Techniques of Water-Resources Investigations of the United States Geological Survey

Chapter A6

QUALITY ASSURANCE PRACTICES FOR THE CHEMICAL AND BIOLOGICAL ANALYSES OF WATER AND FLUVIAL SEDIMENTS

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Book 5

Laboratory Analysis

Materials Evaluation

In order to have an effective analytical data quality assurance program, it is necessary that materials used in sample collection be of adequate and uniform or known quality. Procedures which can be used in designing a plan to be used in testing materials are described here.

Selection of Sample

1. Application or scope

1.1 This practice describes random sampling from systematically packed material and from bulk packed material. Random sampling techniques must be used to select the sample to be tested.

2. Practice

2.1 Systematically-packed material

2.1.1 Assign a sequence number to each carton, to each tray within a carton and to each row and column within a tray. For example, a sample in the first row and third column of the fifth box in the second carton opened (or received) could be assigned the number 2–5–1–3.

2.1.2 Since most vendors pack a specific material the same way each time it is sent, the numbers can be assigned once and records retained for subsequent shippings. Thus if it is known that there are always five rows and five columns in every box and that there are always six boxes in one carton, the item assigned 2–5–1–3 will always be in the same spot of the second carton opened (or second carton received in shipment).

2.2 Bulk-packed material

2.2.1 Assign each carton a separate number.

2.2.2 Arrange items within a carton in 10 groups (for example, if carton contains 1,000 items, arrange 100 in each group). Assign each item a number or arrange in columns and rows and assign the rows and columns a number.

2.2.3 Since vendors usually pack the same number of items in each carton, the numbers can be assigned once and the records retained for subsequent shippings.

2.3 Random sampling

2.3.1 Use a table of random numbers (available in most statistics books), or a calculator or computer to generate random numbers.

2.3.2 If using a table, arbitrarily put a finger on the table and record subsequent numbers.

2.3.3 Select the items indicated by the table and use for quality assurance testing. Thus, the number 2,513 would designate the 2nd carton, 5th box, 1st row, and 3rd column.

Selected References

Dixon, W. J. and Massey, F. J., Jr., 1969, Introduction to statistical analyses (3d ed.): New York, McGraw-Hill, p. 39–42.

Wiesen, J. M., 1974, Sampling by attributes, *in* Juran, J. M., and others, eds., Quality Control Handbook (3d ed.): New York, McGraw-Hill, p. 24–1—24–44.

Single Sampling With Operating Characteristic Curves

1. Application or scope

1.1 This practice describes the calculations needed to prepare operating characteristic (OC) curves for "single sampling" plans and gives some examples. It can be used in setting up a plan to test the quality of materials.

1.2 Although the number of items which must be tested will usually be greater using a single sampling plan than with other sampling plans (such as double or sequential plans), record keeping will usually be simpler and less time-consuming. Single sampling plans are particularly useful when test analyses are time-consuming and results are desired immediately.

1.3 An OC curve gives the best description of the sampling plan (Miller and Freund, 1977). It will define the risks associated with accepting a "bad" lot or rejecting a "good" lot. Most published sampling plans, such as the Dodge-Romig plans (Dodge and Romig, 1959) and those in Mil-Std-105D (U.S. Department of Defense, 1963) show the applicable OC curve.

2. Practice

2.1 Calculation of OC curve.

2.1.1 An OC curve can be calculated using a hypergeometric distribution. The hypergeometric distribution can be (and, in acceptance sampling, usually is) approximated by the binomial distribution if the sample size n, is small compared to the lot size N, that is $n<\frac{1}{10}N$ (Miller and Freund, 1977).

2.1.2 Calculate the points for the OC curve:

$$A(d;n,p)=\sum_{k=0}^{d}\binom{n}{k}p^k(1-p)^{n-k}$$

$$=\sum_{k=0}^{d}\frac{n!}{k!(n-k)!}p^k(1-p)^{n-k} \qquad (45)$$

where

$A(d;\ n,p)$ = probability of accepting the lot,
k = number of defectives in the sample,
d = maximum number of defectives in sample,
n = number of items in sample, and
p = proportion of lot which is defective.

2.1.3 The values are tabulated for $n=20$ in Miller and Freund (1977).

2.1.4 Plot the probability of accepting the lot on the vertical axis and the proportion of lot which is defective on the horizontal axis to obtain the OC curve (see figs. 28–31).

2.2 Sampling risks

2.2.1 The risk of rejecting a "good" lot must be decided and the percent of "bad" items which will be allowed in satisfactory lots must be determined. This risk is also called the "producer's risk" and the percent is the "acceptable quality level" (AQL) (NOTE 1).

NOTE 1. In most cases, the AQL to be used initially will have to be set arbitrarily. However, after preliminary data are obtained it may be found that, due to costs involved or other factors, it is necessary to change the AQL.

2.2.2 The risk of accepting a bad lot must be decided and the percent of "bad" items which makes the lot bad must be determined. This risk is also called the "consumer's risk" and the percent is the "lot tolerance percent defective" (LTPD) (NOTE 2).

NOTE 2. The consumer's risk does not give the probability that the consumer will actually receive a product of the specified LTPD. Obviously, if there are zero defects in the lot, there will be no defects.

2.3 Figures 28–31 show examples of OC curves for several sample sizes.

2.3.1 Figure 28 shows OC curves for a sample size of 10 and requires a minimum lot size of 100. If no defective items are allowed in the sample, there is a 10 percent chance of

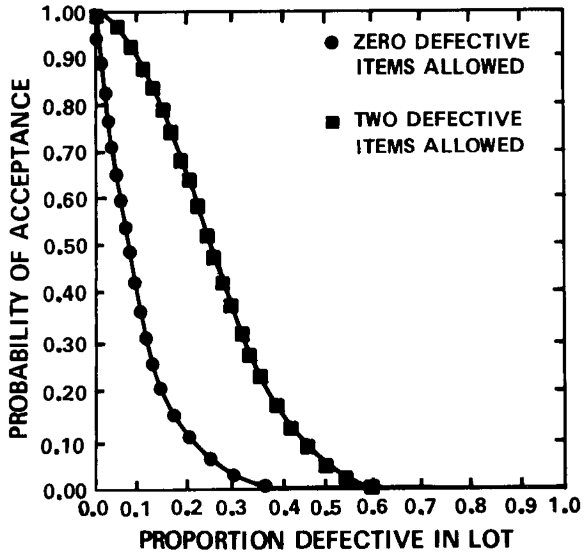

Figure 28.—Operating characteristic curves for sample size of 10.

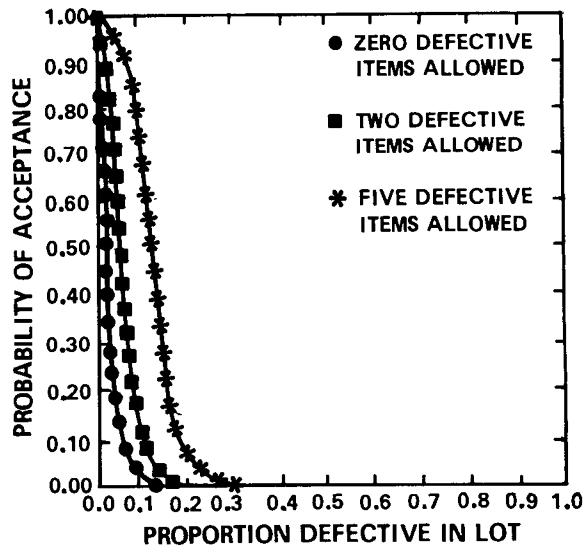

Figure 30.—Operating characteristic curves for sample size of 50.

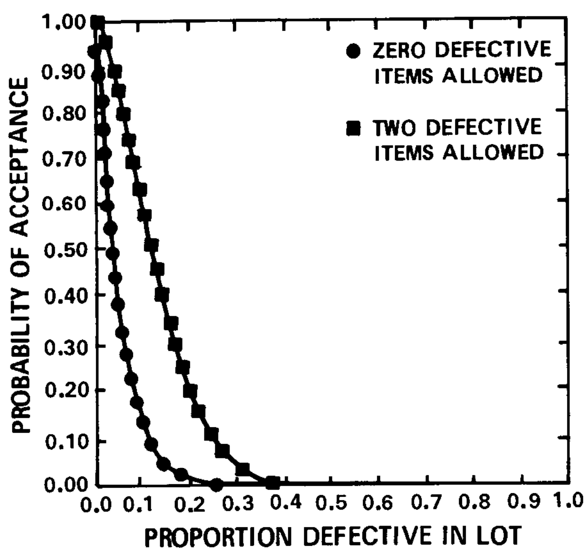

Figure 29.—Operating characteristic curves for sample size of 20.

