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Chapter B1

LOW-FLOW INVESTIGATIONS

By H. C. Riggs

Book 4

HYDROLOGIC ANALYSIS AND INTERPRETATION

UNITED STATES DEPARTMENT OF THE INTERIOR

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PREFACE

The series of manuals on techniques describes procedures for planning and executing specialized work in water-resources investigations. The material is grouped under major subject headings called books and further subdivided into sections and chapters; Section B of Book 4 is on surface water.

The unit of publication, the chapter, is limited to a narrow field of subject matter. This format permits flexibility in revision and publication as the need arises.

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LOW-FLOW INVESTIGATIONS

H. C. Riggs

Abstract

This manual describes methods of defining the low-flow characteristics of streams, shows how certain basin characteristics influence the mean and variability of annual low flows, and recommends procedures for data collection, analysis, and reporting.

Introduction

The adequacy of streamflow to supply requirements for disposal of liquid wastes, municipal or industrial supplies, supplemental irrigation, and maintenance of suitable conditions for fish is commonly evaluated in terms of low-flow characteristics. Certain of these low-flow characteristics also are useful as parameters in regional draft-storage studies, as the basis for forecasting seasonal low flows, and as indicators of the amount of ground-water flow to the stream. In some States the legal index for pollution control is tied to a low-flow characteristic.

Low-flow characteristics at a gaging station may be described by frequency curves of annual or seasonal minimum flows, by duration curves, and by base-flow recession curves. Estimates of low-flow characteristics at ungaged sites are generally quite inaccurate because low flows are highly dependent on the lithology and structure of the rock formations and on the amount of evapotranspiration, neither of which have been adequately described by indices except in a few basins. However, acceptable estimates of low-flow characteristics at an ungaged site may be made if a few discharge measurements of base flow during one or more low-flow seasons are available.

This manual describes the procedures for defining and evaluating low-flow frequency curves for a gaging station, describes methods for estimating low-flow characteristics for sites where little or no discharge information is available, and suggests methods for designing a data-collection program and for reporting results. The subject of recession curves is not covered.

Defining Low-Flow Frequency Characteristics At Gaged Sites

Frequency curves from gaging station records

Annual low flows can be extracted by hand from records of daily discharge at gaging stations but are most efficiently obtained through a digital-computer program. Many gaging station records have already been processed to give, for each year beginning April 1, the lowest mean discharge for 1, 3, 7, 14, 30, 60, 90, 120, and 183 days. A low-flow frequency curve can be prepared from the list of annual values for each period of days, graphically, as follows:

1. Array the values in order of magnitude and assign order numbers beginning with the smallest as number 1 (this step is done by the computer).
2. Compute the recurrence interval (R.I.) of each value by the formula R.I. = $(n+1)/m$, where n is the number of years (values) and m is the order number.
3. Plot each value against its computed recurrence interval on form 9–179b (logarithmic ordinate scale).
4. Draw a smooth curve which properly interprets the plotted points.

An example of computation of plotting position is given in Techniques of Water-Resources Investigations, Book 4, Chapter A2, "Fre-

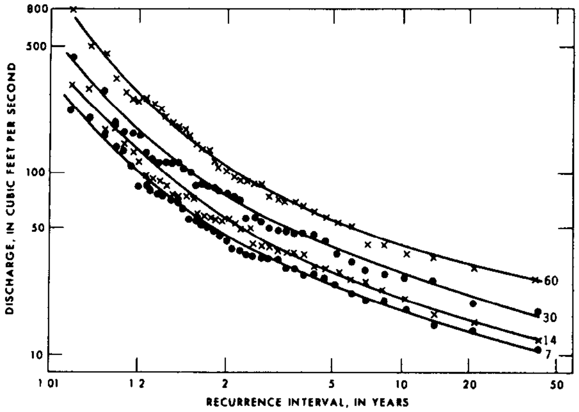


Figure 1.—Typical frequency curves of annual lowest mean discharge for indicated numbers of consecutive days, Spoon River at Seville, Ill.

quency curves" (Riggs, 1968, table 3). Typical low-flow frequency curves for 7-, 14-, 30-, and 60-day periods are shown in figure 1.

Low-flow frequency curves may be fitted mathematically by assuming a theoretical frequency distribution of the data. The following theoretical distributions have been used for low flows: Gumbel's limited distribution of the smallest value, the log normal, the three-parameter log normal, Pearson Type III, and Pearson Type IV. Matalas (1963) found the Gumbel and Pearson Type III distributions about equal in their ability to match the graphical interpretation. O'Conner (1964) indicated a preference for the log normal but suggested that a graphical curve would be adequate.

Fitting a theoretical three-parameter frequency curve is done by solving the equation

$$X = \overline{X} + KS,$$

where X is a point on the frequency curve, $\overline{X}$ and S are the mean and standard deviation, respectively, of the data, and K is the frequency factor taken from a table, for selected recurrence intervals, at the value of skew computed from the data. For an example of a frequency-factor table, see table 1 of Riggs (1968). Frequency factors generally are tabulated for use in defining a frequency curve of events greater than the one indicated, but a low-flow frequency curve expresses the recurrence interval of events less than the one indicated. However, the fitting process for low flows is exactly the same as for high flows except that one must change the recurrence intervals in the frequency-factor table to the corresponding recurrence intervals for events less than. If we let

$$P[X > (\overline{X} + KS)] = a$$

then

$$P[X < (\overline{X} + KS)] = 1 - a.$$

Conversion is made as follows:

R.I. $(X >)$	$P(X >) =$ $1/\text{R.I.}\ (X >)$	$P(X <) =$ $1 - P(X >)$	R.I. $(X <) =$ $1/P(X <)$
100	0.01	0.99	1.01
20	.05	.95	1.05
10	.10	.90	1.11
3.33	.30	.70	1.43
2.00	.50	.50	2.00
1.43	.70	.30	3.33
1.11	.90	.10	10
1.05	.95	.05	20
1.01	.99	.01	100

Low flows computed using tabulated frequency factors should be related to the fourth column recurrence intervals.

Most low-flow frequency curves for streams that do not go dry are smooth curves that are concave upward on the log-Gumbel plot. The curves in figure 1 are typical. Although the shape of a low-flow frequency curve is rather insensitive to some basin characteristics (a regulated stream may have a typical low-flow frequency curve), substantial differences from the typical shape are found.

The probability distribution of annual low flows at a particular site on a stream depends on the distribution of precipitation over the basin in time and space; on the temperature regimen, which may permit the storage of water as snow for considerable periods and which also influences the evapotranspiration rate; and on the soil and geologic characteristics, which govern the recharge and discharge rates in the basin. In addition, at a given site some of the annual low flows may be derived entirely from ground-water inflow to the stream, whereas others may include some flow from reduction of upstream storage during those years when frequent rains occur during the low-water season. Furthermore, the base flow of a stream may be derived from several aquifers, not all of which contribute at all times. Consequently, not all low-frequency curves will have a typical concave upward shape, nor should the high annual minimum flows at a site necessarily be considered as belonging to the same population as the smaller ones.

The unusual shape of the frequency curve for Suwannee River (fig. 2) reflects geologic differences within the basin. The annual minimum flow of this stream consists of two principal parts: (1) a flow ranging from several hundred cubic feet per second to zero from the headwaters in Okefenokee Swamp and (2) an inflow of 5–10 cubic feet per second from a limestone aquifer in a lower reach of the river.

Figure 3 shows frequency curves for two streams affected by diversions. The shape of the Uwharrie River curve is, at least in part, due to a diversion for a city supply. A heavy draft by evapotranspiration (of a more or less constant amount, independent of discharge) may also help to produce the convex shape. The curve for Haw River appears normal except for the extremely low value for 1954. It would be very unusual to find a natural basin which would produce such an anomalous value. In fact, the 1954 annual minimum was caused by emergency withdrawal of water from the stream for municipal supply. None of the flows for other years were so affected.

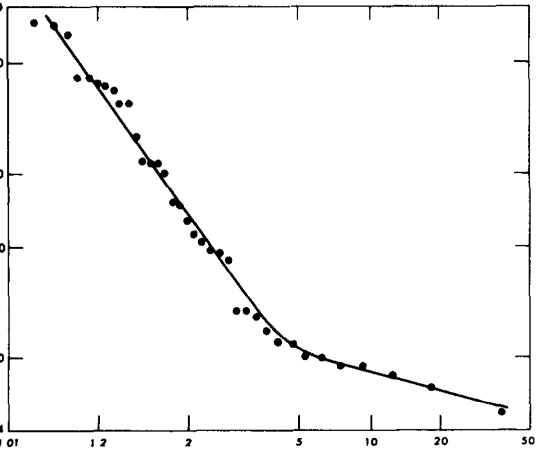


Figure 2.—Low-flow frequency curve (of annual minimum 7-day means) for Suwannee River at White Springs, Fla.

