



Techniques of Water-Resources Investigations of the United States Geological Survey

Chapter B7

ANALYTICAL SOLUTIONS FOR ONE-, TWO-, AND THREE-DIMENSIONAL SOLUTE TRANSPORT IN GROUND-WATER SYSTEMS WITH UNIFORM FLOW

By Eliezer J. Wexler

Book 3

APPLICATIONS OF HYDRAULICS

Attachment 2.—Program source-code listings

FINITE
SEMINF
POINT2
STRIPF
STRIPI
GAUSS
POINT3
PATCHF
PATCHI

```

C
C *****
C * * * * *
C *          **** FINITE ****
C * * * * *
C * ONE-DIMENSIONAL GROUND-WATER SOLUTE-TRANSPORT MODEL *
C * * * * *
C *   FOR A FINITE SYSTEM WITH A FIRST- OR THIRD-TYPE
C * * * * *
C *           BOUNDARY CONDITION AT X=0
C * * * * *
C *           VERSION CURRENT AS OF 04/01/90
C * * * * *
C *****
C
C THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE
C GREATER THAN THOSE GIVEN HERE.
C   MAXX = MAXIMUM NUMBER OF X-VALUES
C   MAXT = MAXIMUM NUMBER OF TIME VALUES
C   MAXRT = MAXIMUM NUMBER OF ROOTS USED IN THE SERIES SUMMATION
C PARAMETER MAXX=100,MAXT=20,MAXRT=1000
C
C IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C REAL XP(MAXX),CP(MAXX),TP,XSCLP
C CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
C CHARACTER*1 IERR(MAXX,MAXT)
C DIMENSION CXT(MAXX,MAXT),X(MAXX),T(MAXT)
C DIMENSION ROOT(MAXRT)
C COMMON /IOUNIT/ IN,IO
C
C PROGRAM VARIABLES
C
C   NOTE: ANY CONSISTANT SET OF UNITS MAY BE USED IN THE
C MODEL. NO FORMAT STATEMENTS NEED TO BE CHANGED AS
C LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.
C
C CO      SOLUTE CONCENTRATION AT THE INFLOW BOUNDARY [M/L**3]
C DX      LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
C VX      GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
C DK      FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
C X       X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C T       TIME AT WHICH CONCENTRATION IS EVALUATED [T]
C CN      NORMALIZED CONCENTRATION C/CO [DIMENSIONLESS]
C CXT     SOLUTE CONCENTRATION C(X,T) [M/L**3]
C XL      LENGTH OF THE FLOW SYSTEM [L]
C ROOT(N) ROOTS OF EQ. USED IN INFINITE SERIES SUMMATION
C
C NBC     SOURCE BOUNDARY CONDITION TYPE (1 OR 3)
C NX      NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED
C NT      NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED
C NROOT   NUMBER OF ROOTS USED IN INFINITE SERIES SUMMATION
C IPLT    PLOT CONTROL. IF IPLT>0, CONCENTRATION PROFILES ARE PLOTT
C

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C	CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS	54