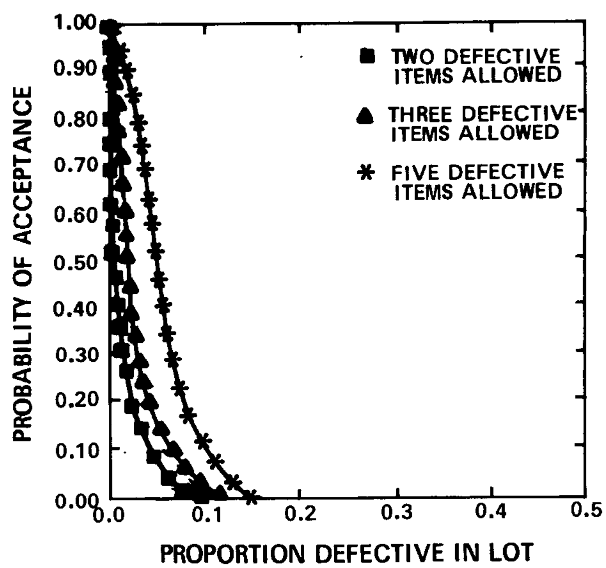

Figure 31.—Operating characteristic curves for sample size of 100.

accepting a lot with 21 percent defective items and a 10 percent chance of rejecting a lot with only 1 percent defective items. If two defective items are allowed in the sample, there is a 10 percent chance of accepting a lot with 45 percent defective items. There is essentially no chance of rejecting a lot with only 1 percent defective items.

2.3.2 By contrast, figure 29 shows OC curves for a sample size of 20 and requires a minimum lot size of 200. If no defective items are allowed in the sample, there is a 10 percent chance of accepting a lot with 11 percent defective items and a 10 percent chance of rejecting a lot with only 1 percent defective items. If two defective items are allowed in the sample, there is a 10 percent chance of accepting a lot with 24 percent defective items. There is a 10 percent chance of rejecting a lot with only 5 percent defective items, but essentially no chance of rejecting a lot with only 1 percent defective items.

2.3.3 Figure 30 shows OC curves for a sample size of 50 and requires a minimum lot size of 500 and figure 31 shows OC curves for a sample size of 100 and requires a minimum lot size of 1,000. As can be seen by the examples, a change in sample size or a change in the number of defective samples allowed can make a considerable difference in the OC curve and in the risks of accepting a bad lot or rejecting a good lot.

References

Crow, E. L. and others, 1960, Statistics Manual: New York, Dover Publications p. 209–226.

Dixon, W. J., and Massey, F. J. Jr., 1969, Introduction to statistics: New York, McGraw-Hill, p. 410–415.

Dodge, H. F. and Romig, H. G., 1959, Sampling inspection tables, single and double sampling (2d ed.): New York, John Wiley, 224 p.

Enrick, N. L., 1972, Quality control and reliability, (6th ed.): New York, Industrial Press, p. 306.

Miller, Irwin, and Freund, J. F., 1977, Probability and statistics for engineers: Englewood Cliffs, New Jersey, Prentice-Hall, p. 60–63, 436–446, 477–481.

U.S. Department of Defense, 1963, American national standard sampling procedure and tables for inspection by attributes: Mil-Std-105D, 64 p.

Weisen, J. M., 1974, Sampling by attributes, *in* Juran, J. M., and others, eds., Quality control handbook (3rd ed.): New York, McGraw-Hill, p. 24-1—24-44.

Single Sampling Plans, to Obtain Lots of Acceptable Quality

1. Application or scope

1.1 This practice can be used in setting up a specific plan to test the quality of materials. The sampling plan detailed here is a portion of the Military Standard 105D plan. For lot quality which is equal to the acceptable quality level (AQL) specified, the probability of accepting the lot will range from 89 to 99.5 percent (Juran, 1965). This practice is particularly useful when test analyses are time-consuming and results are desired immediately.

1.2 Although the number of items which must be tested will usually be greater using a single sampling plan than with other sampling plans (such as double or sequential plans), record keeping usually will be simpler and less time-consuming.

2. Practice

2.1 Choose the acceptable quality level (AQL) by choosing the maximum percent of "bad" items which will be allowed for satisfactory lots (NOTE 1).

NOTE 1. The AQL choosen may have to be a compromise between what is desirable and what is economically possible to attain. See practice "Single sampling with operating characteristic curves" for more information on the AQL.

2.2 Depending upon the size of the lot, randomly select the number of samples specified in table 25. If sample size specified exceeds lot size, do 100 percent inspection.

2.3 If number of "defective" items found is ≥ value tabulated as an "R" value, reject the

Table 25.—Excerpt from Mil-Std-105D, single sampling plan

		Acceptable quality levels																							
		.10		.15		.25		.40		.65		1.0		1.5		2.5		4.0		6.5		10		15	
Lot size	Sample size	A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R
26 to 50	8	↓		↓		↓		↓		↓		↓		0	1	↓		↓		1	2	2	3	3	4
51 to 90	13	↓		↓		↓		↓		↓		0	1	↑		↓		1	2	2	3	3	4	5	6
91 to 150	20	↓		↓		↓		↓		0	1	↑		↓		1	2	2	3	3	4	5	6	7	8
151 to 280	32	↓		↓		↓		0	1	↑		↓		1	2	2	3	3	4	5	6	7	8	10	11
281 to 500	50	↓		↓		0	1	↑		↓		1	2	2	3	3	4	5	6	7	8	10	11	14	15
501 to 1,200	80	↓		0	1	↑		↓		1	2	2	3	3	4	5	6	7	8	10	11	14	15	21	22
1,201 to 3,200	125	0	1	↑		↓		1	2	2	3	3	4	5	6	7	8	10	11	14	15	21	22	↑	
3,201 to 10,000	200	↑		↓		1	2	2	3	3	4	5	6	7	8	10	11	14	15	21	22	↑		↑	
10,001 to 35,000	315	↓		1	2	2	3	3	4	5	6	7	8	10	11	14	15	21	22	↑		↑		↑	
35,001 to 150,000	500	1	2	2	3	3	4	5	6	7	8	10	11	14	15	21	22	↑		↑		↑		↑	
150,001 to 500,000	800	2	3	3	4	5	6	7	8	10	11	14	15	21	22	↑		↑		↑		↑		↑	

↓ = Use sampling plan below arrow. If sample size exceeds lot size, do 100 percent inspection.

↑ = Use sampling plan above arrow.

A = Accept lot if number of defective items ≤ number tabulated.

R = Reject lot if number of defective items ≥ number tabulated.

lot. If number of "defective items found is ≤ value tabulated as an "A" value, accept the lot.

2.4 If 10 lots have been inspected and have not been rejected, refer to Military Standard 105D for procedures to follow to reduce inspection (U.S. Department of Defense, 1963). See also practice "Reducing sample inspection."

References

Juran, J. M. and others, eds.,1965, Quality Control Handbook: New York, McGraw-Hill, p. 24-1—24-44.

Miller, Irwin, and Freund, J. F., 1977, Probability and statistics for engineers: Englewood Cliffs, New Jersey, Prentice-Hall, p. 436-446.

U.S. Department of Defense, 1963, American national standard sampling procedures and tables for inspection by attributes: Mil-Std-105D, 64 p.

Double Sampling Plans, to Obtain Lots of Acceptable Quality

1. Application or scope

1.1 This practice can be used in setting up a specific plan to test the quality of materials. The sampling plan detailed here is a portion of the Military Standrd 105D plan.

1.2 A smaller-sized sample is initially used than would be used in a single sampling plan. If it is possible to accept or reject the lot based on this sample, the overall sample size will be smaller than for single sampling, otherwise a second sample must be selected and the overall sample size will be larger.

2. Practice

2.1 Choose the acceptable quality level (AQL) by choosing the maximum percent of "bad" items which will be allowed for satisfactory lots (NOTE 1).

NOTE 1. The AQL chosen may have to be a compromise between what is desirable and what is economically possible to attain. See practice "Single sampling with operating characteristic curves" for more information on the AQL.

2.2 Depending upon the size of the lot, randomly select the number of samples specified

Table 26.—Excerpt from Mil-Std-105D, double sampling plan

Lot size	Sample set	Sample size	Acceptable quality levels: .10		.15		.25		.40		.65		1.0		1.5		2.5		4.0		6.5		10		15	
			A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R
26 to 50	First	5													.		.				0	2	0	3	1	4
	Second	5											↓						↓		1	2	3	4	4	5
51 to 90	First	8											.		↑				0	2	0	3	1	4	2	5
	Second	8									↓						↓		1	2	3	4	4	5	6	7
91 to 150	First	13									.		↑				0	2	0	3	1	4	2	5	3	7
	Second	13							↓						↓		1	2	3	4	4	5	6	7	8	9
151 to 280	First	20							.		↑				0	2	0	3	1	4	2	5	3	7	5	9
	Second	20					↓						↓		1	2	3	4	4	5	6	7	8	9	12	13
281 to 500	First	32					.		↑				0	2	0	3	1	4	2	5	3	7	5	9	7	11
	Second	32			↓						↓		1	2	3	4	4	5	6	7	8	9	12	13	18	19
501 to 1,200	First	50			.		↑				0	2	0	3	1	4	2	5	3	7	5	9	7	11	11	16
	Second	50	↓						↓		1	2	3	4	4	5	6	7	8	9	12	13	18	19	26	27
1,201 to 3,200	First	80	.		↑				0	2	0	3	1	4	2	5	3	7	5	9	7	11	11	16	↑	
	Second	80					↓		1	2	3	4	4	5	6	7	8	9	12	13	18	19	26	27		
3,201 to 10,000	First	125	↑				0	2	0	3	1	4	2	5	3	7	5	9	7	11	11	16	↑			
	Second	125			↓		1	2	3	4	4	5	6	7	8	9	12	13	18	19	26	27				
10,001 to 35,000	First	200			0	2	0	3	1	4	2	5	3	7	5	9	7	11	11	16	↑					
	Second	200	↓		1	2	3	4	4	5	6	7	8	9	12	13	18	19	26	27						
35,001 to 150,000	First	315	0	2	0	3	1	4	2	5	3	7	5	9	7	11	11	16	↑							
	Second	315	1	2	3	4	4	5	6	7	8	9	12	13	18	19	26	27								
150,001 to 500,000	First	500	0	3	1	4	2	5	3	7	5	9	7	11	11	16	↑									
	Second	500	3	4	4	5	6	7	8	9	12	13	18	19	26	27										

↓ = Use sampling plan below arrow. If sample size exceeds lot size, do 100 percent inspection.