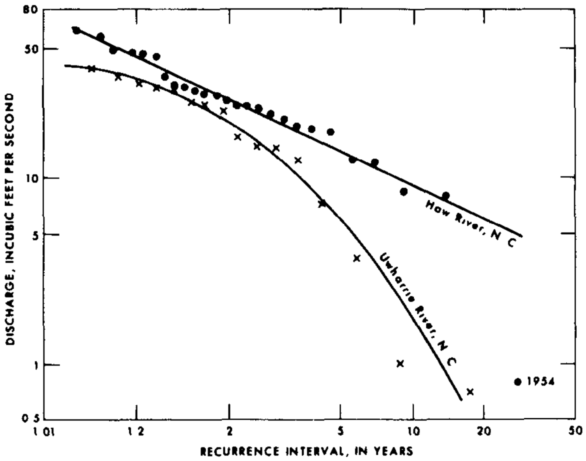


Figure 3.—Frequency curves of annual minimum 7-day means showing effect of diversions.

The frequency curves of figures 1–3 have been derived graphically, but a computer program is available which will select the annual minimum flows; compute the mean, standard deviation, and coefficient of skew; plot the fitted log Pearson type III probability distribution; and plot the individual annual minimum flows versus recurrence interval using the plotting position formula:

$$\text{Recurrence interval} = (n+1)/m$$

where n is the number of items (years) and m is the order number when the annual events are arrayed according to size. Computer fitting saves much time and produces adequate fits to many sets of low-flow data. A typical computer output is shown in figure 4. Note that the probability scale is normal, not Gumbel, on the computer plot and that the recurrence interval scale should be changed as shown (the tabulated recurrence intervals are correct). However, certain low-flow frequency curves cannot be adequately fitted by a three-parameter distribution. For example, figure 5 shows the difference between the graphical and the mathematically-fitted frequency curves for the Suwannee River data plotted in figure 2.

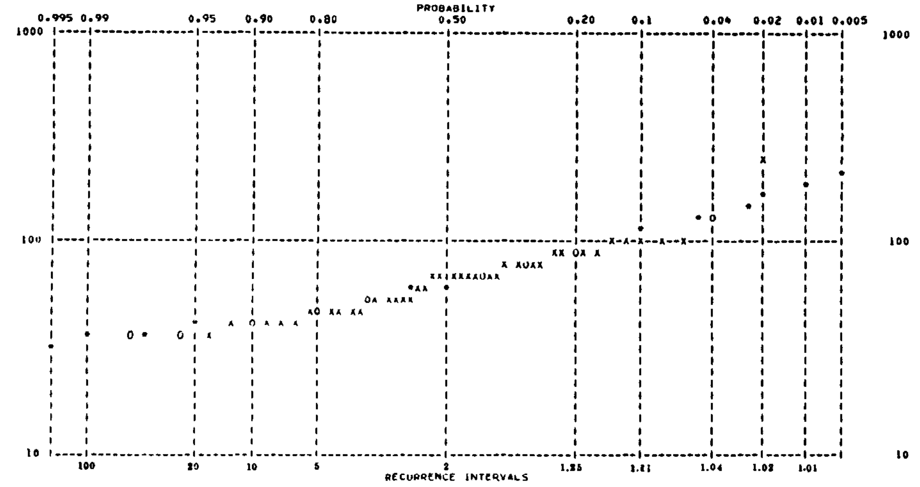
The graphical curve should be considered the basic frequency curve for annual low flows. (See

```
 RED RIVER NR. ADAMS. TENN.                                    N =     48                    STATION  03435500
1922-1969.  12 MON PERIOD ENDING MARCH 31
  7  DAY LOW FLOW

INPUT DATA (ZERO VALUES OMITTED)
      75.000      72.600     102.000      49.100      38.300      71.900      80.300     131.000      65.600      35.700
      54.900      93.300      82.300     101.000      91.900      52.100      66.100      99.000      51.300      45.300
      48.600      87.000      49.000      67.700      63.600      42.900      48.300      37.400     106.000     237.000
      87.100      49.400      39.300      52.000      52.700      41.900      61.700      87.600      78.000      64.300
      66.400      77.400      42.700      41.700      55.300      71.900      95.300      69.300
MEAN =        70.421
VARIANCE =   =    1079.221
STANDARD DEVIATION =        32.852
SKEWNESS =        2.942
STANDARD ERROR OF SKEWNESS =        0.343
SERIAL CORRELATION COEFFICIENT =        0.264
COEFFICIENT OF VARIATION =        0.467
MEAN LOGS =        1.815
VARIANCE LOGS =        0.026
STANDARD DEVIATION LOGS =        0.163
SKEWNESS LOGS =        0.791
STANDARD ERROR OF SKEWNESS LOGS =        0.343
SERIAL CORRELATION COEFFICIENT LOGS =        0.315
COEFFICIENT OF VARIATION LOGS =        0.090
NON EXCEED PROB    RECURRENCE INTERVAL    DISCHARGE

     0.0100             100.00             34.022
     0.0500              20.00             38.751
     0.1000              10.00             42.147
     0.2000               5.00             47.356
     0.5000               2.00             62.139
     0.8000               1.25             87.405
     0.9000               1.11            107.604
     0.9600               1.04            137.487
     0.9800               1.02            163.209
     0.9900               1.01            192.155
     0.9950               1.01            224.825
```

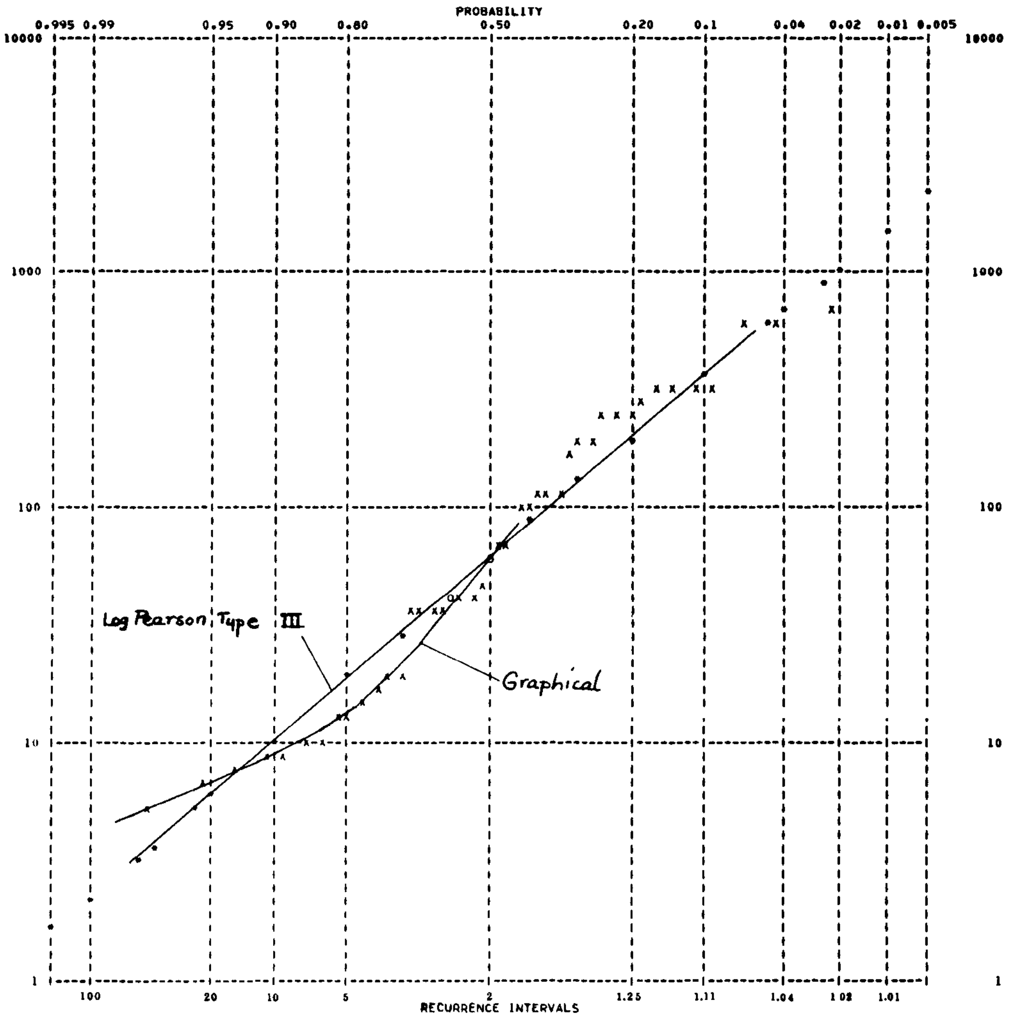
```
 RED RIVER NR. ADAMS. TENN.
                                                               N =     48                    STATION  03435500
1922-1969.  12 MON PERIOD ENDING MARCH 31
  7  DAY LOW FLOW
```



```
THE FOLLOWING SYMBOLS MAY APPEAR IN THE PLOT
X - AN INPUT DATA VALUE
* - A CALCULATED VALUE
O - A CALCULATED VALUE AND ONE DATA VALUE AT SAME POSITION
2 - TWO INPUT DATA VALUES PLOTTED AT SAME POSITION
3 - THREE INPUT DATA VALUES PLOTTED AT SAME POSITION
A - A CALCULATED VALUE AND TWO DATA VALUES AT SAME POSITION
B - A CALCULATED VALUE AND THREE DATA VALUES AT SAME POSITION
```

