Particle-Tracking Analysis of Contributing Areas of Public-Supply Wells in Simple and Complex Flow Systems, Cape Cod, Massachusetts

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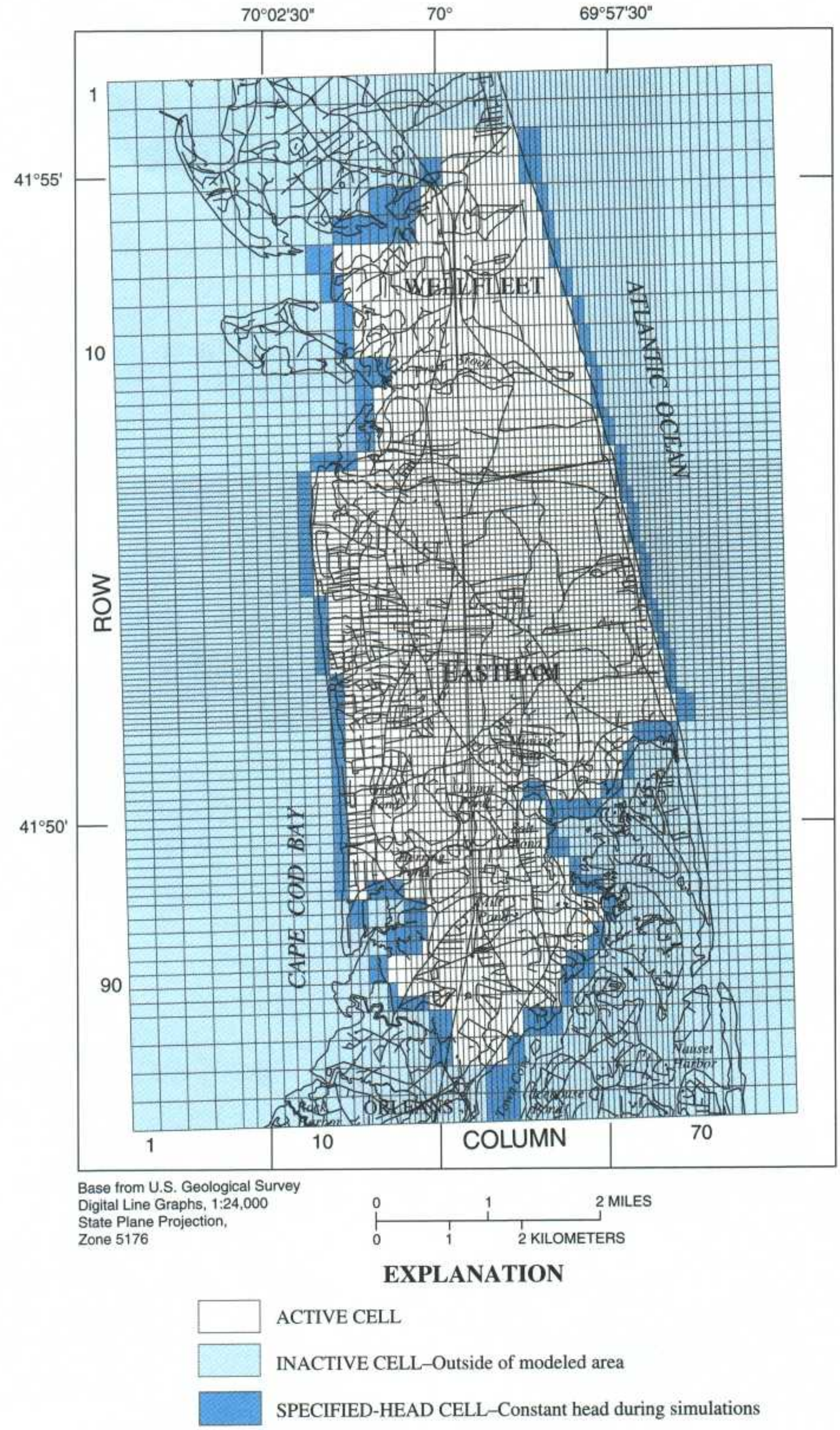


Figure 4. Model grid and lateral boundary conditions for the two-dimensional flow model and the top layer of the three-dimensional flow model of the simple flow system.

