17.69	18.37	17.31	-.68	.38
1	95	45	B 469	20.36	19.03	18.29	1.33	2.07
4	85	54	B 463	25.40	24.17	24.61	1.23	.79
1	77	57	B 464	31.81	31.16	30.05	.65	1.76
2	95	62	B 467	15.39	15.76	13.95	-.37	1.44
1	39	50	B 473	31.50	29.73	30.68	1.77	.82
1	54	66	B 474	31.82	29.65	31.75	2.17	.07
1	53	70	B 475	30.08	27.78	30.22	2.30	-.14
2	55	68	B 476	31.00	28.63	30.90	2.37	.10
2	56	66	B 477	31.93	29.62	31.68	2.31	.25
2	60	66	B 478	31.59	29.39	31.39	2.20	.20
1	51	32	B 479	34.74	34.37	35.12	.37	-.38
1	46	38	B 480	32.80	32.91	33.47	-.11	-.67
1	66	68	B 481	30.45	28.35	29.26	2.10	1.19
1	70	68	B 482	28.46	27.53	27.83	.93	.63
1	64	66	B 483	31.19	30.04	30.81	1.15	.38
1	66	71	B 484	28.96	25.93	27.63	3.03	1.33
1	33	91	B 485	22.00	24.57	26.04	-2.57	-4.04
1	30	87	B 486	18.77	23.99	24.14	-5.22	-5.37
1	57	98	Y 205	26.41	22.87	27.15	3.54	-.74
1	57	73	B 487	28.59	25.54	27.84	3.05	.75
1	47	108	Y 206	24.71	23.07	29.76	1.64	-5.05

Table 8. Observed heads and heads calculated by the three- and two-dimensional models of the complex flow system—*Continued*

Model node			Well No. or pond name	Observed water level, in feet above sea level	Calculated water level, in feet above sea level		Difference between observed and calculated head, in feet	
Layer	Row	Column			3-D model	2-D model	3-D model	2-D model
2	51	101	Y 85	21.31	23.13	29.11	-1.82	-7.80
2	34	50	B 295	28.38	28.25	27.65	.13	.73
1	37	50	B 290	30.18	29.22	29.59	0.96	0.59
2	54	42	B 293	35.28	33.19	35.82	2.09	-.54
1	74	80	B 230	19.27	18.84	20.55	.43	-1.28
6	118	46	B 322	.43	2.69	2.66	-2.26	-2.23
1	29	78	B 247	21.28	24.72	23.50	-3.44	-2.22
1	52	46	B 292	35.33	33.73	35.27	1.60	.06
2	26	36	B 294	19.18	26.78	21.99	-7.60	-2.81
1	84	38	B 306	29.07	26.98	26.74	2.09	2.33
6	16	83	B 318	9.94	14.60	.13	-4.66	9.81
1	81	102	Y 123	8.52	9.84	8.85	-1.32	-.33
4	100	33	B 226	14.23	14.59	14.14	-.36	.09
2	103	33	B 227	11.93	12.03	11.24	-.10	.69
2	102	33	B 368	12.40	12.92	12.12	-.52	.28
1	54	130	Y 89	18.35	15.94	22.45	2.41	-4.10
1	78	140	Y 96	5.46	5.77	8.01	-.31	-2.55
2	72	3	B 254	35.56	33.01	34.27	2.55	1.29
1	60	33	Shallow Pond	34.63	34.77	35.35	-.14	-.72
1	88	21	Long Pond	25.80	22.06	23.66	3.74	2.14
1	62	80	Mary Dunn Pond	23.96	20.06	23.72	3.90	.24

Table 9. Calculated water budgets for the three- and two-dimensional flow models of the complex flow system

[All values are cubic foot per second]

	Three-dimensional model	Two-dimensional model
Inflow:		
Recharge	77.2	77.2
Leakage from Wequaquet Lake	2.2	2.5
Total inflow	79.4	79.7
Outflow:		
Discharge to coastal boundaries	62.7	60.2
Leakage to Wequaquet Lake	2.0	3.9
Wells	8.0	8.0
Streams	6.9	7.7
Total outflow	79.6	79.8
Inflow minus outflow (numerical error)	-0.2	-0.1

A sensitivity analysis was completed for the three-dimensional model to determine the response of calculated heads and flow rates to changes in the values of model parameters. As noted above, the horizontal and vertical hydraulic conductivities assigned for silt and clay in the central part of the study area significantly affect calculated heads. Model results also were sensitive to changes in the recharge rate and hydraulic conductivity of the top layer of the model (fig.10*A*).