Figure 4.—Partial computer output for Red River near Adams, Tenn. (Discharge in cubic feet per second and recurrence interval in years).

```
SUWANNEE RIVER AT WHITE SPRINGS, FLORIDA                    N =     43                    STATION  02315500
1908-1969,  12 MON PERIOD ENDING MARCH 31
  7  DAY LOW FLOW
```



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THE FOLLOWING SYMBOLS MAY APPEAR IN THE PLOT
x - AN INPUT DATA VALUE
* - A CALCULATED VALUE
O - A CALCULATED VALUE AND ONE DATA VALUE AT SAME POSITION
2 - TWO INPUT DATA VALUES PLOTTED AT SAME POSITION
3 - THREE INPUT DATA VALUES PLOTTED AT SAME POSITION
A - A CALCULATED VALUE AND TWO DATA VALUES AT SAME POSITION
B - A CALCULATED VALUE AND THREE DATA VALUES AT SAME POSITION
```

Figure 5.—Computer plot of 7-day annual minimums for Suwannee River at White Springs, Fla. Frequency curves are drawn on the plot. (Discharge in cubic feet per second and recurrence interval in years).

Riggs, 1971.) But it is recommended that the computer plot be obtained and the log Pearson Type III curve used if it is an adequate fit. If not (as in fig. 5), a graphical interpretation should be made on the computer printout. In evaluating the adequacy of fit and (or) in graphical fitting, one should concentrate on fitting the smaller low flows, for the larger ones may not belong to the same population. From the preceding discussion and the examples given in figures 2–5, it should be apparent that a purely statistical analysis of annual minimum flows may give misleading information.

Extrapolation of a low-flow frequency curve is particularly dangerous if made without knowledge of the basin characteristics and without knowing whether the low flows are affected by the works of man. An observation of zero flow or local information that a stream has or has not been dry is a much firmer basis for extrapolation than the extension of a graphical curve or a mathematically-fitted distribution. However, statements of local residents about streamflow tend to be unreliable, particularly in humid regions.

The reliability of a low-flow frequency curve based on natural flows is closely related to the length of record used. A period of record representative of long-term flow characteristics is desirable, but unfortunately there is no way of judging the representativeness of the available sample. However, the inclusion in the record of a period covering a substantial drought makes one more confident that the sample is reasonably representative.

Plans for utilization of low flows of a stream commonly are based on the low flows that have recurrence intervals of not more than 20 years. To define the 20-year recurrence-interval annual minimum flow adequately, more than 20 years of record are considered desirable. Estimates of the 20-year low flow from a record of less than 20 years (say 10) can be made in two ways. The simplest way is to prepare a frequency curve based on the 10-year record and extend it graphically to 20 years. The second method, a more rigorous one, involves estimating low flows for additional years by regression on a longer record and using both estimates and flows of record to define the frequency curve. Regression estimates should be used in preparing the frequency curve only if the regression from which they were obtained has a correlation coefficient greater than 0.8. Otherwise their use probably will result in a less reliable frequency curve than the curve based only on flows of record (Fiering, 1963).

The restriction that any regression used to estimate annual low flows have a correlation coefficient greater than 0.8 generally limits the use of the second method to regions where the low-flow characteristics of nearby streams are very similar. Furthermore, the inclusion of regression estimates with the flows of record will significantly change the frequency curve at the lower end only (1) if the regression estimates are for a drought period more severe than that experienced in the period of record or (2) if the extended record is much longer than the station record. Therefore an examination of flows for the period to be covered by the extended record will usually indicate whether use of estimated values would change the frequency curve appreciably. One practical limitation of extending annual low-flow records at a site is the lack of a suitable longer record closely correlated with the record to be extended. If my search is indicative, suitable examples are few. But assuming a suitable longer record is available, how much practical, as distinguished from statistical, improvement can be obtained? The following example in which parts of the two records shown in table 1 are analyzed will partly answer the question. Suppose we had a record only for 1932–42 on Shoal Creek and wanted to extend that record to obtain an improved frequency curve. The relation of Shoal Creek to Buffalo River annual minimum flows for that period is shown in figure 6; the correlation coefficient is about 0.94. Estimates of Shoal Creek annual minimum flows for years 1926–31 and 1943–59, based on the Buffalo River record and on this relation, are shown in table 1. These estimates were used with the 1932–42 record to define one of the frequency curves of figure 7. Also shown in figure 7 are frequency curves based on records for 1932–42 and 1926–59. The estimates of the annual minimum flow at 20-year recurrence interval are

1. 62 cfs from frequency curve based on 1932–42 record,

2. 69 cfs from frequency curve based on 1932–42 record plus estimates for 1926–31 and 1943–59,
3. 62 cfs from frequency curve based on 1926–59 record.

In this example the use of records extended by regression did not produce as good an estimate of the 20-year event as that obtained by graphical extension of the frequency curve based on 11 years of record. Whether use of regression will generally improve the estimate of the lower end of a frequency curve is not known. The analysis by Fiering (1963) showed that if the correlation coefficient was greater than about 0.8, use of regression estimates would improve the estimate of variance; however, this result presumably could be accomplished without improving the position of the lower end of the frequency curve. Fiering had to work with a theoretical distribution, but low-flow frequency curves often do not resemble any theoretical distribution.