4. Ground-water flow is virtually horizontal in the aquifer except near divides and discharge boundaries where vertical flow dominates. Vertical leakage occurs between the aquifer and underlying confining unit; however, little water flows through the confining unit because of its low permeability.

Three-Dimensional Model

Hydrogeologic data, well-design characteristics, and the conceptual model of ground-water flow were used to develop the three-dimensional model of the simple flow system. The model was calibrated to heads measured in the flow system during average water-level conditions.

Grid

The three-dimensional flow model consists of five layers (table 4) that were chosen on the basis of available information on the lithology of the flow system and were designed for adequate representation of the contact between the upper coarse-grained and lower fine-grained units. More than one model layer was used to simulate the coarse-grained unit in order to vary the location of the well screens of the hypothetical wells. The model grid consists of 96 rows and 78 columns and was aligned to conform as closely as possible to natural boundaries of the aquifer (fig. 4). Grid cells are smallest (264 ft by 264 ft) in the area of proposed well sites, to simulate as accurately as possible the movement of water particles near those wells.

Table 4. Vertical layering, horizontal hydraulic conductivity, and vertical conductance of the calibrated three- and two-dimensional flow models of the simple flow system

[The bottom altitude and horizontal hydraulic conductivity of layer 5 of the three-dimensional model were not specified because transmissivity was used for the layer; --, vertical conductance was not specified for either layer 5 of the three-dimensional model or for the single layer of the two-dimensional model]

Model layer	Altitude of layer bottom, in feet below sea level	Horizontal hydraulic conductivity, in feet per day	Vertical conductance, in day^{-1}
Three-dimensional model:			
1	10	[1]100–150	0.005–1.0
2	35	50–100	0.0001–1.0
3	60	0.001–100	0.00001
4	90	0.001–100	0.00001
Two-dimensional model:			
1	90	25–75	--

[1] Grid cells underlying ponds were assigned a horizontal hydraulic conductivity of 50,000 feet per day.

Boundary Conditions

Specified-head boundaries were used in the top layer of the model to simulate saltwater discharge areas that surround much of the flow system (fig. 4). Active model cells underlie the specified-head boundaries in layers 2 through 5. Equivalent freshwater heads were computed and used at the specified-head boundaries to account for the higher density saltwater that overlies freshwater at the seabed discharge boundaries. Because freshwater and saltwater heads are equal at the discharge boundary, an equivalent freshwater head can be determined by equation 1:

$$Z(f) = \frac{\rho(s) - \rho(f)}{\rho(f)} Z(s) , \quad (1)$$

where

$Z(f)$ is equivalent freshwater head, in feet;

$Z(s)$ is distance from sea level to seabed, in feet (from bathymetric maps of the area);

$\rho(s)$ is density of saltwater, assumed to be 1.025 grams per cubic centimeter (gm/cm^3); and

$\rho(f)$ is density of freshwater, assumed to be 1.000 gm/cm^3.

A stream-surface boundary (or no-flow boundary) was specified along parts of the northern and southern boundaries of the flow system to coincide with the natural ground-water-flow divides that separate the flow system from adjacent flow systems. A stream-surface boundary also was used to simulate the contact between glacial sediments and underlying bedrock.

A recharge rate of 17.4 in/yr was specified to the top layer of the model on the basis of estimates made by LeBlanc and others (1986, pl. 2). Pond cells were assigned a recharge rate of only 12.4 in/yr because of evaporation losses from pond surfaces. Return flow from domestic and commercial septic systems was assumed to be equal to the amount of water withdrawn by wells at each site; therefore, net ground-water withdrawal for public supply was assumed to be zero.