C	CUNITS UNITS OF CONCENTRATION (M/L**3)	55
C	VUNITS UNITS OF GROUND-WATER VELOCITY (L/T)	56
C	DUNITS UNITS OF DISPERSION COEFFICIENT (L**2/T)	57
C	KUNITS UNITS OF SOLUTE DECAY CONSTANT (1/T)	58
C	LUNITS UNITS OF LENGTH (L)	59
C	TUNITS UNITS OF TIME (T)	60
C		61
C	DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE	62
	CALL OFILE	63
	CALL TITLE	64
	WRITE(IO,201)	65
C		66
C	READ IN MODEL PARAMETERS	67
	READ(IN,101) NBC,NX,NT,NROOT,IPLT	68
	IF(NBC.EQ.1) WRITE(IO,202)	69
	IF(NBC.EQ.3) WRITE(IO,203)	70
	WRITE(IO,205) NX,NT,NROOT	71
	READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS	72
	READ(IN,110) CO,VX,DX,DK,XL,XSCLP	73
	WRITE(IO,210) CO,CUNITS,VX,VUNITS,DX,DUNITS,DK,KUNITS,XL,LUNITS,	74
	1 XSCLP	75
	READ(IN,110) (X(I),I=1,NX)	76
	WRITE(IO,215) LUNITS	77
	WRITE(IO,220) (X(I),I=1,NX)	78
	READ(IN,110) (T(I),I=1,NT)	79
	WRITE(IO,225) TUNITS	80
	WRITE(IO,220) (T(I),I=1,NT)	81
C		82
C	GET EIGENVALUES (BETA) USED IN SERIES SUMMATION BY SOLVING FOR	83
C	THE POSITIVE ROOTS OF: BETA*COTAN(BETA)+VX*XL/(2*DX)=0.0	84
C	FOR A FIRST-TYPE SOURCE BOUNDARY CONDITION,	85
C	OR: BETA*COTAN(BETA)-BETA**2*DX/(VX*XL)+VX*XL/(4*DX)=0.0	86
C	FOR A THIRD-TYPE SOURCE BOUNDARY CONDITION.	87
C		88
	IF (NBC.EQ.1) THEN	89
	C=VX*XL/(2.0D0*DX)	90
	CALL ROOT1(C,ROOT,NROOT)	91
	ELSE	92
	A=0.250D0*VX*XL/DX	93
	C=DX/(XL*VX)	94
	CALL ROOT3(A,C,ROOT,NROOT)	95
	END IF	96
C		97
C	BEGIN TIME LOOP	98
	DO 40 IT=1,NT	99
C		100
C	BEGIN X-COORDINATE LOOP	101
	DO 50 IX=1,NX	102
C		103
C	CALL ROUTINE TO CALCULATE NORMALIZED CONCENTRATION	104
C	BASED ON TYPE OF BOUNDARY CONDITION SPECIFIED	105
	IF(NBC.EQ.1) CALL CNRML1(XL,T(IT),X(IX),DX,VX,DK,ROOT,CN,NROOT,	106

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1 IERR(IX,IT)) 107
IF(NBC.EQ.3) CALL CNRML3(XL,T(IT),X(IX),DX,VX,DK,ROOT,CN,NROOT, 108
1 IERR(IX,IT)) 109
CXT(IX,IT)=CN*CO 110
50 CONTINUE 111
C 112
C CONVERT X AND C TO SINGLE PRECISION AND DIVIDE BY CO TO 113
C PLOT NORMALIZED CONCENTRATION PROFILE FOR EACH TIME VALUE. 114
IF(IPLT.LT.1) GO TO 40 115
DO 60 I=1,NX 116
XP(I)=SNGL(X(I)) 117
60 CP(I)=SNGL(CXT(I,IT)/CO) 118
TP=SNGL(T(IT)) 119
CALL PLOT1D(XP,CP,NX,TP,IT,NT,TUNITS,LUNITS,XSCLP) 120
40 CONTINUE 121
C 122
C PRINT OUT TABLES OF CONCENTRATION VALUES 123
NPAGE=1+(NT-1)/9 124
DO 80 NP=1,NPAGE 125
IF(NP.EQ.1) WRITE(IO,230) TUNITS 126
IF(NP.NE.1) WRITE(IO,231) TUNITS 127
NP1=(NP-1)*9 128
NP2=9 129