Table 1.—Annual minimum 7-day average flows, two Tennessee streams, and estimates of one for part of period based on the relation of figure 6

Year	Recorded flows (cfs)		Estimated flow of Shoal Creek using 1932–42 record only
	Shoal Creek	Buffalo River	
1926	99	99	75
1927	90	145	112
1928	116	130	100
1929	142	159	125
1930	99	110	83
1931	63	96	72
1932	128	168	
1933	126	149	
1934	93	125	
1935	83	122	
1936	83	115	
1937	111	146	
1938	112	155	
1939	127	149	
1940	97	124	
1941	71	93	
1942	84	102	
1943	56	97	72
1944	108	131	100
1945	123	170	134
1946	120	182	143
1947	116	146	114
1948	98	120	92
1949	145	200	160
1950	202	226	180
1951	133	208	165
1952	111	164	130
1953	101	146	114
1954	72	112	86
1955	80	112	86
1956	80	120	92
1957	97	156	122
1958	125	166	132
1959	124	165	132

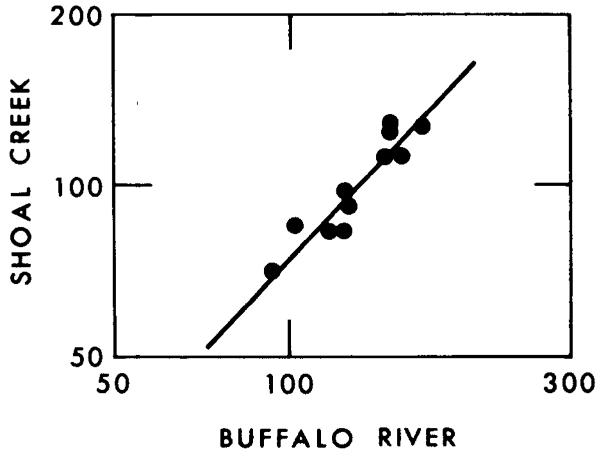


Figure 6.—Relation of annual minimum flows (in cubic feet per second), 1932–42. (Data from table 1.)

Referring to figure 6, it is possible that a different sample of concurrent annual minimum flows might define a relation of appreciably different slope and yet have a correlation greater than 0.8. The slope of the relation line is important because a downward extrapolation is usually required, and the estimates made from this extrapolation need to be accurate if they are to define a reasonable extension of the frequency curve. Thus it appears that extension of a low-flow frequency curve by regression is rarely justified.

Recommendations for use of short records to estimate annual minimum flows at 20-year recurrence interval are

1. Define the frequency curve from the recorded events if the record is 10 years or longer. Extend the defined frequency curve graphically to 20 years.
2. If the record is less than 10 years, estimate the 20-year event by the method recommended in the section, "Frequency curves from short records."

So far only annual minimum flows have been considered because the majority of streams in the United States reach their annual minimums in late summer or fall; the year beginning April 1, commonly used for low flows, encompasses this period. But annual minimums of Florida streams usually occur in the late spring although they may occasionally occur in late fall. Annual minimums of some northern

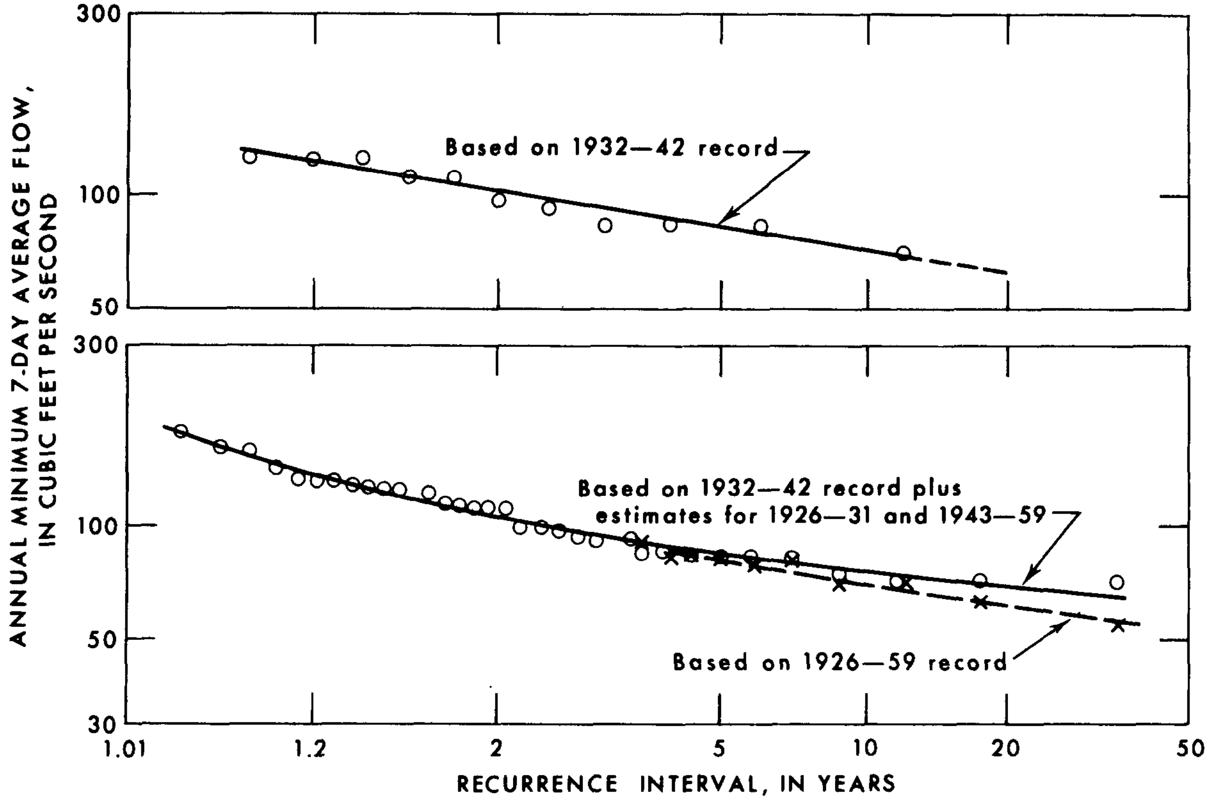


Figure 7.—Low-flow frequency curves, Shoal Creek, Tenn.

streams may occur either in late summer-fall or during winter. A frequency curve based on flows from different seasons applies to no one season but to the entire year. Such a frequency curve would be suitable for a problem in which water demand was constant throughout the year, but water demand is usually greater in summer or fall than in winter. For that condition, the appropriate low-flow frequency curve should be based on one value from each summer-fall season of record. Seasonal annual minimum flows can be abstracted from daily discharge records by computer.

In some regions minimum-flow frequency curves by calendar months may be justified. For example in the Northwest, west of the Cascades, need for supplemental irrigation occasionally arises during July and August, but the annual minimum flows usually occur later in the season. Here a frequency curve of July–August minimum flows would provide information needed to allocate water for irrigation, and a frequency curve for September–October minimum flows would describe the most critical period.

Frequency characteristics from short records

Less than 10 annual events are usually inadequate to define a relation of better than 0.8 correlation with the concurrent part of a longer record. And such a record is too short to define a frequency curve worthy of extension. However, the relation between concurrent events (either annual minimums, daily mean base flows, or discharge measurements of base flows) can be used along with a frequency curve for the gaged stream site that has the longer record, to approximate the 20-year annual minimum flow (or the 10 year) without preparing a frequency curve for the short-record site. In this approach we are no longer bound by the criterion that the correlation coefficient, *r*, be more than 0.8, for we are not attempting to improve an existing frequency curve; we develop no frequency curve, but only seek to estimate one or two points on a frequency curve. Thus any estimate, logically arrived at, should be better than none, although it is obvious that the better the relation between concurrent flows, the better the estimates of flow for any recurrence interval. The following examples detail the procedure.

Suppose streamflow records were available only for 1932–36 on Shoal Creek (table 1). A frequency curve based on these five points would not provide a reliable estimate of the 20-year event, but a plot of the annual minimum flows against the corresponding ones for Buffalo River (the left curve of fig. 8) defines a line whose slope can be extended downward with some confidence. The 20-year annual minimum flow for Buffalo (95 cfs, based on 34-years of record), when transferred through the left relation of figure 8, gives an estimate of the 20-year annual minimum flow for Shoal Creek of about 65 cfs. This estimate is in close agreement with the value of 62 cfs determined from a 34-year record; in fact it is much closer than one should expect. A minor variation in placement of the left relation line of figure 8, however, could change the estimate by 10 percent. A different 5 years of concurrent record might define a considerably different line, as shown by the center graph in figure 8, based on records for 1944–48. It is apparent that if a record were available only for 1944–48, the estimate of the 20-year annual minimum flow from the relation based on the five annual minimums would be greatly in error. However, the 1944–48 record includes daily mean base flows which can be used to define a better relation. The concurrent base flows of Shoal and Buffalo, given in table 2 and plotted in figure 8, were selected to cover a considerable range in discharge so as to define the slope of the relation more accurately. It is of course possible that a straight-line downward extension of the relation line defined by medium to high base flows is not correct, but in the ab-