↑ = Use sampling plan above arrow.

A = Acceptance number.

R = Rejection number.

. = Use corresponding single sampling plan.

in table 26. If sample size specified exceeds lot size, do 100 percent inspection.

2.3 If for the first sample set the numbers of "defective" items found is ≥ value tabulated as an "R" value, reject the lot. If number of "defective" items found is ≤ value tabulated as an "A" value, accept the lot. If the number of "defective" items found is between the "A" and "R" values, select a second set of samples.

2.4 If the number of "defective" items found in first and second set combined is ≥ the value tabulated as an "R" value, reject the lot. If the number is ≤ the value tabulated as an "A" value, accept the lot.

2.5 If 10 lots have been inspected and have not been rejected, refer to Military Standard 105D for procedures to follow to reduce the number of items to be inspected (U.S. Dept. of Defense, 1963). See also practice, "Reducing sample inspection."

References

Juran, J. M. and others, eds., 1965, Quality Control Handbook: New York, McGraw-Hill, p. 24-1—24-44.

Miller, Irwin, and Freund, J. F., 1977, Probability and statistics for engineers: New Jersey, Prentice-Hall, p. 436-446.

U.S. Department of Defense, 1963, American national standard sampling procedures and tables for inspection by attributes: Mil-Std-105D, 64 p.

Sequential Sampling Plans, to Obtain Lots of Acceptable Quality

1. Application or scope

1.1 This practice can be used for testing the quality of materials where it is practical to test one item at a time.

1.2 It may be used to test physical properties of materials prior to accepting their delivery (for example, in testing pesticide bottles which must meet certain size criteria to be used in samplers). Since results from each test must be evaluated before deciding whether to test the next sample, this practice should not be used when time is a critical factor and it is inconvenient or costly to wait for the results from tests of one item at a time.

2. Practice

2.1 Construction of graph

2.1.1 Indicate the number of items tested (n) along the horizontal axis and the number of defective items found (d) along the vertical axis (fig. 32).

2.1.2 Calculate and draw parallel lines, d_1 and d_2, to define areas of acceptance and rejection (Grant and Leavenworth, 1974):

$$d_1 = sn - h_1 \tag{46}$$

$$d_2 = sn + h_2 \tag{47}$$

where

d_1 = lower line, below which is region of acceptance,

d_2 = upper line, above which is region of rejection,

n = number of items tested,

$$h_1 = \frac{\log \dfrac{1-\alpha}{\beta}}{\log \dfrac{P_2}{P_1} + \log \dfrac{1-P_1}{1-P_2}},$$

$$h_2 = \frac{\log \dfrac{1-\beta}{\alpha}}{\log \dfrac{P_2}{P_1} + \log \dfrac{1-P_1}{1-P_2}},$$

$$s = \frac{\log \dfrac{1-P_1}{1-P_2}}{\log \dfrac{P_2}{P_1} + \log \dfrac{1-P_1}{1-P_2}},$$

where

α = probability of rejection of a "good" lot (producer's risk or alpha error),

β = probability of accepting a "bad" lot (consumer's risk or beta error),

P_1 = acceptable quality level (AQL), and

P_2 = lot tolerance percent defective (LTPD)

2.2 Examples of graphs

2.2.1 Figure 33 can be used when willing to take a 2 percent chance of rejecting a lot with 5 percent defective pieces and a 5 percent chance of rejecting a lot with 1 percent defective pieces.

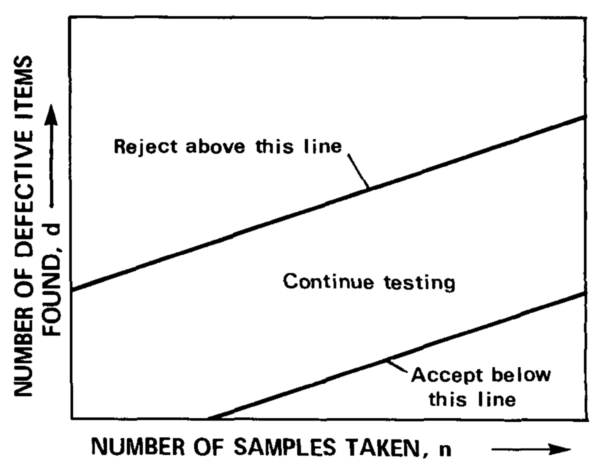

Figure 32.—Sequential sampling.

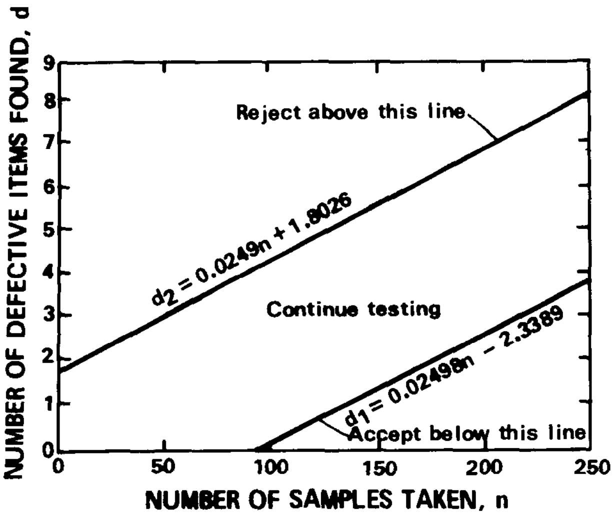

Figure 33.—Sequential sampling: 2-percent chance of accepting a lot with 5-percent defective items, and 5-percent chance of rejecting a lot with 1- percent defective items.

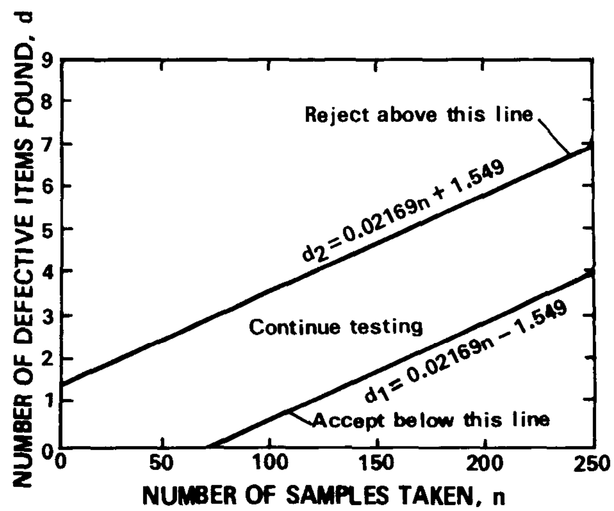

Figure 35.—Sequential sampling: 10-percent chance of accepting a lot with 4-percent defective items, and 10-percent chance of rejecting a lot with 1- percent defective items.

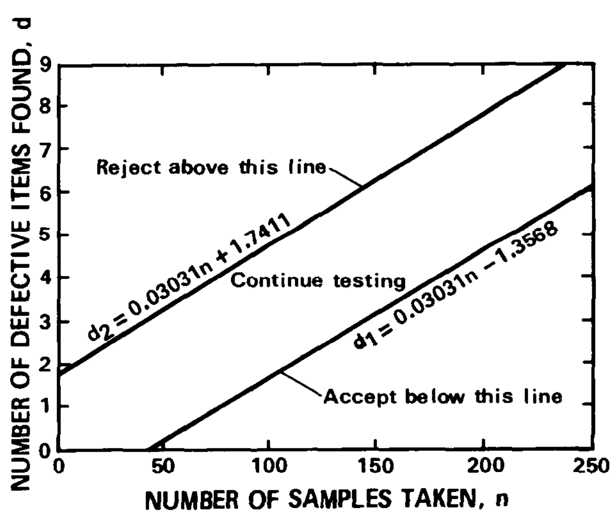

Figure 34.—Sequential sampling: 10-percent chance of accepting a lot with 5-percent defective items, and 5-percent chance of rejecting a lot with 1- percent defective items.

Figure 34 can be used when willing to take a 10 percent chance of accepting a lot with 5 percent defective pieces and a 5 percent chance of rejecting a lot with 1 percent defective pieces. Figure 35 can be used when willing to take a 10 percent chance of accepting a lot with 4 percent defective pieces and a 10 percent chance of rejecting a lot with 1 percent defective.

2.3 Maximum n

2.3.1 It would be possible, particularly if the lot is of "borderline" quality, to continue sequential sampling indefinitely. If an extremely large sample is undesirable, a value for n may be selected at which sampling is to stop.

2.3.2 One value which may be used (Crow and others, 1960) is to calculate and agree to stop sampling at a maximum n such that :

$$n_{max} = \frac{3\left(\log\frac{1-\beta}{\alpha}\right)\left(\frac{1-\alpha}{\beta}\right)}{\left(\log\frac{P_2}{P_1}\right)\left(\log\frac{1-P_1}{1-P_2}\right)} \quad (48)$$

where

n_{max}=maximum number of samples to be taken, and α, β, P_1 and P_2 are as previously defined (NOTE 1).