IF((NP1+NP2).GT.NT) NP2=NT-NP1 130
WRITE(IO,235) (T(NP1+J),J=1,NP2) 131
WRITE(IO,236) CUNITS,LUNITS 132
DO 70 IX=1,NX 133
WRITE(IO,240) X(IX),(CXT(IX,NP1+J),IERR(IX,NP1+J),J=1,NP2) 134
IF(MOD(IX,45).NE.0) GO TO 70 135
WRITE(IO,231) TUNITS 136
WRITE(IO,235) (T(NP1+J),J=1,NP2) 137
WRITE(IO,236) CUNITS,LUNITS 138
70 IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(IO,241) 139
80 CONTINUE 140
C 141
CLOSE (IN) 142
CLOSE (IO) 143
STOP 144
C 145
C FORMAT STATEMENTS 146
101 FORMAT(20I4) 147
105 FORMAT(8A10) 148
110 FORMAT(8F10.0) 149
201 FORMAT(/////1H ,30X,'ANALYTICAL SOLUTION TO THE ONE-DIMENSIONAL'/
1 1H ,28X,'ADVECTIVE-DISPERSIVE SOLUTE-TRANSPORT EQUATION'/ 151
2 1H ,36X,'FOR A SYSTEM OF FINITE LENGTH'///1H0,40X,'INPUT DATA'/ 152
3 1H ,40X,10(1H-)) 153
202 FORMAT(1H0,25X,'FIRST-TYPE BOUNDARY CONDITION AT X = 0.0') 154
203 FORMAT(1H0,25X,'THIRD-TYPE BOUNDARY CONDITION AT X = 0.0') 155
205 FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',I4/1H ,25X, 156
1 'NUMBER OF TIME VALUES (NT) = ',I4/1H ,25X,'NUMBER OF ROOTS ', 157
2 'USED IN INFINITE SERIES SUMMATION (NROOT) = ',I4) 158
210 FORMAT(1H0,25X,'SOLUTE CONCENTRATION ON MODEL BOUNDARY (CO) =', 159

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1 1P1E13.6,1X,A10/1H ,25X, 160
2 'GROUND-WATER VELOCITY IN X-DIRECTION (VX) =',1P1E13.6,1X,A10/ 161
3 1H ,25X,'DISPERSION IN THE X-DIRECTION (DX) =',1P1E13.6,1X,A10/ 162
4 1H ,25X,'FIRST-ORDER SOLUTE-DECAY RATE (DK) =',1P1E13.6,1X,A10/ 163
5 1H ,25X,'LENGTH OF FINITE FLOW SYSTEM (XL) =',1P1E13.6,1X,A10/ 164
6 1H ,25X,'PLOT SCALING FACTOR (XSCLP) =',1P1E13.6) 165
215 FORMAT(1H0,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ', 166
1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/) 167
220 FORMAT(1H ,5X,8F12.4) 168
225 FORMAT(1H0,25X,'TIMES AT WHICH SOLUTE CONCENTRATIONS ' 169
1 'WILL BE CALCULATED, IN ',A10/1H ,25X,70(1H-)/) 170
230 FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AS A FUNCTION OF TIME', 171
1 15X,'* INDICATES SOLUTION DID NOT CONVERGE'/ 172
2 1H0,25X,'TIME VALUES, IN ',A10) 173
231 FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AS A FUNCTION OF TIME =', 174
1 5X,'(CONTINUED)'/ 175
2 1H0,25X,'TIME VALUES, IN ',A10) 176
235 FORMAT(1H ,20X,9F12.4) 177
236 FORMAT(1H ,19X,'*',108(1H-)/ 178
1 1H ,4X,'X-COORDINATE,',2X,'!',44X,'SOLUTE CONCENTRATION, IN ' 179
2 A10/1H ,4X,'IN ',A10,2X,1H!/1H ,19X,'!') 180
240 FORMAT(1H ,5X,F12.4,2X,'! ',9(F11.5,A1)) 181