Hydraulic Properties

Horizontal hydraulic conductivity and vertical conductance were determined for the top four layers of the model at 31 sites by comparison of lithologic logs of test holes at these sites to generalized hydraulic conductivities for glacial sediments (table 3). Each lithologic log was divided into layers corresponding to those of the three-dimensional model. An equivalent horizontal hydraulic conductivity, K_h, was then computed for each layer of the log, according to the method described by Freeze and Cherry (1979, p. 34):

$$K_h = \frac{\sum_{i=1}^{n} K_i b_i}{\sum_{i=1}^{n} b_i}, \qquad (2)$$

where

K_h is equivalent horizontal hydraulic conductivity for the layer, in feet per day;

K_i is horizontal hydraulic conductivity of the ith hydrogeologic unit of the layer, in feet per day (estimated from table 3);

b_i is thickness of the ith hydrogeologic unit in the layer, in feet; and

n is the number of hydrogeologic units within the layer; the top of the uppermost layer coincides with the position of the water table.

Vertical conductance is specified between vertically adjacent nodes of the McDonald and Harbaugh (1988, p. 5–11) model. Vertical conductance was determined for each log from the relation:

$$Vcont_{(k+1/2)} = \frac{1}{\sum_{i=1}^{n} \frac{b_i}{K_{v_i}}} \qquad (3)$$

where

$Vcont_{(k+1/2)}$ is vertical conductance between layers k and $k+1$, in day^{-1};

K_{v_i} is vertical hydraulic conductivity of the ith hydrogeologic unit between layers k and $k+1$, in feet per day;

b_i is thickness of the ith hydrogeologic unit between layers k and $k+1$, in feet; and

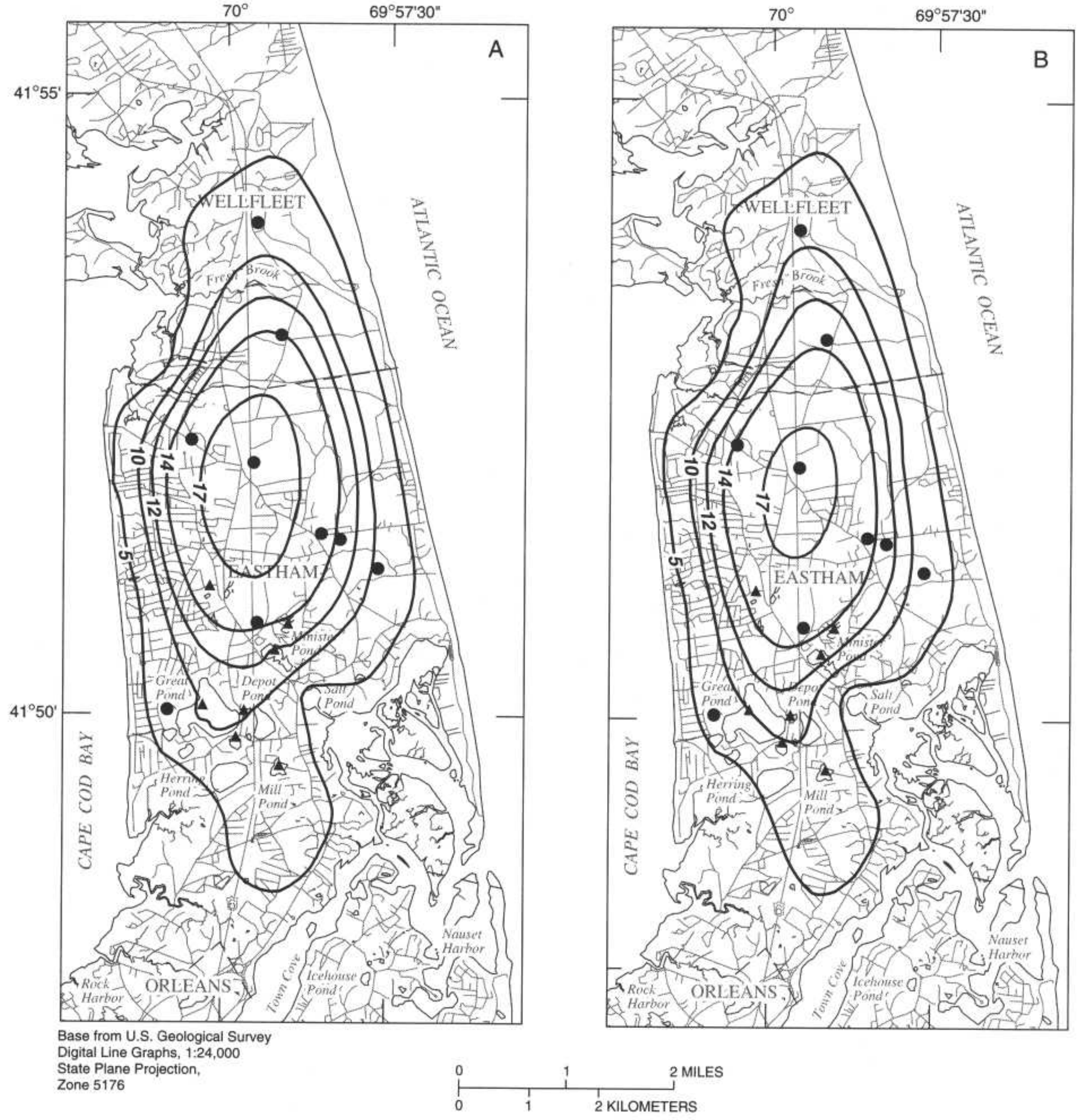
n is the number of hydrogeologic units between layer k and $k+1$.