NOTE 1. Maximum values would be 517, 241 and 339 for the examples given in figures 33, 34, and 35.

2.3.3 When n_{max} is reached, accept the lot if the distance between n_{max} and the lower line, d_1, is less than the distance between n_{max} and the upper line, d_2. Otherwise, reject the lot (NOTE 2).

NOTE 2. Rarely will it be necessary to test n_{max} number of samples before deciding whether to accept or reject the lot (Crow and others, 1960).

References

Crow, E. L., Davis, F. A., and Maxfield, M. W., 1960, Statistics manual: New York, Dover Publications, p. 209–226.

Dixon, W. J., and Massey, F. J., Jr., 1969, Introduction to statistics: New York, McGraw-Hill, p. 262–375.

Enrick, N. L., 1972, Quality control and reliability (6th ed.): New York, Industrial Press, p. 306

Grant, E. I., and Leavenworth, R. S., 1972, Statistical quality control (4th ed.): New York, McGraw-Hill, p. 456–459.

Miller, Irwin, and Freund, J. F., 1977, Probability and statistics for engineers: New Jersey, Prentice-Hall, p. 436–446.

Reducing Sample Inspection

1. Application or scope

1.1 This practice describes methods which can be used to reduce the size of the sample (and cost) necessary for quality assurance testing when lots are received repeatedly from the same vendor (NOTE 1).

NOTE 1. If vendor cooperation is necessary, it usually will be necessary to specify the type of cooperation needed from the vendor in any contract issued.

2. Practice

2.1 Vendor selection of sample

2.1.1 Require the vendor to select the sample. The size of sample, frequency of sampling, definition of a lot, and so forth, must be agreed upon beforehand (Fitzgibbons, 1974).

2.1.2 Require the vendor to identify the sample as to its lot and give any other pertinent information. Have the sample shipped prior to or along with shipment of the lot.

Table 27.—Excerpt from Mil-Std-105D, maximum number of defective items allowed for reduced inspection

Number of sample units from last ten lots or batches	Acceptable quality level									
	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15
20-29	.	.	.	.	.	.	.	.	0	0
30-49	.	.	.	.	.	.	.	0	0	1
50-79	.	.	.	.	.	.	0	0	2	3
80-129	.	.	.	.	.	0	0	2	4	7
130-199	.	.	.	.	0	0	2	4	7	13
200-319	.	.	.	0	0	2	4	8	14	22
320-499	.	.	0	0	1	4	8	14	24	39
500-799	.	0	0	2	3	7	14	25	40	63
800-1,249	0	0	2	4	7	14	24	42	68	105
1,250-1,999	0	2	4	7	13	24	40	69	110	169
2,000-3,149	2	4	8	14	22	40	68	115	181	
3,150-1,999	4	8	14	24	38	67	111	106		
5,000-7,999	7	14	25	40	63	110	181			
8,000-12,499	14	24	42	68	105	181				
12,500-19,999	24	40	69	110	169					
20,000-31,499	40	68	115	181						
31,500-49,999	67	111	186							
50,000 > Over	110	181	301							

. Denotes that the number of sample units from the last ten lots or batches is not sufficient for reduced inspection for this AQL.

Table 28.—Excerpt from Mil-Std-105D, reduced inspection

		Acceptable quality levels (percent)																			
		.25		.40		.65		1.0		1.5		2.5		4.0		6.5		10		15	
Lot size	Sample size	A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R
26 to 90	2	↓		↓		↓		↓		↓		0	1	↓		↓		0	2	1	3
91 to 150	3	↓		↓		↓		↓		0	1	↑		↓		0	2	1	3	1	4
151 to 280	5	↓		↓		↓		0	1	↑		↓		0	2	1	3	1	4	2	5
281 to 500	8	↓		↓		0	1	↑		↓		0	2	1	3	1	4	2	5	3	6
501 to 1,200	13	↓		0	1	↑		↓		0	2	1	3	1	4	2	5	3	6	5	8
1,201 to 3,200	20	0	1	↑		↓		0	2	1	3	1	4	2	5	3	6	5	8	7	10
3,201 to 10,000	32	↑		↓		0	2	1	3	1	4	2	5	3	6	5	8	7	10	10	13
10,001 to 35,000	50	↓		0	2	1	3	1	4	2	5	3	6	5	8	7	10	10	13	↑	
35,001 to 150,000	80	0	2	1	3	1	4	2	5	3	6	5	8	7	10	10	13	↑		↑	
150,001 to 500,000	125	1	3	1	4	2	5	3	6	5	8	7	10	10	13	↑		↑		↑	

↓ = Use sampling plan below arrow.
↑ = Use sampling plan above arrow.
A = Accept lot if number of defective items ≤ number tabulated.
R = Reject lot if number of defective items ≥ number tabulated.

2.1.3 Initially, draw a separate independent sample from the lot received. Check the independent sample and the vendor's selected sample.

2.1.4 If results from the independent sample and vendor's sample agree for several shipments, assume that the vendor is selecting a representative sample. Then subsample and test the vendor's sample instead of the lot; occasionally also test the lot to ensure results continue to agree.

2.2 Vendor quality control data

2.2.1 Require the vendor's quality control data to be submitted with each lot.

2.2.2 Initially, select a sample to be tested from each lot and compare with the vendor's data.

2.2.3 If results from the samples tested agree with the vendor's data, assume the vendor's data is adequate. Then rely on the vendor's data for most shipments; occasionally test a lot to verify data integrity.

2.3 Military Standard 105D—reduced sampling

2.3.1 If at least 10 consecutive lots from a vendor have been acceptable and the total number of defective items does not exceed the numbers in table 27, reduced sampling may be used for the Military Standard 105D plan.

2.3.2 Using table 28, select the sample size depending on the size of the lot. Accept the lot if the number of "bad" samples is less than or equal to the value under A. Reject the lot if the number of "bad" samples is equal to or greater than the value under R.

2.3.3 If a lot is rejected, resume normal sampling for Military Standard 105D (See practice "Single sampling plans, to obtain lots of acceptable quality").

References

Fitzgibbons, R. G., 1974, Vendor relations, *in* Juran, J. M., and others, eds., Quality control handbook (30th ed): New York, McGraw-Hill, p. 10–1—10–35.

Juran, J. M., and Gryna, F. M., Jr., 1976, Quality planning and analysis: New York, McGraw-Hill, 684 p.

U.S. Department of Defense, 1963, American national standard sampling procedures and tables for inspection by attributes: Mil-Std-105D, 64 p.

Appendix
Statistical Tables

Table A1.—Critical Values for T[a]

Number of observations, n	2.5 Percent significance level[b]	5 Percent significance level[b]
3	1.155	1.153
4	1.481	1.463
5	1.715	1.672
6	1.887	1.822
7	2.020	1.938
8	2.126	2.032
9	2.215	2.110
10	2.290	2.176
11	2.355	2.234
12	2.412	2.285
13	2.462	2.331
14	2.507	2.371
15	2.549	2.409
16	2.585	2.443
17	2.620	2.475
18	2.651	2.504
19	2.681	2.532
20	2.709	2.557
21	2.733	2.580
22	2.758	2.603
23	2.781	2.624
24	2.802	2.644
25	2.822	2.663
26	2.841	2.681
27	2.859	2.698
28	2.876	2.714
29	2.893	2.730
30	2.908	2.745
31	2.924	2.759
32	2.938	2.773
33	2.952	2.786
34	2.965	2.799
35	2.979	2.811
36	2.991	2.823
37	3.003	2.835
38	3.014	2.846
39	3.025	2.857
40	3.036	2.866
41	3.046	2.877
42	3.057	2.887
43	3.067	2.896
44	3.075	2.905
45	3.085	2.914
46	3.094	2.923
47	3.103	2.931
48	3.111	2.940
49	3.120	2.948
50	3.128	2.956
51	3.136	2.964
52	3.143	2.971
53	3.151	2.978
54	3.158	2.986
55	3.166	2.992
56	3.172	3.000
57	3.180	3.006
58	3.186	3.013
59	3.193	3.019
60	3.199	3.025
61	3.205	3.032
62	3.212	3.037
63	3.218	3.044
64	3.224	3.049
65	3.230	3.055
66	3.235	3.061
67	3.241	3.066
68	3.246	3.071
69	3.252	3.076
70	3.257	3.082
71	3.262	3.087
72	3.267	3.092
73	3.272	3.098
74	3.278	3.102
75	3.282	3.107
76	3.287	3.111
77	3.291	3.117
78	3.297	3.121
79	3.301	3.125
80	3.305	3.130
81	3.309	3.134
82	3.315	3.139
83	3.319	3.143
84	3.323	3.147
85	3.327	3.151
86	3.331	3.155
87	3.335	3.160
88	3.339	3.163
89	3.343	3.167
90	3.347	3.171
91	3.350	3.174
92	3.355	3.179
93	3.358	3.182
94	3.362	3.186
95	3.365	3.189
96	3.369	3.193
97	3.372	3.196
98	3.377	3.201
99	3.380	3.204
100	3.383	3.207
101	3.386	3.210
102	3.390	3.214
103	3.393	3.217
104	3.397	3.220
105	3.400	3.224
106	3.403	3.227
107	3.406	3.230
108	3.409	3.233
109	3.412	3.236
110	3.415	3.239
111	3.418	3.242
112	3.422	3.245
113	3.424	3.248
114	3.427	3.251
115	3.430	3.254
116	3.433	3.257
117	3.435	3.259
118	3.438	3.262
119	3.441	3.265
120	3.444	3.267
121	3.447	3.270
122	3.450	3.274
123	3.452	3.276
124	3.455	3.279
125	3.457	3.281
126	3.460	3.284
127	3.462	3.286
128	3.465	3.289
129	3.467	3.291
130	3.470	3.294
131	3.473	3.296
132	3.475	3.298
133	3.478	3.302
134	3,480	3,304
135	3.482	3.306
136	3.484	3.309
137	3.487	3.311
138	3.489	3.313
139	3.491	3.315
140	3.493	3.318
141	3.497	3.320
142	3.499	3.322
143	3.501	3.324
144	3.503	3.326
145	3.505	3.328
146	3.507	3.331
147	3.509	3.334

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b/For testing either positive or negative side of the distribution (not both sides).