Table 2.—Selected daily mean base flows, 1944–48, for Shoal Creek and Buffalo River

Date	Shoal Creek (cfs)	Buffalo River (cfs)
9-16-44	141	221
7-20-44	115	141
5-25-44	271	314
8-15-45	173	252
9-7-45	124	166
7-11-45	180	252
10-15-45	120	208
7-29-46	132	205
7-29-47	161	213
9-5-47	143	172
7-6-48	112	151
9-8-46	120	179
8-15-46	154	220
6-5-46	223	330
6-10-47	283	366
5-15-48	274	298

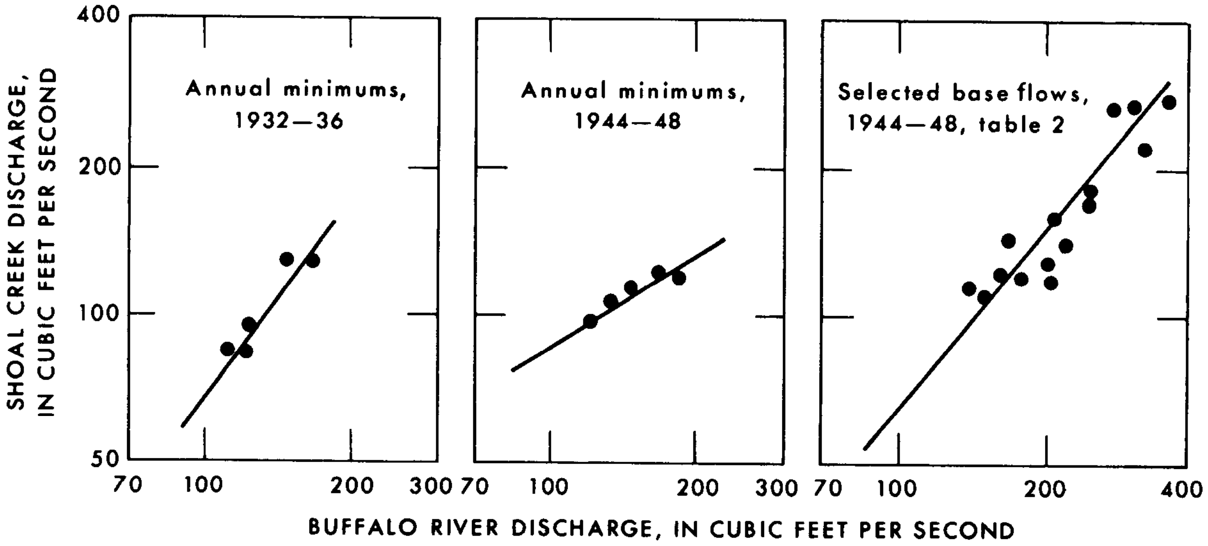


Figure 8.—Plots of concurrent flows for Shoal Creek and Buffalo River, Tenn. (Data from tables 1 and 2).

sence of information to the contrary it seems to be the best interpretation. In figure 8 the position of the relation line defined by concurrent daily mean base flows selected during the period 1944–48 is much more nearly correct than the position of the relation line based on five annual minimum flows for the same period. One must conclude, therefore, that the daily record during periods of base-flow recession is more useful than a few annual minimum flows for estimating the low-flow characteristics.

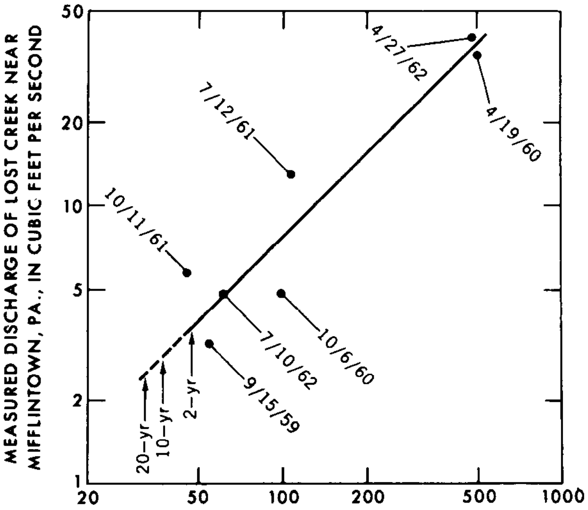


Figure 9.—Relation of base-flow measurements of Lost Creek to concurrent daily mean flows of Penn Creek (gaged).

Frequency characteristics from discharge measurements

If, as concluded in the previous paragraphs, concurrent base flows provide the best definition of a relation, we need only to obtain base-flow discharge measurements rather than a continuous flow record. Thus, low-flow partial-record stations, at which one or more discharge measurements of base flow are made each year, should provide nearly as much low-flow information as a complete flow record of a few years length. The operator of a low-flow partial-record station may ask how long the station should be operated. Eight or ten measurements made on different recessions and in more than one year should provide adequate data to define a relation with concurrent flows at a long-record gaging station. Ordinarily those relations having close to unit slope will be better defined and produce better estimates of low-flow characteristics than relations having other slopes, because a unit-slope relation indicates that the two streams have similar flow characteristics. Relations between similar streams can be defined with fewer measurements than relations between dissimilar ones. Only a few measurements are needed if some of them were made during a significant drought. On the other hand there is a practical limit to the number of measurements justified at any site. For example, the seven measurements plotted in figure 9 define a relation line that could be extended to define the 20-year low flow for Lost Creek; it would give about $2\frac{1}{2}$ cfs, although that low flow could be anywhere between $1\frac{1}{2}$ and 4 cfs. Even though additional measurements would improve the definition of the relation line slightly, the scatter of points about the line cannot be reduced by additional measurements, for that scatter is due to dissimilar flow characteristics of the two streams as well as to differences in antecedent rainfall. Because further measurements will probably not reduce the dispersion, the opportunity for improving the location of the relation line appears small; thus additional measurements probably are not warranted.

A relation between concurrent base flows (such as that shown in figure 9) should not be used to define an entire frequency curve at the partial-record site; to do so would imply a greater accuracy than is warranted. Ordinarily only the discharges at a couple of recurrence intervals would be estimated.

Frequency Characteristics At Ungaged Sites

That low-flow characteristics of streams in adjacent basins may be greatly different may not be discernable from known basin characteristics. The principal terrestrial influence on low flows is geology (the lithology and structure of the rock formations), and the principal meteorological influence is precipitation. So far it has not been possible adequately to describe the effect of geology on low flow by an index, although the effect may be clearly shown for selected streams. Precipitation can be described where known, but evapotranspiration loss, which may be a significant factor, is not easily described by an index. Thus estimation of low flow characteristics at sites without discharge measurements has met with only limited success. Exceptions are on streams in a region homogeneous with respect to geology, topography, and climate in which it should be possible to define a range of flow per square mile for a given recurrence interval. A following section on "Regionalization" covers this subject in more detail.

Geologic homogeneity with respect to base flow usually cannot be identified from field or geologic-map examination. But homogeneity may be indicated by plots of concurrent base flows at gaging stations. For example, figure 10 shows relations of concurrent base flows at four different dates to drainage area. The points rather closely define the curves, indicating little variation due to basin characteristics other than drainage area. To determine whether the indicated homogeneity extends to the smaller basins, four series of discharge measurements were made on small streams in the Hazel River subbasin. The results, plotted against drainage area in figure 11, are somewhat more variable than those of figure 10 but indicate that the curves of figure 10 may be extrapolated to small drainage areas with moderate confidence.

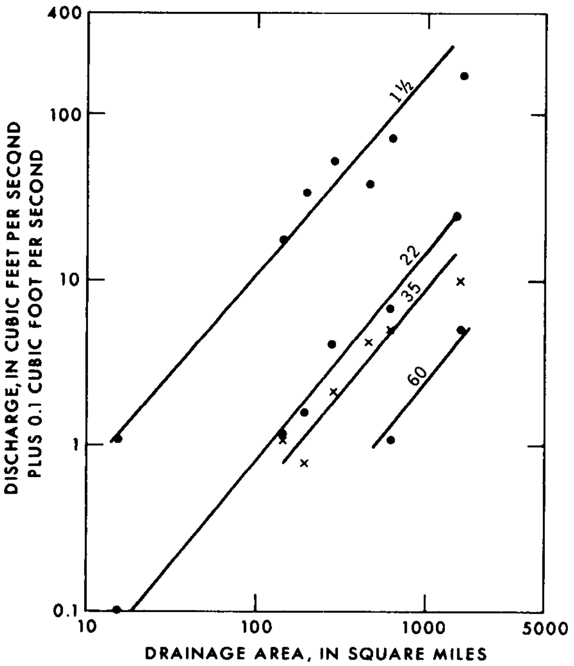


Figure 10.—Relations of concurrent discharges at gaging stations to drainage area, Rappahannock River basin, Virginia. Parameter is recurrence interval, in years, of flows less than indicated values.

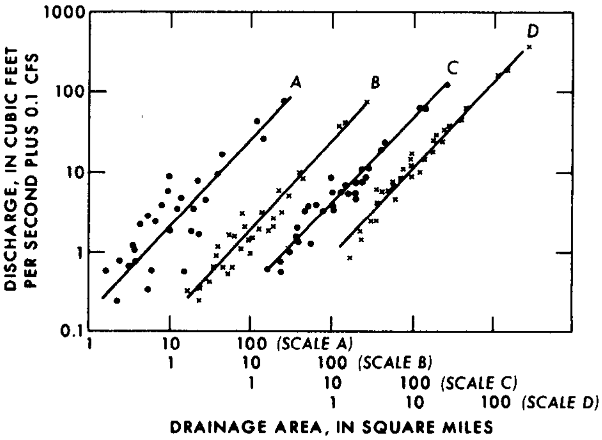


Figure 11.—Relation of measured base flows of small streams to drainage area in part of Rappahannock River basin.