Maps of the spatial variation of horizontal hydraulic conductivity and vertical conductance were made for layers 1 through 4 by use of the values of horizontal hydraulic conductivity and vertical conductance computed for each layer of the 31 logs. These maps served as initial estimates of horizontal hydraulic conductivity and vertical conductance of the three-dimensional model. Transmissivity of layer 5 (0.378 ft^2/d) was determined by multiplying the generalized horizontal hydraulic conductivity of silt and clay determined through laboratory permeameter tests (0.001 ft/d, table 3) by the total thickness of the deposit at test hole Eastham 45 (378 ft).

Calibration and Sensitivity

The model was calibrated by comparison of calculated heads to heads measured at 19 locations in the aquifer in May 1988 and 3 additional locations reported by LeBlanc and others (1986, pl. 4). Heads at this time were used for model calibration because they are considered representative of average (steady-state) conditions. Initial estimates of recharge, horizontal hydraulic conductivity, and vertical conductance were adjusted during model calibration. Horizontal hydraulic conductivity and vertical conductance were decreased by a maximum of 30 percent during model calibration, and recharge was increased over the initial estimate by 5 percent. Differences between initial estimates of horizontal hydraulic conductivity and calibrated model values were greatest for coarse and very coarse sand deposits. The decrease in both horizontal hydraulic conductivity and vertical conductance with increasing depth of each model layer (table 4) is consistent with the lithology of the flow system.

A map of calculated water-table altitudes for the top layer of the model is shown in figure 5*A*. Generally, agreement between observed and calculated heads is close at each of the observation points (table 5). The mean error of the absolute value of observed minus calculated heads is 0.8 ft, which corresponds to approximately 4 percent of the total relief of the water table in the simple flow system.



EXPLANATION

—5— WATER-TABLE CONTOUR–Shows calculated altitude of water table. Contour interval, in feet, is variable. Datum is sea level.

● OBSERVATION WELL–Site where calculated and observed water levels were compared during calibration

▲ POND–Pond where calculated and observed water levels were compared during calibration

Figure 5. Calculated water-table configurations for (*A*) the top layer of the three-dimensional model and (*B*) the two-dimensional model of the simple flow system.