Table A2.—Criteria for testing outlying value [a]

n	95 percentile	99 percentile
3	.941	.988
4	.765	.889
5	.642	.780
6	.560	.698
7	.507	.637
8	.554	.683
9	.512	.635
10	.477	.597
11	.576	.679
12	.546	.642
13	.521	.615
14	.546	.641
15	.525	.616
16	.507	.595
17	.490	.577
18	.475	.561
19	.462	.547
20	.450	.535
21	.440	.524
22	.430	.514
23	.421	.505
24	.413	.497
25	.406	.489

a/From "Introduction to Statistical Analysis" by Wilfred J. Dixon and Frank J. Massey, Jr. Copyright (c) 1951, 1957, 1969 by McGraw-Hill, Inc. Used with the permission of McGraw-Hill Book Company.

Table A3.—Significant values for $\sqrt{b}$ [a]

n	1 percent significance level	5 percent significance level
5	1.34	1.05
10	1.31	0.92
15	1.20	0.84
20	1.11	0.79
25	1.06	0.71
30	0.98	0.66
35	0.92	0.62
40	0.87	0.59
50	0.79	0.53
60	0.72	0.49

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Table A4.—Significant values for b_2 [a]

n	1 percent significance level	5 percent significance level
5	3.11	2.89
10	4.83	3.85
15	5.08	4.07
20	5.23	4.15
25	5.00	4.00
50	4.88	3.99
75	4.59	3.87
100	4.39	3.77

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Table A5.—Approximate 5 percent limits for ranking scores[a] (two- sided test)

Number of labs, n	Number of materials, M[b]												
	3	4	5	6	7	8	9	10	11	12	13	14	15
3		4	5	7	8	10	12	13	15	17	19	20	22
		12	15	17	20	22	24	27	29	31	33	36	38
4		4	6	8	10	12	14	16	18	20	22	24	26
		16	19	22	25	28	31	34	37	40	43	46	49
5		5	7	9	11	13	16	18	21	23	26	28	31
		19	23	27	31	35	38	42	45	49	52	56	59
6	3	5	7	10	12	15	18	21	23	26	29	32	35
	18	23	28	32	37	41	45	49	54	58	62	66	70
7	3	5	8	11	14	17	20	23	26	29	32	36	39
	21	27	32	37	42	47	52	57	62	67	72	76	81
8	3	6	9	12	15	18	22	25	29	32	36	39	43
	24	30	36	42	48	54	59	65	70	76	81	87	92
9	3	6	9	13	16	20	24	27	31	35	39	43	47
	27	34	41	47	54	60	66	73	79	85	91	97	103
10	4	7	10	14	17	21	26	30	34	38	43	47	51
	29	37	45	52	60	67	73	80	87	94	100	107	114
11	4	7	11	15	19	23	27	32	36	41	46	51	55
	32	41	49	57	65	73	81	88	96	103	110	117	125
12	4	7	11	15	20	24	29	34	39	44	49	54	59
	35	45	54	63	71	80	88	96	104	112	120	128	136
13	4	8	12	16	21	26	31	36	42	47	52	58	63
	38	48	58	68	77	86	95	104	112	121	130	138	147
14	4	8	12	17	22	27	33	38	44	50	56	61	67
	41	52	63	73	83	93	102	112	121	130	139	149	158
15	4	8	13	18	23	29	35	41	47	53	59	65	71
	44	56	67	78	89	99	109	119	129	139	149	159	169

[a]From W. J. Youden's "Statistical techniques for collaborative tests" in the Statistical Manual of the AOAC," 1975. Copyright 1975 by the Association of Official Analytical Chemists. Reprinted with permission.

[b]Assign ranks 1 to n for each material. Sum the ranks to get the score for each laboratory. The mean score is $M(n + 1)/2$. The entries are lower and upper limits that are included in the approximate 5 percent critical region.

Table A6a.—Five percent values for the distribution of F

Degrees of freedom (for denominator)	Degrees of freedom (for numerator) 1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	∞
1	161	200	216	225	230	234	237	239	241	242	243	244	245	246	248	249	250	251	252	253	253	254	254	254
2	18.51	19.00	19.16	19.25	19.30	19.33	19.36	19.37	19.38	19.39	19.40	19.41	19.42	19.43	19.44	19.45	19.46	19.47	19.47	19.48	19.49	19.49	19.50	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.88	8.84	8.81	8.78	8.76	8.74	8.71	8.69	8.66	8.64	8.62	8.60	8.58	8.57	8.56	8.54	8.54	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.93	5.91	5.87	5.84	5.80	5.77	5.74	5.71	5.70	5.68	5.66	5.65	5.64	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.78	4.74	4.70	4.68	4.64	4.60	4.56	4.53	4.50	4.46	4.44	4.42	4.40	4.38	4.37	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00	3.96	3.92	3.87	3.84	3.81	3.77	3.75	3.72	3.71	3.69	3.68	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.63	3.60	3.57	3.52	3.49	3.44	3.41	3.38	3.34	3.32	3.29	3.28	3.25	3.24	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.34	3.31	3.28	3.23	3.20	3.15	3.12	3.08	3.05	3.03	3.00	2.98	2.96	2.94	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.13	3.10	3.07	3.02	2.98	2.93	2.90	2.86	2.82	2.80	2.77	2.76	2.73	2.72	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.97	2.94	2.91	2.86	2.82	2.77	2.74	2.70	2.67	2.64	2.61	2.59	2.56	2.55	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.86	2.82	2.79	2.74	2.70	2.65	2.61	2.57	2.53	2.50	2.47	2.45	2.42	2.41	2.40
12	4.75	3.88	3.49	3.26	3.11	3.00	2.92	2.85	2.80	2.76	2.72	2.69	2.64	2.60	2.54	2.50	2.46	2.42	2.40	2.36	2.35	2.32	2.31	2.30
13	4.67	3.80	3.41	3.18	3.02	2.92	2.84	2.77	2.72	2.67	2.63	2.60	2.55	2.51	2.46	2.42	2.38	2.34	2.32	2.28	2.26	2.24	2.22	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.77	2.70	2.65	2.60	2.56	2.53	2.48	2.44	2.39	2.35	2.31	2.27	2.24	2.21	2.19	2.16	2.14	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.70	2.64	2.59	2.55	2.51	2.48	2.43	2.39	2.33	2.29	2.25	2.21	2.18	2.15	2.12	2.10	2.08	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.45	2.42	2.37	2.33	2.28	2.24	2.20	2.16	2.13	2.09	2.07	2.04	2.02	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.62	2.55	2.50	2.45	2.41	2.38	2.33	2.29	2.23	2.19	2.15	2.11	2.08	2.04	2.02	1.99	1.97	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34	2.29	2.25	2.19	2.15	2.11	2.07	2.04	2.00	1.98	1.95	1.93	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.55	2.48	2.43	2.38	2.34	2.31	2.26	2.21	2.15	2.11	2.07	2.02	2.00	1.96	1.94	1.91	1.90	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.52	2.45	2.40	2.35	2.31	2.28	2.23	2.18	2.12	2.08	2.04	1.99	1.96	1.92	1.90	1.87	1.85	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.28	2.25	2.20	2.15	2.09	2.05	2.00	1.96	1.93	1.89	1.87	1.84	1.82	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.47	2.40	2.35	2.30	2.26	2.23	2.18	2.13	2.07	2.03	1.98	1.93	1.91	1.87	1.84	1.81	1.80	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.45	2.38	2.32	2.28	2.24	2.20	2.14	2.10	2.04	2.00	1.96	1.91	1.88	1.84	1.82	1.79	1.77	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.43	2.36	2.30	2.26	2.22	2.18	2.13	2.09	2.02	1.98	1.94	1.89	1.86	1.82	1.80	1.76	1.74	1.73
25	4.24	3.38	2.99	2.76	2.60	2.49	2.41	2.34	2.28	2.24	2.20	2.16	2.11	2.06	2.00	1.96	1.92	1.87	1.84	1.80	1.77	1.74	1.72	1.71