However, the four sets of measurements in figure 11 would be assigned recurrence intervals of less than 2 years on the basis of their positions in the family of curves of figure 10. At large recurrence intervals the effects of differences in geology and evapotranspiration become more significant, as shown by the plot of measurements made August 23–24, 1966 (fig. 12). Based on gaging station records, the recurrence interval applicable to these measurements is about 15 years, but the wide scatter of points for drainage areas less than 10 square miles reduces our confidence in that conclusion. Nevertheless one set of measurements during a significant drought period would give considerable information for the measured sites. Lack of homogeneity with respect to area, as shown in figure 12, does not permit estimates at ungaged sites beyond the general one that the 15-year event will be near zero for drainage areas less than about 10 square miles in that basin.

Seepage runs

Seepage runs sometimes point out significant channel gains or losses and thus help in interpretation of other data. A seepage run consists

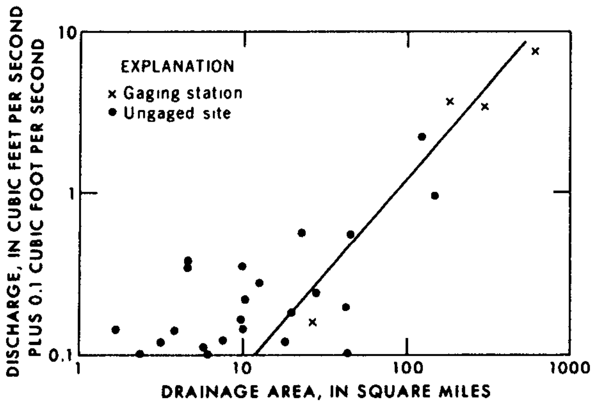
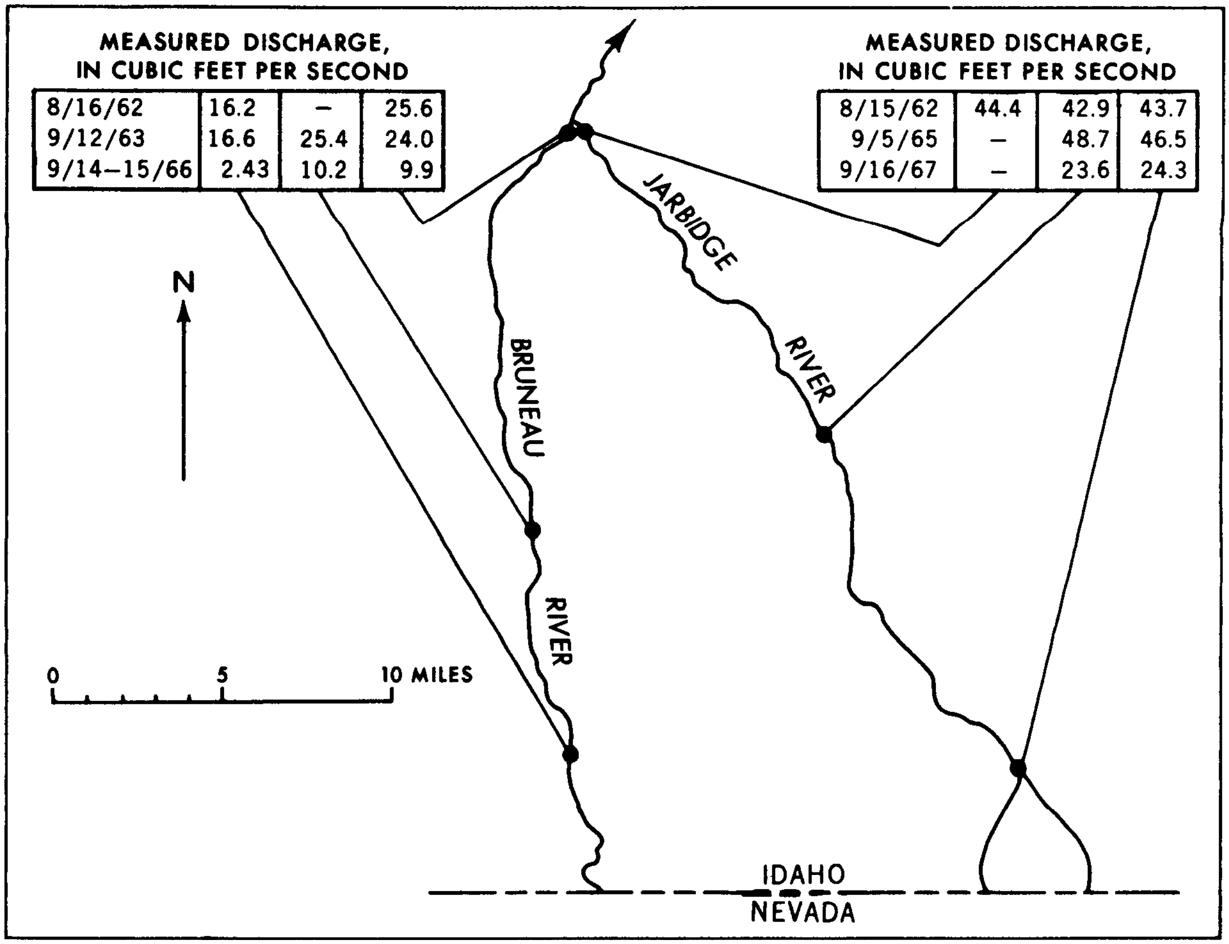


Figure 12.—Discharges of August 23–24, 1966 (about 15-year recurrence interval) in Rappahannock River basin.

of measuring discharge at intervals along a channel reach during a period of base flow. Measurements of specific conductance and temperature made concurrently add to the usefulness of the flow data. Figure 13 shows measured discharges on two channels in an arid region; inflow from tributaries in these reaches is usually negligible at any time of the year. These seepage runs indicate an accretion of 7–9 cfs from ground water between the two upstream measuring sites on Bruneau River. This information will permit a better definition of the base-flow relation of the Bruneau and Jarbidge Rivers (fig. 14) than would be obtained only from the concurrent discharges, by indicating that Bruneau River discharge would be about 8 cfs when Jarbidge River discharge is zero. Small differences in measured flow at different sites on Jarbidge River may be due to errors inherent in discharge measurements.

Seepage runs may give different results at different times. For example, during the dry summer of 1963 two seepage runs were made on


MEASURED DISCHARGE, IN CUBIC FEET PER SECOND

8/16/62	16.2	–	25.6
9/12/63	16.6	25.4	24.0
9/14–15/66	2.43	10.2	9.9

MEASURED DISCHARGE, IN CUBIC FEET PER SECOND

8/15/62	44.4	42.9	43.7
9/5/65	–	48.7	46.5
9/16/67	–	23.6	24.3

Figure 13.—Map of part of Bruneau River basin, Idaho, showing results of seepage runs.

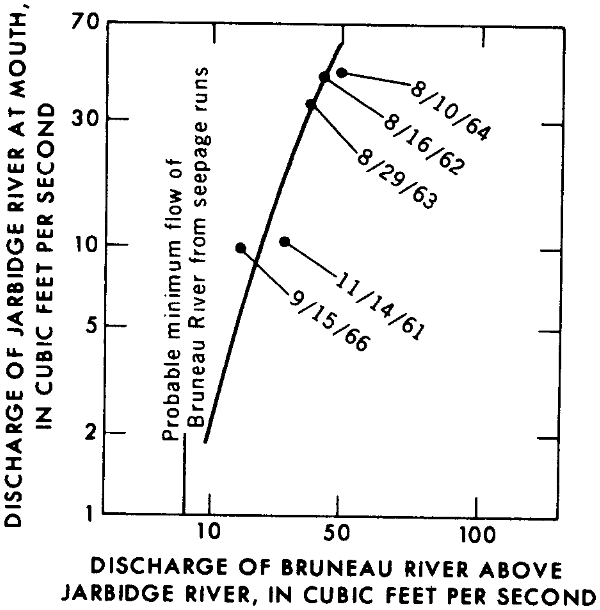


Figure 14.—Plot showing use of information from seepage runs (fig. 13) in defining a base-flow relation.

Thornton River, Va., from Rush River to Hazel River. Throughout most of that reach the channel is wide and shallow; parts of the reach are bordered by a low, heavily wooded flood plain. Plots of the seepage run data (fig. 15) show a gain in flow through most subreaches in early August but a loss in some subreaches in September. The ground-water inflow to the channel was greater than the evapotranspiration loss in early August but was less than evapotranspiration loss by the middle of September. It seems unlikely that evapotranspiration was greater in September than in August, so ground-water inflow must have decreased substantially during the dry summer.

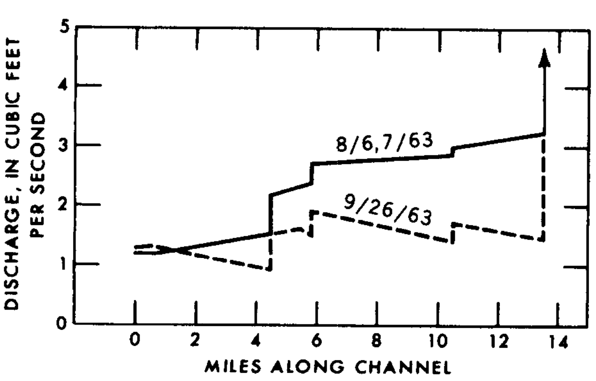


Figure 15.—Two seepage runs on Thornton River, Va., from mouth of Rush River to Hazel River.

A seepage run also provides information at unmeasured sites on the channel and, to some extent, on tributary flows.

Regionalization

Although estimates of low-flow characteristics from basin characteristics generally are of low accuracy, the demand for such estimates on short notice justifies development of regional relations where conditions are favorable. For example, if all significant basin characteristics except drainage area in a region have extremely limited ranges, one would expect a good relation between a low-flow characteristic and drainage area, such as that shown in figure 16, which is a plot of 7-day 20-year minimum flows against drainage area for Snohomish River basin, Washington (data from Collings, 1971). With the exception of two points, the agreement is good.

Attempts have been made to regionalize low-flow characteristics by multiple regression on several basin characteristics, including geologic indexes. Some of these regressions showed the geologic parameters to be statistically significant, but the standard errors of these regres-

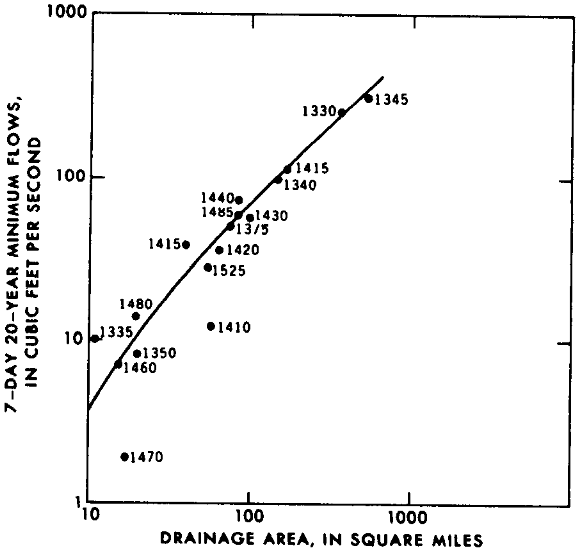


Figure 16.—Relation of a low-flow characteristic to drainage area, Snohomish River basin, Washington.

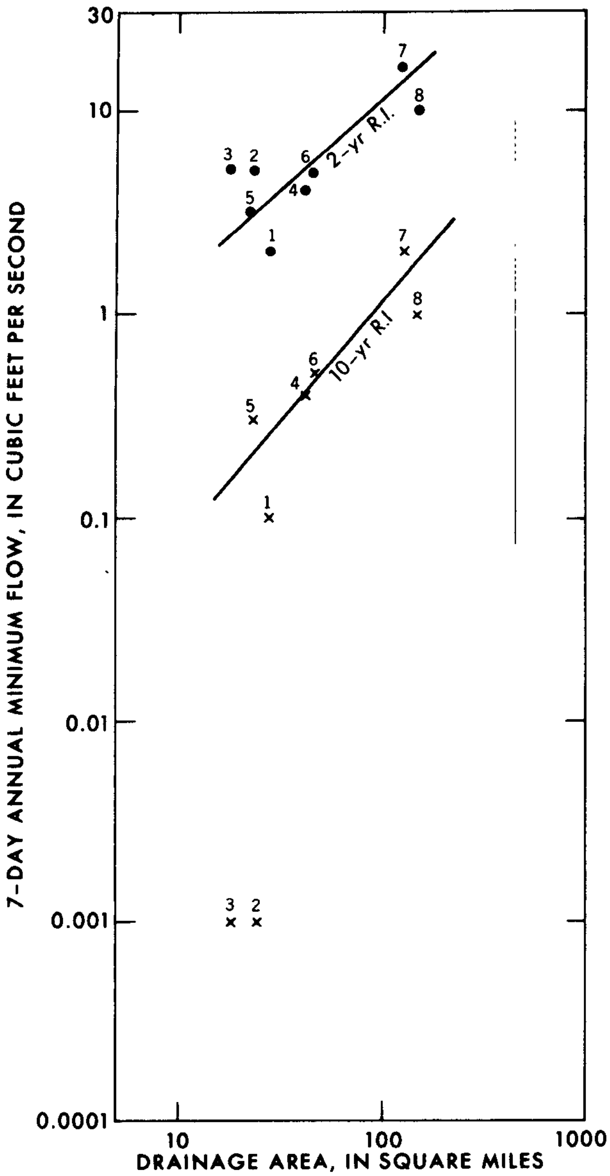


Figure 17.—Data for Hazel River basin, Virginia, showing inconsistent behavior of streams 2 and 3.

sions were too large to justify application of the relations to ungaged sites. One of the better regressions, which was derived for Connecticut, related 7-day 10-year low flow to drainage area, mean basin elevation, and percentage of basin covered by stratified drift; it has a standard error of 68 percent (Thomas and Cervione, 1970).

The principal roadblock to regionalization of low-flow characteristics is our inability to describe quantitatively the effects of various geological formations on low flows—even where detailed geologic maps are available. Furthermore, one index of a geologic formation may not be adequate to describe the range of low-flow characteristics. For example, in figure 17, note that points 2 and 3 plot high relative to the others at the 2-year recurrence interval but that their values are virtually zero at the 10-year recurrence interval. The reason for the behavior of these two streams is not apparent from the geologic map of Virginia, but inspection of the basins indicates the presence of some alluvial material along the lower reaches. This material presumably transmits water to the channels in normal years and extracts water from them during substantial droughts. This loss and gain is in agreement with the results of the two seepage runs shown in figure 15. The difficulty of describing this hydrologic effect quantitatively in terms of the extent of the alluvium is apparent.