Table 5. Observed heads and heads calculated by the three- and two-dimensional models of the simple flow system

[No., number; W, Wellfleet; E, Eastham; 3-D, three dimensional; 2-D, two dimensional; --, value not calculated by two-dimensional model]

Model node			Well No. or pond name	Observed water level, in feet above sea level	Calculated water level, in feet above sea level		Difference between observed and calculated head, in feet	
Layer	Row	Column			3-D model	2-D model	3-D model	2-D model
2	9	32	W17	9.1	9.2	9.9	-0.1	-0.8
1	14	37	W112	13.8	13.5	13.1	.3	.7
2	14	37	W113	13.7	13.4	--	.3	--
3	14	37	W114	13.7	13.4	--	.3	--
1	29	20	E49	16.9	15.6	14.1	1.3	2.8
2	29	20	E50	16.9	15.3	--	1.6	--
4	29	20	E51	17.2	15.3	--	1.9	--
1	35	33	E46	17.6	18.3	17.7	-.7	-.1
2	35	33	E47	17.6	18.3	--	-.7	--
3	35	33	E48	17.5	18.3	--	-.8	--
1	49	48	E32	12.8	13.3	12.7	-.5	.1
1	48	44	E36	14.3	15.2	14.6	-.9	-.3
1	75	17	E37	8.5	8.3	7.3	.2	1.2
1	65	33	E39	13.9	14.2	14.4	-.3	-.5
2	55	55	E40	8.6	9.1	8.4	-.5	.2
1	81	37	Mill Pond	10.5	8.1	8.6	2.4	1.9
1	65	38	Molls Pond	13.6	13.5	13.4	.1	.2
1	75	31	Priscilla Pond	9.3	10.1	10.7	-.8	-1.4
1	78	29	Jemima Pond	9.7	9.7	9.9	.0	-.2
1	59	23	Briggs Pond	15.9	15.3	14.8	.6	1.1
1	64	36	Minister Pond	12.7	11.9	12.7	.8	.0
1	75	23	Great Pond	8.6	10.0	10.1	-1.4	-1.5

The errors are distributed around a mean value of 0.14 ft, indicating a small positive bias in calculated heads but no significant systematic error in calibrated model parameters. Total steady-state recharge to the calibrated model is 22.1 ft^3/s, or 14.3 Mgal/d, nearly 50 percent of which is discharged from layer 1 to specified-head boundaries at the coast without flowing to layers 2 through 5. Layer 5 receives much less than 1 percent of available recharge to the model.

A sensitivity analysis was done to determine the response of calculated heads to changes in model parameters. The analysis was done because model parameters (horizontal hydraulic conductivity, transmissivity, vertical conductance, and recharge) are imprecise estimates of the true values. The uncertainty associated with each parameter can have a significant effect on calculated heads and flows, and, therefore, on the contributing areas delineated for supply wells. The sensitivity analysis can identify those parameters to which model results are most sensitive and can guide future data collection toward improved definition of those parameters.

During the sensitivity analysis, each model parameter was uniformly increased or decreased and the model was then rerun. The mean error of the absolute value of the difference between observed and calculated heads was then determined. Calculated heads were found to be most sensitive to recharge and to the horizontal hydraulic conductivity of the top layer of the model (fig. 6*A*). Heads were less sensitive to horizontal hydraulic conductivity of layers 2 through 4; heads were nearly insensitive to increases or decreases in the transmissivity of layer 5 (fig. 6*B*). In general, heads were less sensitive to variations in vertical conductance than to variations in horizontal