241 FORMAT(1H ,19X,'!') 182
END 183
SUBROUTINE CNRML1(XL,T,X,D,V,DK,ROOT,CN,NROOT,IERR) 184
IMPLICIT DOUBLE PRECISION (A-H,O-Z) 185
CHARACTER*1 IERR 186
DIMENSION ROOT(NROOT) 187
C 188
C SOLUTION FOR THE ONE-DIMENSIONAL SOLUTE-TRANSPORT EQUATION 189
C FOR A SYSTEM OF FINITE LENGTH WITH A FIRST-TYPE SOURCE 190
C BOUNDARY CONDITION. VALUE RETURNED IS THE NORMALIZED SOLUTE 191
C CONCENTRATION AT A GIVEN X-COORDINATE AND TIME VALUE. 192
C FOR NO SOLUTE DECAY, A SIMPLIFIED SOLUTION IS USED. 193
C 194
IERR=' ' 195
XL2=XL*XL 196
V2D=V/(2.0D0*D) 197
VX2D=V2D*X 198
VL2D=V2D*XL 199
VL2D2=VL2D*VL2D 200
DKL2D=DK*XL*XL/D 201
VSQT4D=V*V*T/(4.0D0*D) 202
IF(DK.EQ.0.0D0) GO TO 20 203
C 204
C BEGIN SERIES SUMMATION FOR SOLUTE WITH DECAY 205
SIGMA=0.0 206
DO 10 N=1,NROOT 207
BETA=ROOT(N) 208
BETA2=BETA*BETA 209
C 210
C TERM 1 211
X1=(BETA2+VL2D2)*DEXP(-BETA2*D*T/XL2) 212

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C		213
C	TERM 2	214
	DENOM=(BETA2+VL2D2+VL2D)*(BETA2+VL2D2+DKL2D)	215
	X2=BETA*DSIN(BETA*X/XL)/DENOM	216
	SIGMA=SIGMA+X1*X2	217
C		218
C	CHECK FOR CONVERGENCE OF SERIES	219
	IF(N.GT.25 .AND. DABS(X1*X2).LT.1.0D-14) GO TO 15	220
10	CONTINUE	221
	IERR='*'	222
15	CONTINUE	223
C		224
C	TERM 3	225
	U=DSQRT(V*V+4.0D0*DK*D)	226
	VMU=V-U	227
	VPU=V+U	228
	VUPM=(U-V)/VPU	229
	D2=D*2.0D0	230
	X3=DEXP(VMU*X/D2)+VUPM*DEXP((VPU*X-2.0D0*U*XL)/D2)	231
	X3=X3/(1.0D0+VUPM*DEXP(-U*XL/D))	232
	CN=X3-2.0D0*DEXP(VX2D-VSQT4D-DK*T)*SIGMA	233
	RETURN	234
C		235
C	BEGIN SERIES SUMMATION FOR SOLUTE WITH NO DECAY	236
20	SIGMA=0.0	237
	DO 30 N=1,NROOT	238
	BETA=ROOT(N)	239
	BETA2=BETA*BETA	240
C		241
C	TERM 1	242
	DENOM=BETA2+VL2D2+VL2D	243
	X1=BETA*DSIN(BETA*X/XL)*DEXP(-BETA2*D*T/XL2)	244
	X1=X1/DENOM	245
	TERM=X1	246
	SIGMA=SIGMA+X1	247
	IF(N.GT.25 .AND. DABS(X1).LT.1.0D-14) GO TO 35	248
30	CONTINUE	249
	IERR='*'	250
35	CONTINUE	251
	CN=1.0D0-2.0D0*DEXP(VX2D-VSQT4D)*SIGMA	252
	RETURN	253
	END	254
	SUBROUTINE CNRML3(XL,T,X,D,V,DK,ROOT,CN,NROOT,IERR)	255
	IMPLICIT DOUBLE PRECISION (A-H,O-Z)	256
	CHARACTER*1 IERR	257
	DIMENSION ROOT(NROOT)	258
C		259
C	SOLUTION FOR THE ONE DIMENSIONAL SOLUTE-TRANSPORT EQUATION	260
C	FOR A SYSTEM OF FINITE LENGTH WITH A THIRD-TYPE SOURCE	261
C	BOUNDARY CONDITION. VALUE RETURNED IS THE NORMALIZED SOLUTE	262
C	CONCENTRATION AT A GIVEN X-COORDINATE AND TIME VALUE.	263
C	FOR NO SOLUTE DECAY, A SIMPLIFIED SOLUTION IS USED.	264
C		265