Special Applications

Regulated streams

If the pattern of regulation of a stream has been consistent for several years and is expected to continue so, low-flow frequency curves based on the record for those years may be useful. Whether these curves should be annual, seasonal, or monthly would depend on the character of the regulation. Such curves can only be made for sites having a continuous record; occasional discharge measurements of an ungaged regulated stream cannot be used to estimate the flow characteristics of that stream.

The data and analysis necessary to define the low-flow characteristics of a stream that does not have a long discharge record depend on the natural characteristics of the stream and on the regulation. It may be necessary to route historical flows through a model of the present system in order to define the flow characteristics. And the pattern of regulation may be such that a description other than conventional low-flow frequency curves is more suitable.

In streams regulated to maintain a legal minimum flow, that flow may be used after it has been verified that the legal requirements are being met.

Use of the duration curve

The lower end of the duration curve is an expression of the low-flow characteristics of a stream, but it provides less information than a low-flow frequency curve, because the duration curve applies to the period of record rather than to a year. Nevertheless, frequent requests for duration curves indicate that they are used as a tool in water-related studies. Furthermore, station data for any period can be economically arranged in duration form by the computer, and tabular presentation of values from the duration curve requires only one line per station in a report. The period or periods selected for computation of duration for a given station should correspond to the period of natural flow or to a period that represents particular conditions imposed on the basin.

If necessary, points on a duration curve may be estimated for an ungaged stream by using the same procedures outlined for estimating the frequency of annual events. However, it is suggested that duration data in reports be given only for gaging stations unless there is a valid reason for doing otherwise.

The duration curve and the low-flow frequency curve are related, as W. J. Schneider showed (written commun., 1959). Some of his results are shown in table 3. They indicate different relations in different States; these differences are thought to reflect differences in low-flow characteristics.

The Data-Collection System

Long records of unregulated flow at gaging stations constitute the framework of any data-collection system. These records provide the necessary sampling in time but usually represent a limited sample in space. Spatial sampling can be increased as much as desired by operation of partial-record stations and by collection of one or two base-flow measurements at each of many other sites.

A partial-record station is a site at which enough base-flow measurements will be obtained to define an adequate relation with concurrent flows at a nearby gaging station. See the section on "Frequency characteristics from discharge measurements." The conclusion in that section is that a continuous record offers little advantage over base-flow measurements at carefully selected times for defining low-flow characteristics. However, the continuous record would show natural diurnal fluctuations and effects of regulation which might otherwise be overlooked. Thus if only base-flow measurements are to be obtained at a site it may be desirable to record the stage for a short period, preferably with an analog recorder, to assure that the flow is not affected by artificial regulation. The existence of a substantial diurnal fluctuation can be determined by occasional stage measurements throughout one sunny summer day of base flow. If a large diurnal fluctuation is found, the discharge measurements should be made at time of day of the daily mean flow, as indicated by the pattern of regulation. Alternatively, the measurement could be made on a cloudy day, but the field schedule may not permit this flexibility.

It may be possible to obtain adequate data (six or seven base-flow measurements) at a partial-record station in one low-flow season, but plans usually call for operation of a station over

Table 3.—Relation of duration curves to low-flow frequency curves

State	Number of stations	Percent duration of 7-day minimum flow at indicated recurrence interval			
		Mean		Range	
		2 years	20 years	2 years	20 years
Virginia	27	93.2	99.79	91.3–94.7	99.10–99.95
North Carolina	4	92.8	99.88	92.4–93.2	99.84–99.92
Illinois	3	95.2	99.92	94.7–96.0	99.88–99.97
Ohio	3	93.5	99.88	92.3–94.9	99.83–99.93
California	9	93.2	99.76	92.2–95.1	99.31–99.94
Kansas	11	87.8	99.58	82.8–92.0	99.17–99.85

a period of several years with only a few measurements each year. The latter schedule generally produces better results. However, an early need for information would call for a one-season operation. A severe drought would permit good definition of the relation in one season, but of course the probability of a severe drought during any one season of sampling will be slight.

Gaging stations used as index stations for transferring low-flow characteristics to partial-record sites need not have more than 20 years of record because (1) estimates of low-flow characteristics at partial-record sites usually are not made for recurrence intervals larger than 20 years and (2) the 20-year low flow is adequately defined by 20 years of record. A discontinued index station will serve as well as an active one if its frequency curve is well defined and if a discharge measurement is made at the index station on the same day as the partial-record station measurement.