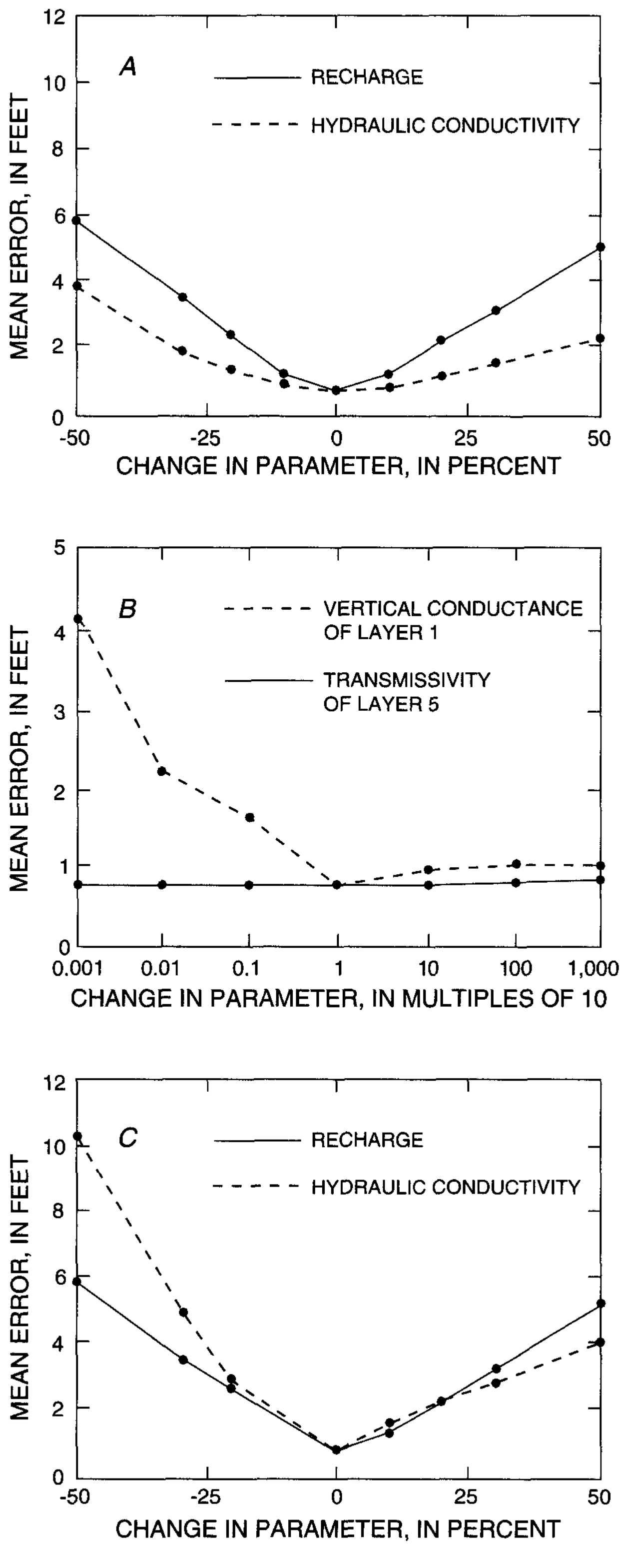


Figure 6. Sensitivity of the three-dimensional model of the simple flow system to changes in (*A*) recharge and horizontal hydraulic conductivity of the top layer of the model (*B*) vertical conductance of layer 1 and transmissivity of layer 5, and (*C*) sensitivity of the two-dimensional model of the simple flow system to changes in recharge and horizontal hydraulic conductivity.

hydraulic conductivity. The model is nearly insensitive to increases in the vertical conductance of the top layer (fig. 6*B*) because the initial estimates of vertical hydraulic conductivity of the sand and gravel stratified drift of the top layer were already very large; increases in the calibrated value of vertical conductance of the top layer had very little effect on heads or the flow of water between layers 1 and 2.

Although heads are sensitive to changes in the individual values of recharge and horizontal hydraulic conductivity of the top layer, heads were less sensitive to changes in these parameters when they were varied simultaneously (fig. 7). Figure 7 shows that an infinite number of combinations of recharge and horizontal hydraulic conductivity of layer 1 are possible that result in a mean error between observed and calculated heads of 0.8 ft. Because neither horizontal hydraulic conductivity nor recharge are known precisely, it is not possible to determine a unique combination of these parameters through model calibration. The results have important implications to the delineation of contributing areas for public-supply wells because uncertainty in the true value of each parameter results in a range of possible contributing areas for each well.