Degrees of freedom (for denominator)	Degrees of freedom (for numerator) 1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	∞
26	4.22	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15	2.10	2.05	1.99	1.95	1.90	1.85	1.82	1.78	1.76	1.72	1.70	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.30	2.25	2.20	2.16	2.13	2.08	2.03	1.97	1.93	1.88	1.84	1.80	1.76	1.74	1.71	1.68	1.67
28	4.20	3.34	2.95	2.71	2.56	2.44	2.36	2.29	2.24	2.19	2.15	2.12	2.06	2.02	1.96	1.91	1.87	1.81	1.78	1.75	1.72	1.69	1.67	1.65
29	4.18	3.33	2.93	2.70	2.54	2.43	2.35	2.28	2.22	2.18	2.14	2.10	2.05	2.00	1.94	1.90	1.85	1.80	1.77	1.73	1.71	1.68	1.65	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.34	2.27	2.21	2.16	2.12	2.09	2.04	1.99	1.93	1.89	1.84	1.79	1.76	1.72	1.69	1.66	1.64	1.62
32	4.15	3.30	2.90	2.67	2.51	2.40	2.32	2.25	2.19	2.14	2.10	2.07	2.02	1.97	1.91	1.86	1.82	1.76	1.74	1.69	1.67	1.64	1.61	1.59
34	4.13	3.28	2.88	2.65	2.49	2.38	2.30	2.23	2.17	2.12	2.08	2.05	2.00	1.95	1.89	1.84	1.80	1.74	1.71	1.67	1.64	1.61	1.50	1.57
36	4.11	3.26	2.86	2.63	2.48	2.36	2.28	2.21	2.15	2.10	2.06	2.03	1.98	1.93	1.87	1.82	1.78	1.72	1.69	1.65	1.62	1.59	1.56	1.55
38	4.10	3.25	2.85	2.62	2.46	2.35	2.26	2.19	2.14	2.09	2.05	2.02	1.96	1.92	1.85	1.80	1.76	1.71	1.67	1.63	1.60	1.57	1.54	1.53
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.07	2.04	2.00	1.95	1.90	1.84	1.79	1.74	1.69	1.66	1.61	1.59	1.55	1.53	1.51
42	4.07	3.22	2.83	2.59	2.44	2.32	2.24	2.17	2.11	2.06	2.02	1.99	1.94	1.89	1.82	1.78	1.73	1.68	1.64	1.60	1.57	1.54	1.51	1.49
44	4.06	3.21	2.82	2.58	2.43	2.31	2.23	2.16	2.10	2.05	2.01	1.98	1.92	1.88	1.81	1.76	1.72	1.66	1.63	1.58	1.56	1.52	1.50	1.48
46	5.05	3.20	2.81	2.57	2.42	2.30	2.22	2.14	2.09	2.04	2.00	1.97	1.91	1.87	1.80	1.75	1.71	1.65	1.62	1.57	1.54	1.51	1.48	1.46
48	4.04	3.19	2.80	2.56	2.41	2.30	2.21	2.14	2.08	2.03	1.99	1.96	1.90	1.86	1.79	1.74	1.70	1.64	1.61	1.56	1.53	1.50	1.47	1.45
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.02	1.98	1.95	1.90	1.85	1.78	1.74	1.69	1.63	1.60	1.55	1.52	1.48	1.46	1.44
55	4.02	3.17	2.78	2.54	2.38	2.27	2.18	2.11	2.05	2.00	1.97	1.93	1.88	1.83	1.76	1.72	1.67	1.61	1.58	1.52	1.50	1.46	1.43	1.41
60	4.00	3.15	2.76	2.52	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92	1.86	1.81	1.75	1.70	1.65	1.59	1.56	1.50	1.48	1.44	1.41	1.39
65	3.99	3.14	2.75	2.51	2.36	2.24	2.15	2.08	2.02	1.98	1.94	1.90	1.85	1.80	1.73	1.68	1.63	1.57	1.54	1.49	1.46	1.42	1.39	1.37
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.01	1.97	1.93	1.89	1.84	1.79	1.72	1.67	1.62	1.56	1.53	1.47	1.45	1.40	1.37	1.35
80	3.96	3.11	2.72	2.48	2.33	2.21	2.12	2.05	1.99	1.95	1.91	1.88	1.82	1.77	1.70	1.65	1.60	1.54	1.51	1.45	1.42	1.38	1.35	1.32
100	3.94	3.09	2.70	2.46	2.30	2.19	2.10	2.03	1.97	1.92	1.88	1.85	1.79	1.75	1.68	1.63	1.57	1.51	1.48	1.42	1.39	1.34	1.30	1.28
125	3.92	3.07	2.68	2.44	2.29	2.17	2.08	2.01	1.95	1.90	1.86	1.83	1.77	1.72	1.65	1.60	1.55	1.49	1.45	1.39	1.36	1.31	1.27	1.25
150	3.91	3.06	2.67	2.43	2.27	2.16	2.07	2.00	1.94	1.89	1.85	1.82	1.76	1.71	1.64	1.59	1.54	1.47	1.44	1.37	1.34	1.29	1.25	1.22
200	3.89	3.04	2.65	2.41	2.26	2.14	2.05	1.98	1.92	1.87	1.83	1.80	1.74	1.69	1.62	1.57	1.52	1.45	1.42	1.35	1.32	1.26	1.22	1.19
400	3.86	3.02	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85	1.81	1.78	1.72	1.67	1.60	1.54	1.49	1.42	1.38	1.32	1.28	1.22	1.16	1.13
1000	3.85	3.00	2.61	2.38	2.22	2.10	2.02	1.95	1.89	1.84	1.80	1.76	1.70	1.65	1.58	1.53	1.47	1.41	1.36	1.30	1.26	1.19	1.13	1.08
∞	3.84	2.99	2.60	2.37	2.21	2.09	2.01	1.94	1.88	1.83	1.79	1.75	1.69	1.64	1.57	1.52	1.46	1.40	1.35	1.28	1.24	1.17	1.11	1.00

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Table A6b.—One percent values for the distribution of F[a]

Degrees of freedom (for denominator)	Degrees of freedom (for numerator) 1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	∞
1	4,052	4,999	5,403	5,625	5,764	5,859	5,928	5,981	6,022	6,056	6,082	6,106	6,142	6,169	6,208	6,234	6,258	6,286	6,302	6,323	6,334	6,352	6,361	6,366
2	98.49	99.00	99.17	99.25	99.30	99.33	99.34	99.36	99.38	99.40	99.41	99.42	99.43	99.44	99.45	99.46	99.47	99.48	99.48	99.49	99.49	99.49	99.50	99.50
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.34	27.23	27.13	27.05	26.92	26.83	26.69	26.60	26.50	26.41	26.35	26.27	26.23	26.18	26.14	26.12
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.54	14.45	14.37	14.24	14.15	14.02	13.93	13.83	13.74	13.69	13.61	13.57	13.52	13.48	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.45	10.27	10.15	10.05	9.96	9.89	9.77	9.68	9.55	9.47	9.38	9.29	9.24	9.17	9.13	9.07	9.04	9.02
6	13.74	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72	7.60	7.52	7.39	7.31	7.23	7.14	7.09	7.02	6.99	6.94	6.90	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	7.00	6.84	6.71	6.62	6.54	6.47	6.35	6.27	6.15	6.07	5.98	5.90	5.85	5.78	5.75	5.70	5.67	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.19	6.03	5.91	5.82	5.74	5.67	5.56	5.48	5.36	5.28	5.20	5.11	5.06	5.00	4.96	4.91	4.88	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.62	5.47	5.35	5.26	5.18	5.11	5.00	4.92	4.80	4.73	4.64	4.56	4.51	4.45	4.41	4.36	4.33	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.21	5.06	4.95	4.85	4.78	4.71	4.60	4.52	4.41	4.33	4.25	4.17	4.12	4.05	4.01	3.96	3.93	3.91
11	9.65	7.20	6.22	5.67	5.32	5.07	4.88	4.74	4.63	4.54	4.46	4.40	4.29	4.21	4.10	4.02	3.94	3.86	3.80	3.74	3.70	3.66	3.62	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.65	4.50	4.39	4.30	4.22	4.16	4.05	3.98	3.86	3.78	3.70	3.61	3.56	3.49	3.46	3.41	3.38	3.36
13	9.07	6.70	5.74	5.20	4.86	4.62	4.44	4.30	4.19	4.10	4.02	3.96	3.85	3.78	3.67	3.59	3.51	3.42	3.37	3.30	3.27	3.21	3.18	3.16
14	8.86	6.51	5.56	5.03	4.69	4.46	4.28	4.14	4.03	3.94	3.86	3.80	3.70	3.62	3.51	3.43	3.34	3.26	3.21	3.14	3.11	3.06	3.02	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67	3.56	3.48	3.36	3.29	3.20	3.12	3.07	3.00	2.97	2.92	2.89	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.61	3.55	3.45	3.37	3.25	3.18	3.10	3.01	2.96	2.89	2.86	2.80	2.77	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.52	3.45	3.35	3.27	3.16	3.08	3.00	2.92	2.86	2.79	2.76	2.70	2.67	2.65
18	8.28	6.01	5.09	4.58	4.25	4.01	3.85	3.71	3.60	3.51	3.44	3.37	3.27	3.19	3.07	3.00	2.91	2.83	2.78	2.71	2.68	2.62	2.59	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.36	3.30	3.19	3.12	3.00	2.92	2.84	2.76	2.70	2.63	2.60	2.54	2.51	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.71	3.56	3.45	3.37	3.30	3.23	3.13	3.05	2.94	2.86	2.77	2.69	2.63	2.56	2.53	2.47	2.44	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.65	3.51	3.40	3.31	3.24	3.17	3.07	2.99	2.88	2.80	2.72	2.63	2.58	2.51	2.47	2.42	2.38	2.36
22	7.94	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12	3.02	2.94	2.83	2.75	2.67	2.58	2.53	2.46	2.42	2.37	2.33	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.14	3.07	2.97	2.89	2.78	2.70	2.62	2.53	2.48	2.41	2.37	2.32	2.28	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.25	3.17	3.09	3.03	2.93	2.85	2.74	2.66	2.58	2.49	2.44	2.46	2.33	2.27	2.23	2.21
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.21	3.13	3.05	2.99	2.89	2.81	2.70	2.62	2.54	2.45	2.40	2.32	2.29	2.23	2.19	2.17