In addition to regular and partial-record stations, the data-collection program should include a third level of activity in which a base-flow measurement is obtained at each of many sites, all within a few days during a period of low base flow. Several of these sets of measurements at the same sites under various drought conditions will permit description of the flow characteristics at those sites. In the event of a severe drought, measurements of base flow or observations of no flow (or of dry) should be made on as many streams as possible. A single measurement at such a time is much more valuable than many measurements of more common low flows. Seepage runs, as previously described, should also be included in the data-collection program.

In addition to documenting natural changes in base flow throughout a reach by seepage runs, an inventory of manmade diversions, return flows, and regulation is needed to properly interpret the low-flow data. The existence of some of these flow modifications may be apparent from the chemical characteristics of the water or from the shape of the low-flow frequency curve.

It is good practice to measure specific conductance and water temperature when a base-flow measurement is made. These additional observations may indicate the presence of a pollutant or, if the flow is natural, they tell something about the geology of the basin and the suitability of the water for use.

Design of the data-collection system should consist of the following steps:

1. State objectives.
2. Analyze available low-flow data to find what information is already known, what methods of analysis are suitable, and what additional data are required.
3. Determine whether an adequate network of index stations is available and establish additional stations if needed.
4. Select partial-record sites and schedule discharge measurements (usually 2 or 3 years is long enough at one site; it is better to define flow characteristics at many sites than to achieve higher accuracy at a few).
5. Plan (a) to make base-flow measurements at many sites if a significant drought occurs, (b) for seepage runs, and (c) for other types of data as needed.
6. Plan to analyze the data periodically and modify the data-collection program if so indicated. Use regionalization methods where feasible.
7. Preliminary results should be made available to the public promptly even though they are incomplete and (or) not very accurate.

Reporting Results

The principal objectives of a report on low flows should be (1) to present the low-flow characteristics at sites where flow data have been collected and (2) to present a method of estimating those characteristics at ungaged sites.

The flow characteristics should be presented in such a way that the reader can find information at or close to his site of interest rapidly. One method of presentation, used by Busch and Shaw (1966), shows on a map the location of each site for which low-flow information is available and refers the map site to a table giving the low-flow information. Another method of presentation is to show all the information on a map. (See fig. 18.) If roads and other cultural features are shown on the map, the locations of the sites are adequately identified, and a land line description is not needed. A good example of a preliminary report is the one by Gebert (1971).

In a more comprehensive report frequency characteristics at gaging stations should be given in more detail than can readily be shown on a map such as figure 18. The frequency curves for 7, 14, 30, and 60 days should be plotted for one or a few stations and the characteristics at the other gaging stations tabulated, as, for example, they were by Busch and Shaw (1966). Information given for each gaging station may also include a brief station description and a tabulation of selected points on the duration curve (if meaningful).

An advantage of the presentation of figure 18 is that a user may approximate the low-flow characteristics at an unmeasured site by interpolation between two points on the same channel. For example, the 2-year recurrence interval low flow of Hypothetical River halfway between the confluence of East and West Hypo Creeks and the mouth of Thetical River is about 1.3 cfs. Selection of partial-record sites to facilitate such interpolation is desirable. Seepage runs on the larger streams will add information permitting more accurate interpolation.

Seasonal low-flow characteristics may be shown in the same manner as annual ones.

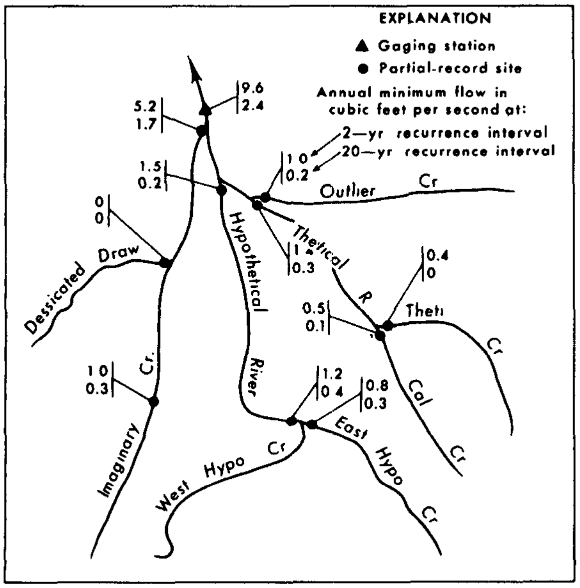


Figure 18.—Map of Hypothetical River basin showing frequency characteristics of low flows.

The drainage maps should show every site at which a base-flow measurement (or field estimate of very small flow or observation of zero flow) has been obtained. The report should show the estimated low-flow-frequency characteristics at each site and list the measured discharges. Thus the low-flow report will contain all the base-flow measurements made in the area covered by the report in order to make the report of value to those planning collection of additional data as well as to the principal users; in effect, the report is a compilation of base-flow measurements scattered throughout the annual data reports.

Results of acceptable regional analyses should be presented for use in estimating the characteristics at ungaged sites. The method of application and an estimate of the reliability of the result should be included.

Where regionalization is not successful, a relation between a low-flow characteristic and drainage area may be presented, or a statement included giving the range in base flow in cubic feet per second per square mile throughout a limited region.

Another generalization, of wider application, is a statement that essentially all streams draining less than *x* square miles in a region will be dry at the *n*-year recurrence interval. The basis for such a statement should be an analysis such as that shown in figures 10 and 12.

The report should describe the procedure for determining low-flow characteristics from base-flow measurements and should state that this will be done at additional sites when a need is foreseen.

The report should identify all diversions and sources of regulation.

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