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IERR=' ' 266
XL2=XL*XL 267
V2D=V/(2.0D0*D) 268
VLD=V*XL/D 269
VX2D=V2D*X 270
VL2D=V2D*XL 271
VL2D2=VL2D*VL2D 272
DKL2D=DK*XL*XL/D 273
VSQT4D=V*V*T/(4.0D0*D) 274
IF(DK.EQ.0.0D0) GO TO 20 275
C 276
C BEGIN SERIES SUMMATION FOR SOLUTE WITH DECAY 277
SIGMA=0.0 278
DO 10 N=1,NROOT 279
BETA=ROOT(N) 280
BETA2=BETA*BETA 281
C 282
C TERM 1 283
BETAXL=BETA*X/XL 284
X1=BETA*(BETA*DCOS(BETAXL)+VL2D*DSIN(BETAXL)) 285
C 286
C TERM 2 287
DENOM=(BETA2+VL2D2+VLD)*(BETA2+VL2D2+DKL2D) 288
X2=DEXP(-BETA2*D*T/XL2)/DENOM 289
SIGMA=SIGMA+X1*X2 290
C 291
C CHECK FOR CONVERGENCE OF SERIES 292
IF(N.GT.25 .AND. DABS(X1*X2).LT.1.0D-14) GO TO 15 293
10 CONTINUE 294
IERR='*' 295
15 CONTINUE 296
C 297
C TERM 3 298
U=DSQRT(V*V+4.0D0*DK*D) 299
VMU=V-U 300
VPU=V+U 301
VUPM=(U-V)/VPU 302
D2=D*2.0D0 303
X3=DEXP(VMU*X/D2)+VUPM*DEXP((VPU*X-2.0D0*U*XL)/D2) 304
X3=2.0D0*V*X3/(VPU+VMU*VUPM*DEXP(-U*XL/D)) 305
CN=X3-2.0D0*VLD*DEXP(VX2D-VSQT4D-DK*T)*SIGMA 306
RETURN 307
C 308
C BEGIN SERIES SUMMATION FOR SOLUTE WITH NO DECAY 309
20 SIGMA=0.0 310
DO 30 N=1,NROOT 311
BETA=ROOT(N) 312
BETA2=BETA*BETA 313
C 314
C TERM 1 315
BETAXL=BETA*X/XL 316
X1=BETA*(BETA*DCOS(BETAXL)+VL2D*DSIN(BETAXL)) 317
C 318

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C	TERM 2	319
	DENOM=(BETA2+VL2D2+VLD)*(BETA2+VL2D2)	320
	X2=DEXP(-BETA2*D*T/XL2)/DENOM	321
C		322
	SIGMA=SIGMA+X1*X2	323
	IF(N.GT.25 .AND. DABS(X1*X2).LT.1.0D-14) GO TO 35	324
30	CONTINUE	325
	IERR='*'	326
35	CONTINUE	327
C		328
	CN=1.0D0-2.0D0*VLD*DEXP(VX2D-VSQT4D)*SIGMA	329
	RETURN	330
	END	331
	SUBROUTINE ROOT1 (C,ROOT,NROOT)	332
	IMPLICIT DOUBLE PRECISION (A-H,O-Z)	333
	DIMENSION ROOT(NROOT)	334
	COMMON /IOUNIT/ IN,IO	335
	DATA MAXIT, EPS/50, 1.0D-10/	336
C		337
C	THIS ROUTINE CALCULATES ROOTS OF THE EQUATION: B*COTAN(B)+C=0	338
C	USING NEWTON'S SECOND-ORDER METHOD.	339
C		340
C	PROGRAM VARIABLES	341
C	MAXIT MAXIMUM NUMBER OF ITERATIONS ALLOWED IN ROOT SEARCH	342
C	EPS CONVERGENCE CRITERION	343
C	F1,F2 1ST AND 2ND DERIVATIVES OF THE EQUATION	344
C	H SECOND-ORDER CORRECTION FACTOR	345
C		346
C	FIRST ROOT LIES BETWEEN PI/2 AND PI. START WITH .75*PI	347
	PI=3.14159265359D0	348
	ROOT(1)=0.750D0*PI	349
C		350
C	START LOOP FOR EACH ROOT SEARCH	351
	DO 10 N=1,NROOT	352
C		353
C	BEGIN ITERATIVE LOOP	354
	DO 20 I=1,MAXIT	355
	X=ROOT(N)	356
	SINX2=DSIN(X)*DSIN(X)	357
	COTX=1.0D0/DTAN(X)	358
	F=X*COTX+C	359
C	IF F IS 0.0, EXACT ROOT HAS BEEN FOUND	360