Sensitivity of calculated heads to the value of the conductance term for the head-dependent flux boundaries used to represent Wequaquet Lake and streams also was tested. Each of the conductance terms was increased and decreased by an order of magnitude in four separate simulations. Computed heads were found to be less sensitive to changes in the conductance of the lake sediments than to changes in the conductance of streambed sediments. Mean errors for these four simulations ranged from 1.7 to 1.9 ft. The discharge of ground water to streams was affected by changes in the values of the conductance term for

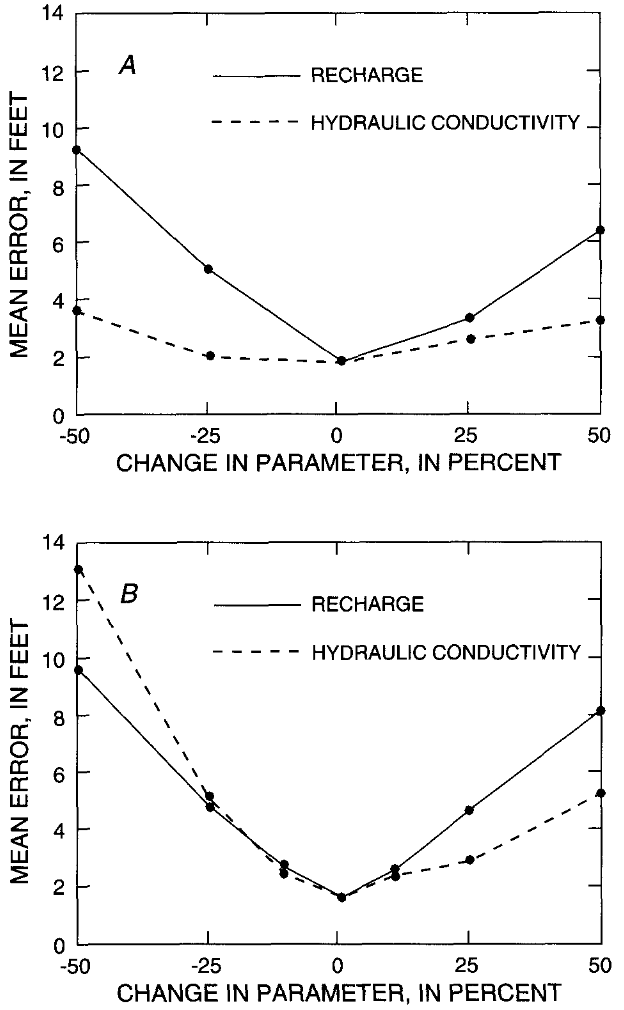


Figure 10. Sensitivity of (*A*) the three-dimensional model and (*B*) the two-dimensional model of the complex flow system to changes in recharge and horizontal conductivity of layer 1 of each model.

stream cells. Ground-water discharge to streams increased from 6.9 to 9.9 ft^3/s when stream conductances were increased by an order of magnitude; ground-water discharge decreased to 1.8 ft^3/s when stream conductances were decreased by an order of magnitude. The discharge of water to Weqauquet Lake was unaffected by changes in the value of the conductance term used to represent the lake sediments. Although the conductance terms used to represent the lake sediments are not well defined, these sensitivity tests indicate that the parameter does not significantly affect calculated heads in the aquifer or the rate of ground-water discharge to the lake.

Calculated heads were also found to be fairly insensitive to the inclusion of the first seven rows of the model, which are in Cape Cod Bay and simulate offshore discharge through lacustrine deposits in layers 2 through 5. The exclusion of these cells resulted in a change of 0.1 ft in the mean error. A simulation also was made on the basis of the 1987 average daily pumping rate of all supply wells in the study area. Model heads determined for that simulation did not differ significantly from those determined on the basis of the pumping rates of October 14, 1987. Calculated heads along the western boundary of the model, where a ground-water-flow line was used to separate the modeled area from the remainder of the flow system, were also found to be insensitive to the pumping rates simulated at each well; therefore, the uncertainty associated with the boundary location is assumed to be insignificant to the delineation of contributing areas for simulated wells.

Two-Dimensional Model

The two-dimensional model of the complex flow system consists of a single layer that extends from the water table to bedrock. The horizontal grid spacing is the same as that of the three-dimensional model (fig. 8). Lateral boundary conditions specified for the two-dimensional model are the same as those specified for the top layer of the three-dimensional model (fig. 8), but inactive cells were specified for the first seven rows of the model because the three-dimensional model was fairly insensitive to their inclusion. In addition, a no-flow boundary was set at the contact between glacial deposits and underlying bedrock. The rate of areal recharge specified for each cell and the pumping rates of simulated wells (table 7) was the same as those specified for the three-dimensional model. An initial estimate of horizontal hydraulic conductivity for each cell of the two-dimensional model was determined by division of the total transmissivity by the saturated thickness of the eight layers of the calibrated three-dimensional model.

Calculated heads were compared to observed heads measured at 66 observation wells and ponds in the study area on October 14, 1987. The mean residual of the absolute value of the difference between observed heads and calculated heads is 1.6 ft, and the mean error is 0.03 ft, indicating no significant systematic bias in the estimates of model parameters. Agreement between calculated heads and observed heads at each of the observation points is generally close (table 8). Water-table contours based on calculated heads are shown in figure 11.