Degrees of freedom (for denominator)	Degrees of freedom (for numerator) 1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	∞
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.17	3.09	3.02	2.96	2.86	2.77	2.66	2.58	2.50	2.41	2.36	2.28	2.25	2.19	2.15	2.13
27	7.68	5.49	4.60	4.11	3.79	3.56	3.39	3.26	3.14	3.06	2.98	2.93	2.83	2.74	2.63	2.55	2.47	2.38	2.33	2.25	2.21	2.16	2.12	2.10
28	7.64	5.45	4.57	4.07	3.76	3.53	3.36	3.23	3.11	3.03	2.95	2.90	2.80	2.71	2.60	2.52	2.44	2.35	2.30	2.22	2.18	2.13	2.09	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.08	3.00	2.92	2.87	2.77	2.68	2.57	2.49	2.41	2.32	2.27	2.19	2.15	2.10	2.06	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.06	2.98	2.90	2.84	2.74	2.66	2.55	2.47	2.38	2.29	2.24	2.16	2.13	2.07	2.03	2.01
32	7.50	5.34	4.46	3.97	3.66	3.42	3.25	3.12	3.01	2.94	2.86	2.80	2.70	2.62	2.51	2.42	2.34	2.25	2.20	2.12	2.08	2.02	1.98	1.96
34	7.44	5.29	4.42	3.93	3.61	3.38	3.21	3.08	2.97	2.89	2.82	2.76	2.66	2.58	2.47	2.38	2.30	2.21	2.15	2.08	2.04	1.98	1.94	1.91
36	7.39	5.25	4.38	3.89	3.58	3.35	3.18	3.04	2.94	2.86	2.78	2.72	2.62	2.54	2.43	2.35	2.26	2.17	2.12	2.04	2.00	1.94	1.90	1.87
38	7.35	5.21	4.34	3.86	3.54	3.32	3.15	3.02	2.91	2.82	2.75	2.69	2.59	2.51	2.40	2.32	2.22	2.14	2.08	2.00	1.97	1.90	1.86	1.84
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.88	2.80	2.73	2.66	2.56	2.49	2.37	2.29	2.20	2.11	2.05	1.97	1.94	1.88	1.84	1.81
42	7.27	5.15	4.29	3.80	3.49	3.26	3.10	2.96	2.86	2.77	2.70	2.64	2.54	2.46	2.35	2.26	2.17	2.08	2.02	1.94	1.91	1.85	1.80	1.78
44	7.24	5.12	4.26	3.78	3.46	3.24	3.07	2.94	2.84	2.75	2.68	2.62	2.52	2.44	2.32	2.24	2.15	2.06	2.00	1.92	1.88	1.82	1.78	1.75
46	7.21	5.10	4.24	3.76	3.44	3.22	3.05	2.92	2.82	2.73	2.66	2.60	2.50	2.42	2.30	2.22	2.13	2.04	1.98	1.90	1.86	1.80	1.76	1.72
48	7.19	5.08	4.22	3.74	3.42	3.20	3.04	2.90	2.80	1.71	2.64	2.58	2.48	2.40	2.28	2.20	2.11	2.02	1.96	1.88	1.84	1.78	1.73	1.70
50	7.17	5.06	4.20	3.72	3.41	3.18	3.02	2.88	2.78	2.70	2.62	2.56	2.46	2.39	2.26	2.18	2.10	2.00	1.94	1.86	1.82	1.76	1.71	1.68
55	7.12	5.01	4.16	3.68	3.37	3.15	2.98	2.85	2.75	2.66	2.59	2.53	2.43	2.35	2.23	2.15	2.06	1.96	1.90	1.82	1.78	1.71	1.66	1.64
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50	2.40	2.32	2.20	2.12	2.03	1.93	1.87	1.79	1.74	1.68	1.63	1.60
65	7.04	4.95	4.10	3.62	3.31	3.09	2.93	2.79	2.70	2.61	2.54	2.47	2.37	2.30	2.18	2.09	2.00	1.90	1.84	1.76	1.71	1.64	1.60	1.56
70	7.01	4.92	4.08	3.60	3.29	3.07	2.91	2.77	2.67	2.59	2.51	2.45	2.35	2.28	2.15	2.07	1.98	1.88	1.82	1.74	1.69	1.62	1.56	1.53
80	6.96	4.88	4.04	3.56	3.25	3.04	2.87	2.74	2.64	2.55	2.48	2.41	2.32	2.24	2.11	2.03	1.94	1.84	1.78	1.70	1.65	1.57	1.52	1.49
100	6.90	4.82	3.98	3.51	3.20	2.99	2.82	2.69	2.59	2.51	2.43	2.36	2.26	2.19	2.06	1.98	1.89	1.79	1.73	1.64	1.59	1.51	1.46	1.43
125	6.84	4.78	3.94	3.47	3.17	2.95	2.79	2.65	2.56	2.47	2.40	2.33	2.23	2.15	2.03	1.94	1.85	1.75	1.68	1.59	1.54	1.46	1.40	1.37
150	6.81	4.75	3.91	3.44	3.14	2.92	2.76	2.62	2.53	2.44	2.37	2.30	2.20	2.12	2.00	1.91	1.83	1.72	1.66	1.56	1.51	1.43	1.37	1.33
200	6.76	4.71	3.88	3.41	3.11	2.90	2.73	2.60	2.50	2.41	2.34	2.28	2.17	2.09	1.97	1.88	1.79	1.69	1.62	1.53	1.48	1.39	1.33	1.28
400	6.70	4.66	3.83	3.36	3.06	2.85	2.69	2.55	2.46	2.37	2.29	2.23	2.12	2.04	1.92	1.84	1.74	1.64	1.57	1.47	1.42	1.32	1.24	1.19
1000	6.66	4.62	3.80	3.34	3.04	2.82	2.66	2.53	2.43	2.34	2.26	2.20	2.09	2.01	1.89	1.81	1.71	1.61	1.54	1.44	1.38	1.28	1.19	1.11
∞	6.64	4.60	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.24	2.18	2.07	1.99	1.87	1.79	1.69	1.59	1.52	1.41	1.36	1.25	1.15	1.00

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Table A7.—Factor c_2, for use in estimating control chart limits[a]

Number of observations in subgroup n	c_2	Number of observations in subgroup n	c_2
		21	0.9638
2	0.5642	22	0.9655
3	0.7236	23	0.9670
4	0.7979	24	0.9684
5	0.8407	25	0.9696
6	0.8686	30	0.9748
7	0.8882	35	0.9784
8	0.9027	40	0.9811
9	0.9139	45	0.9832
10	0.9227	50	0.9849
11	0.9300	55	0.9863
12	0.9359	60	0.9874
13	0.9410	65	0.9884
14	0.9453	70	0.9892
15	0.9490	75	0.9900
16	0.9523	80	0.9906
17	0.9551	85	0.9912
18	0.9576	90	0.9916
19	0.9599	95	0.9921
20	0.9619	100	0.9925

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Table A8.—Factors d_2 and A_2, for use in estimating control chart limits[a]

Number of observations in subgroup n	d_2	$A_2 = 3/d_2\sqrt{n}$
2	1.128	1.88
3	1.693	1.02
4	2.059	0.73
5	2.326	0.58
6	2.534	0.48
7	2.704	0.42
8	2.847	0.37
9	2.970	0.34
10	3.078	0.31
11	3.173	0.29
12	3.258	0.27
13	3.336	0.25
14	3.407	0.24
15	3.472	0.22
16	3.532	0.21
17	3.588	0.20
18	3.640	0.19
19	3.689	0.19
20	3.735	0.18

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Table A9.—Factors B_3 and B_4, for use in estimating control chart limits[a]

Number of observations in subgroup n	Factors for s chart	
	Lower control limit B_3	Upper control limit B_4
2	0	3.27
3	0	2.57
4	0	2.27
5	0	2.09
6	0.03	1.97
7	0.12	1.88
8	0.19	1.81
9	0.24	1.76
10	0.28	1.72
11	0.32	1.68
12	0.35	1.65
13	0.38	1.62
14	0.41	1.59
15	0.43	1.57
16	0.45	1.55
17	0.47	1.53
18	0.48	1.52
19	0.50	1.50
20	0.51	1.49
21	0.52	1.48
22	0.53	1.47
23	0.54	1.46
24	0.55	1.45
25	0.56	1.44
30	0.60	1.40
35	0.63	1.37
40	0.66	1.34
45	0.68	1.32
50	0.70	1.30
55	0.71	1.29
60	0.72	1.28
65	0.73	1.27
70	0.74	1.26
75	0.75	1.25
80	0.76	1.24
85	0.77	1.23
90	0.77	1.23
95	0.78	1.22
100	0.79	1.21