Two-Dimensional Model

The two-dimensional model of the simple flow system consists of a single layer that corresponds generally with coarse-grained sand and gravel stratified drift. The layer extends from the water table to 90 ft below sea level (table 4). The altitude of the base of the layer was chosen because the three-dimensional model indicated that little water flows through the underlying confining unit; therefore, a no-flow boundary was set at 90 ft below sea level. The horizontal grid spacing and lateral boundary conditions of the two-dimensional model are the same as those specified for the top layer of the three-dimensional model (fig. 4). The rates of recharge specified for the model were the same as those specified for the calibrated three-dimensional model; or 18.3 in/yr for the stratified drift and 12.4 in/yr for cells that underlie ponds. An initial estimate of hydraulic conductivity for each cell of the two-dimensional model was determined by dividing the total transmissivity by the saturated thickness of the top four layers of the calibrated three-dimensional model.

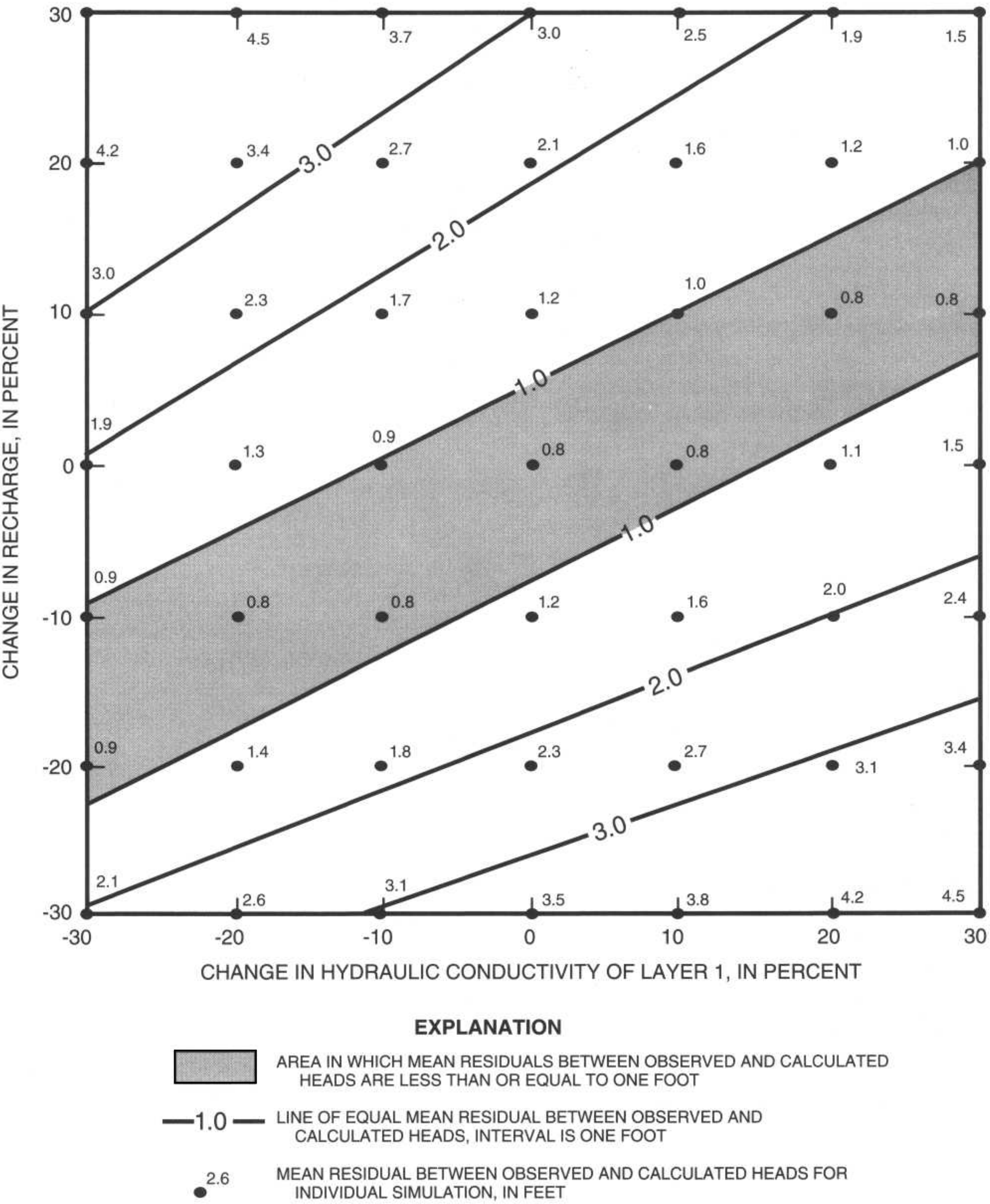


Figure 7. Mean residuals between observed and calculated heads resulting from simultaneous changes to recharge and hydraulic conductivity of layer 1 of the three-dimensional flow model for the simple flow system.

Calculated heads were compared to heads measured at 16 sites in the aquifer. Initial estimates of hydraulic conductivity were reduced over much of the model during calibration. The mean error between the absolute value of the difference between observed and calculated heads for the calibrated model was 0.8 ft, and the mean difference between observed and calculated heads was 0.21 ft, indicating no significant systematic bias in the specification of model parameters. Heads calculated by the two-dimensional model compare favorably with observed heads at each of the observation points (table 5) and are similar to those determined for the top layer of the three-dimensional model (fig. 5*B*). Total recharge to the aquifer is 22.1 ft^3/s, the same as that determined for the three-dimensional model.

A sensitivity analysis completed for the model indicates that calculated heads are sensitive to both recharge and hydraulic conductivity (fig. 6*C*) and that the model is especially sensitive to decreases in hydraulic conductivity. A comparison between the sensitivity of the three-dimensional model to variations in horizontal hydraulic conductivity for the top layer of the model (fig. 6*A*) and the sensitivity of the two-dimensional model to variations in horizontal hydraulic conductivity for the single-layer model (fig. 6*C*) indicates that the single-layer, two-dimensional model is much more sensitive to changes in horizontal hydraulic conductivity.