	IF(F.EQ.0.0) GO TO 30	361
	F1=COTX-X/SINX2	362
	F2=-1.0D0/SINX2-(SINX2-X*DSIN(X*2.0D0))/(SINX2*SINX2)	363
	H=(F2/2.0D0)/F1-F1/F	364
	H=1.0D0/H	365
	ROOT(N)=X+H	366
C		367
C	CHECK FOR CONVERGENCE. IF NOT ACHIEVED, RE-ITERATE	368
	IF(DABS(H).LT.EPS) GO TO 30	369
20	CONTINUE	370
	WRITE(IO,201) MAXIT,N	371

C	NEXT ROOT IS ABOUT PI GREATER THAN LAST ROOT	425
30	IF(N.NE.NROOT) ROOT(N+1)=ROOT(N)+PI	426
10	CONTINUE	427
	RETURN	428
C		429
C	FORMAT STATEMENTS	430
201	FORMAT(1H ,5X,'**** WARNING **** ROOT SEARCH ROUTINE DID NOT',	431
1	'CONVERGE AFTER ',I4,'ITERATIONS WHILE SEARCHING FOR ROOT',I5)	432
	END	433

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C
C ***** 1
C * 2
C * 3
C * **** SEMINF **** * 4
C * * 5
C * ONE-DIMENSIONAL GROUND-WATER SOLUTE-TRANSPORT MODEL * 6
C * * 7
C * FOR A SEMI-INFINITE SYSTEM WITH A FIRST-TYPE OR * 8
C * * 9
C * THIRD-TYPE BOUNDARY CONDITION AT X=0 * 10
C * * 11
C * VERSION CURRENT AS OF 04/01/90 * 12
C * * 13
C ***** 14
C 15
C THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE 16
C GREATER THAN THOSE GIVEN HERE. 17
C MAXX = MAXIMUM NUMBER OF X-VALUES 18
C MAXT = MAXIMUM NUMBER OF TIME VALUES 19
C PARAMETER MAXX=100,MAXT=20 20
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C IMPLICIT DOUBLE PRECISION (A-H,O-Z) 22
C REAL XP(MAXX),CP(MAXX),TP,XSCLP 23
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C DIMENSION CXT(MAXX,MAXT),X(MAXX),T(MAXT) 25
C COMMON /IOUNIT/ IN,IO 26
C 27
C PROGRAM VARIABLES 28
C 29
C NOTE: ANY CONSISTANT SET OF UNITS MAY BE USED IN THE 30
C MODEL. NO FORMAT STATEMENTS NEED TO BE CHANGED AS 31
C LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT. 32
C 33
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C DX LONGITUDINAL DISPERSION COEFFICIENT [L**2/T] 35
C VX GROUND-WATER VELOCITY IN X-DIRECTION [L/T] 36
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C X X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L] 38
C T TIME AT WHICH CONCENTRATION IS EVALUATED [T] 39
C CN NORMALIZED CONCENTRATION C/CO [DIMENSIONLESS] 40
C CXT SOLUTE CONCENTRATION C(X,T) [M/L**3] 41
C 42
C NBC SOURCE BOUNDARY CONDITION TYPE (1 OR 3) 43
C NX NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED 44
C NT NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED 45
C IPLT PLOT CONTROL. IF IPLT>0, CONCENTRATION PROFILES ARE PLOTT 46
C 47
C CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS 48
C CUNITS UNITS OF CONCENTRATION (M/L**3) 49