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Table A10.—Factors D_3 and D_4, for use in estimating control chart limits[a]

Number of observations in subgroup n	Factors for R chart	
	Lower control limit D_3	Upper control limit D_4
2	0	3.27
3	0	2.57
4	0	2.28
5	0	2.11
6	0	2.00
7	0.08	1.92
8	0.14	1.86
9	0.18	1.82
10	0.22	1.78
11	0.26	1.74
12	0.28	1.72
13	0.31	1.69
14	0.33	1.67
15	0.35	1.65
16	0.36	1.64
17	0.38	1.62
18	0.39	1.61
19	0.40	1.60
20	0.41	1.59

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Table A11.—t-distribution[a]

Degrees of freedom	Probability of a value which is greater (when testing both positive and negative sides of the distribution)[b] 0.500	0.200	0.100	0.050	0.020	0.010
1	1.000	3.078	6.314	12.706	31.821	63.657
2	.816	1.886	2.920	4.303	6.965	9.925
3	.765	1.638	2.353	3.182	4.541	5.841
4	.741	1.533	2.132	2.776	3.747	4.604
5	.727	1.476	2.015	2.571	3.365	4.032
6	.718	1.440	1.943	2.447	3.143	3.707
7	.711	1.415	1.895	2.365	2.998	3.499
8	.706	1.397	1.860	2.306	2.896	3.355
9	.703	1.383	1.833	2.262	2.821	3.250
10	.700	1.372	1.812	2.228	2.764	3.169
11	.697	1.363	1.796	2.201	2.718	3.106
12	.695	1.356	1.782	2.179	2.681	3.055
13	.694	1.350	1.771	2.160	2.650	3.012
14	.692	1.345	1.761	2.145	2.624	2.977
15	.691	1.341	1.753	2.131	2.602	2.947
16	.690	1.337	1.746	2.120	2.583	2.921
17	.689	1.333	1.740	2.110	2.567	2.898
18	.688	1.330	1.734	2.101	2.552	2.878
19	.688	1.328	1.729	2.093	2.539	2.861
20	.687	1.325	1.725	2.086	2.528	2.845
21	.686	1.323	1.721	2.080	2.518	2.831
22	.686	1.321	1.717	2.074	2.508	2.819
23	.685	1.319	1.714	2.069	2.500	2.807
24	.685	1.318	1.711	2.064	2.492	2.797
25	.684	1.316	1.708	2.060	2.485	2.787
26	.684	1.315	1.706	2.056	2.479	2.779
27	.684	1.314	1.703	2.052	2.473	2.771
28	.683	1.313	1.701	2.048	2.467	2.763
29	.683	1.311	1.699	2.045	2.462	2.756
30	.683	1.310	1.697	2.042	2.457	2.750
40	.681	1.303	1.684	2.021	2.423	2.704
60	.679	1.296	1.671	2.000	2.390	2.660
120	.677	1.289	1.658	1.980	2.358	2.617
∞	.674	1.282	1.645	1.960	2.326	2.576

a/Data are taken from Table III (Distribution of t) of Fisher and Yates: "Statistical Tables for Biological, Agricultural and Medical Research," published by Longman Group Ltd. London (1974) 6th edition, (previously published by Oliver & Boyd Ltd. Edinburgh) and by permission of the authors and publishers.

b/If testing only positive or negative side of distribution (one-tailed test), probability of a greater value is half that tabulated.

Table A12.—Percentiles of the q distribution[a]

Degrees of freedom	Test level (percent)	Number of values used to compute each mean											
		2	3	4	5	6	7	9	10	12	16	20	24
1	90	8.93	13.4	16.4	18.5	20.2	21.5	22.6	24.5	25.9	28.1	29.7	31.0
	95	18.0	27.0	32.8	37.1	40.4	43.1	45.4	49.1	52.0	56.3	59.6	62.1
	99	90.0	135	164	186	202	216	227	246	260	282	298	311
2	90	4.13	5.73	6.77	7.54	8.14	8.63	9.05	9.73	10.3	11.1	11.7	12.2
	95	6.09	8.33	9.80	10.9	11.7	12.4	13.0	14.0	14.8	16.0	16.8	17.5
	99	14.0	19.0	22.3	24.7	26.6	28.2	29.5	31.7	33.4	36.0	38.0	39.5
3	90	3.33	4.47	5.20	5.74	6.16	6.51	6.81	7.29	7.67	8.25	8.68	9.03
	95	4.50	5.91	6.83	7.50	8.04	8.48	8.85	9.46	9.95	10.7	11.2	11.7
	99	8.26	10.6	12.2	13.3	14.2	15.0	15.6	16.7	17.5	18.8	20.0	20.5
4	90	3.02	3.98	4.59	5.04	5.39	5.68	5.93	6.33	6.65	7.13	7.50	7.79
	95	3.93	5.04	5.76	6.29	6.71	7.05	7.35	7.83	8.21	8.79	9.23	9.58
	99	6.51	8.12	9.17	9.96	10.6	11.1	11.6	12.3	12.8	13.7	14.4	14.9
6	90	2.75	3.56	4.07	4.44	4.73	4.97	5.17	5.50	5.76	6.16	6.47	6.71
	95	3.46	4.34	4.90	5.31	5.63	5.90	6.12	6.49	6.79	7.24	7.59	7.86
	99	5.24	6.33	7.03	7.56	7.97	8.32	8.61	9.10	9.49	10.1	10.5	10.9
8	90	2.63	3.37	3.83	4.17	4.43	4.65	4.83	5.13	5.36	5.72	6.00	6.21
	95	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.92	6.18	6.57	6.87	7.11
	99	4.75	5.64	6.20	6.63	6.96	7.24	7.47	7.86	8.18	8.66	9.03	9.32
10	90	2.56	3.27	3.70	4.02	4.26	4.47	4.64	4.91	5.13	5.47	5.73	5.93
	95	3.15	3.88	4.33	4.65	4.91	5.12	5.31	5.60	5.83	6.19	6.47	6.69
	99	4.48	5.27	5.77	6.14	6.43	6.67	6.88	7.21	7.49	7.91	8.23	8.48
12	90	2.52	3.20	3.62	3.92	4.16	4.35	4.51	4.78	4.99	5.31	5.55	5.74
	95	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.40	5.62	5.95	6.21	6.41
	99	4.32	5.05	5.50	5.84	6.10	6.32	6.51	6.81	7.06	7.44	7.73	7.96
16	90	2.47	3.12	3.52	3.80	4.03	4.21	4.36	4.61	4.81	5.11	5.33	5.52
	95	3.00	3.65	4.05	4.33	4.56	4.74	4.90	5.15	5.35	5.66	5.90	6.08
	99	4.13	4.79	5.19	5.49	5.72	5.92	6.08	6.35	6.56	6.90	7.15	7.36
20	90	2.44	3.08	3.46	3.74	3.95	4.12	4.27	4.51	4.70	4.99	5.21	5.38
	95	2.95	3.58	3.96	4.23	4.45	4.62	4.77	5.01	5.20	5.49	5.71	5.89
	99	4.02	4.64	5.02	5.29	5.51	5.69	5.84	6.09	6.29	6.59	6.82	7.01
24	90	2.42	3.05	3.42	3.69	3.90	4.07	4.21	4.45	4.62	4.91	5.12	5.29
	95	2.92	3.53	3.90	4.17	4.37	4.54	4.68	4.92	5.10	5.38	5.59	5.76
	99	3.96	4.55	4.91	5.17	5.37	5.54	5.69	5.92	6.11	6.39	6.61	6.79
30	90	2.40	3.02	3.39	3.65	3.85	4.02	4.16	4.38	4.56	4.83	5.03	5.20
	95	2.89	3.49	3.85	4.10	4.30	4.46	4.60	4.82	5.00	5.27	5.48	5.64
	99	3.89	4.46	4.80	5.05	5.24	5.40	5.54	5.76	5.93	6.20	6.41	6.57
40	90	2.38	2.99	3.35	3.61	3.80	3.96	4.10	4.32	4.50	4.75	4.95	5.11
	95	2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.74	4.90	5.16	5.36	5.51
	99	3.83	4.37	4.70	4.93	5.11	5.27	5.39	5.60	5.76	6.02	6.21	6.36
60	89	2.36	2.96	3.31	3.56	3.76	3.91	4.04	4.25	4.42	4.68	4.86	5.02
	95	2.83	3.40	3.74	3.98	4.16	4.31	4.44	4.65	4.81	5.06	5.24	5.39
	99	3.76	4.29	4.60	4.82	5.00	5.13	5.25	5.45	5.60	5.84	6.02	6.16
120	90	2.34	2.93	3.28	3.52	3.71	3.86	3.99	4.19	4.35	4.60	4.78	4.92
	95	2.80	3.36	3.69	3.92	4.10	4.24	4.36	4.56	4.71	4.95	5.13	5.27
	99	3.70	4.20	4.50	4.71	4.87	5.01	5.12	5.30	5.44	5.66	5.83	5.96
	90	2.33	2.90	3.24	3.48	3.66	3.81	3.93	4.13	4.29	4.52	4.69	4.83
	95	2.77	3.31	3.63	3.86	4.03	4.17	4.29	4.47	4.62	4.85	5.01	5.14
	99	3.64	4.12	4.40	4.60	4.76	4.88	4.99	5.16	5.29	5.49	5.65	5.77

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