Complex Flow System

Flow models for the complex flow system extend from approximately 1 mi west of Wequaquet Lake in Barnstable to the Bass River area in Yarmouth and from Nantucket Sound north to Cape Cod Bay (fig. 8). The modeled area includes several existing public-supply wells for which contributing areas are delineated in later sections of this report.

Conceptual Model of Ground-Water Flow

The following statements and assumptions describe the conceptual model of the complex flow system.

1. The flow system is bounded in the north by dune, marsh, swamp, lake, and moraine deposits, which are not favorable areas for ground-water development. South of the moraine, the flow system consists of three units in Barnstable and two units in Yarmouth. In Barnstable, these units are an upper sand and gravel unconfined aquifer, a middle, discontinuous fine-grained confining unit of silt and clay, and a lower semiconfined sand aquifer. In Yarmouth, these units are an upper, unconfined aquifer grading downward from sand and gravel to fine sand and silt and an underlying confining unit of silt and clay. Discontinuous lenses of very fine to fine sand, silt, and clay can be found in the aquifers of both towns.
2. The flow system is bounded laterally by saltwater discharge areas at Cape Cod Bay, the Bass River, and Nantucket Sound (fig. 3). The system is bounded to the west by a ground-water-flow line drawn perpendicular to the water-table contours constructed from measurements made on October 14, 1987 (fig. 3), and to the northeast by a ground-water divide between the complex and adjoining flow systems. It is assumed that the flow line and ground-water divide are unaffected by ground-water withdrawals from the system. The flow system is underlain by impermeable, crystalline bedrock.
3. The distribution of ground-water recharge to the water table is nonuniform and consists of precipitation and wastewater return flow. Ground-water withdrawals for public supply are redistributed within and returned to the flow system through septic systems and a wastewater-treatment facility.
4. Ground-water flow is virtually horizontal in most parts of the aquifers and is vertical in the confining unit of Barnstable. Vertical flow components in the aquifers, however, occur near ground-water divides and discharge boundaries (coastal areas, wells, streams, and ponds) and in the vicinity of discontinuous lenses of very fine to fine sand, silt, and clay.