C VUNITS UNITS OF GROUND-WATER VELOCITY (L/T) 50
C DUNITS UNITS OF DISPERSION COEFFICIENT (L**2/T) 51
C KUNITS UNITS OF SOLUTE DECAY CONSTANT (1/T) 52
C LUNITS UNITS OF LENGTH (L) 53

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C	TUNITS UNITS OF TIME (T)	54
C		55
C	DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE	56
	CALL OFILE	57
	CALL TITLE	58
	WRITE(IO,201)	59
C		60
C	READ IN MODEL PARAMETERS	61
	READ(IN,101) NBC,NX,NT,IPLT	62
	IF(NBC.EQ.1) WRITE(IO,202)	63
	IF(NBC.EQ.3) WRITE(IO,203)	64
	WRITE(IO,205) NX,NT	65
	READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS	66
	READ(IN,110) CO,VX,DX,DK,XSCLP	67
	WRITE(IO,210) CO,CUNITS,VX,VUNITS,DX,DUNITS,DK,KUNITS,XSCLP	68
	READ(IN,110) (X(I),I=1,NX)	69
	WRITE(IO,215) LUNITS	70
	WRITE(IO,220) (X(I),I=1,NX)	71
	READ(IN,110) (T(I),I=1,NT)	72
	WRITE(IO,225) TUNITS	73
	WRITE(IO,220) (T(I),I=1,NT)	74
C		75
C	BEGIN TIME LOOP	76
	DO 40 IT=1,NT	77
C		78
C	BEGIN X-COORDINATE LOOP	79
	DO 50 IX=1,NX	80
C		81
C	CALL ROUTINE TO CALCULATE NORMALIZED CONCENTRATION	82
C	BASED ON TYPE OF BOUNDARY CONDITION SPECIFIED	83
	IF(NBC.EQ.1) CALL CNRML1(DK,T(IT),X(IX),DX,VX,CN)	84
	IF(NBC.EQ.3) CALL CNRML3(DK,T(IT),X(IX),DX,VX,CN)	85
	CXT(IX,IT)=CN*CO	86
50	CONTINUE	87
C		88
C	CONVERT X AND C TO SINGLE PRECISION AND DIVIDE BY CO TO	89
C	PLOT NORMALIZED CONCENTRATION PROFILE FOR EACH TIME VALUE.	90
	IF(IPLT.LT.1) GO TO 40	91
	DO 60 I=1,NX	92
	XP(I)=SNGL(X(I))	93
60	CP(I)=SNGL(CXT(I,IT)/CO)	94
	TP=SNGL(T(IT))	95
	CALL PLOT1D(XP,CP,NX,TP,IT,NT,TUNITS,LUNITS,XSCLP)	96
40	CONTINUE	97
C		98
C	PRINT OUT TABLES OF CONCENTRATION VALUES	99
	NPAGE=1+(NT-1)/9	100
	DO 80 NP=1,NPAGE	101
	IF(NP.EQ.1) WRITE(IO,230) TUNITS	102
	IF(NP.NE.1) WRITE(IO,231) TUNITS	103
	NP1=(NP-1)*9	104
	NP2=9	105
	IF((NP1+NP2).GT.NT) NP2=NT-NP1	106

```

WRITE(IO,235) (T(NP1+J),J=1,NP2) 107
WRITE(IO,236) CUNITS,LUNITS 108
DO 70 IX=1,NX 109
WRITE(IO,240) X(IX),(CXT(IX,NP1+J),J=1,NP2) 110
IF(MOD(IX,45).NE.0) GO TO 70 111
WRITE(IO,231) TUNITS 112
WRITE(IO,235) (T(NP1+J),J=1,NP2) 113
WRITE(IO,236) CUNITS,LUNITS 114
70 IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(IO,241) 115
80 CONTINUE 116
C 117
CLOSE (IN) 118
CLOSE (IO) 119
STOP 120
C 121
C FORMAT STATEMENTS 122
101 FORMAT(20I4) 123
105 FORMAT(8A10) 124
110 FORMAT(8F10.0) 125
201 FORMAT(/////1H ,30X,'ANALYTICAL SOLUTION TO THE ONE-DIMENSIONAL'/ 126
1 1H ,28X,'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION'/ 127
2 1H ,38X,'FOR A SEMI-INFINITE SYSTEM'///1H0,40X,'INPUT DATA'/ 128
3 1H ,40X,10(1H-)) 129
202 FORMAT(1H0,25X,'FIRST-TYPE BOUNDARY CONDITION AT X = 0.0') 130
203 FORMAT(1H0,25X,'THIRD-TYPE BOUNDARY CONDITION AT X = 0.0') 131
205 FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',I4/1H ,25X, 132
1 'NUMBER OF TIME VALUES (NT) = ',I4) 133
210 FORMAT(1H0,25X,'SOLUTE CONCENTRATION ON MODEL BOUNDARY (C0) =', 134
1 1P1E13.6,1X,A10/1H ,25X, 135
2 'GROUND-WATER VELOCITY IN X-DIRECTION (VX) =',1P1E13.6,1X,A10/ 136
3 1H ,25X,'DISPERSION IN THE X-DIRECTION (DX) =',1P1E13.6,1X,A10/ 137
4 1H ,25X,'FIRST-ORDER SOLUTE DECAY RATE (DK) =',1P1E13.6,1X,A10/ 138
5 1H ,25X,'PLOT SCALING FACTOR (XSCLP) =',1P1E13.6) 139
215 FORMAT(1H0,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ', 140
1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/) 141
220 FORMAT(1H ,5X,8F12.4) 142
225 FORMAT(1H0,25X,'TIMES AT WHICH SOLUTE CONCENTRATIONS ' 143
1 'WILL BE CALCULATED, IN ',A10/1H ,25X,70(1H-)/) 144
230 FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AS A FUNCTION OF TIME'/ 145
2 1H0,25X,'TIME VALUES, IN ',A10) 146
231 FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AS A FUNCTION OF TIME =', 147
1 5X,'(CONTINUED)'/ 148
2 1H0,25X,'TIME VALUES, IN ',A10) 149
235 FORMAT(1H ,20X,9F12.4) 150
236 FORMAT(1H ,19X,'*',108(1H-)/ 151
1 1H ,4X,'X-COORDINATE,',2X,'!',44X,'SOLUTE CONCENTRATION, IN ' 152
2 A10/1H ,4X,'IN ',A10,2X,1H!/1H ,19X,'!') 153
240 FORMAT(1H ,5X,F12.4,2X,'! ',9F12.5) 154
241 FORMAT(1H ,19X,'!') 155
END 156
SUBROUTINE CNRML1(DK,T,X,D,V,CN) 157
IMPLICIT DOUBLE PRECISION (A-H,O-Z) 158
C 159

```

C	SOLUTION FOR THE ONE-DIMENSIONAL SOLUTE TRANSPORT EQUATION	160
C	FOR A SEMI-INFINITE SYSTEM WITH A FIRST-TYPE SOURCE	161
C	BOUNDARY CONDITION. VALUE RETURNED IS THE NORMALIZED SOLUTE	162
C	CONCENTRATION AT A GIVEN X-COORDINATE AND TIME.	163
C	FOR NO SOLUTE DECAY, A SIMPLIFIED SOLUTION IS USED.	164
C		165
	ALPHA=2.0D0*DSQRT(D*T)	166
	U=DSQRT(V*V+4.0D0*D*DK)	167
	X2D=X/(2.0D0*D)	168
C		169
C	SOLUTION WITH SOLUTE DECAY	170
	IF(DK.EQ.0.0) GO TO 10	171
C		172
C	TERM 1	173
	X1=X2D*(V-U)	174
	Y1=(X-U*T)/ALPHA	175
	CALL EXERFC(X1,Y1,Z1)	176
C		177
C	TERM 2	178
	X2=X2D*(V+U)	179
	Y2=(X+U*T)/ALPHA	180
	CALL EXERFC(X2,Y2,Z2)	181
	CN=(Z1+Z2)/(2.0D0)	182
	RETURN	183
C		184
C	SOLUTION WITH NO SOLUTE DECAY	185
C	TERM 1	186
10	Y1=(X-V*T)/ALPHA	187
	CALL EXERFC(0.0D0,Y1,Z1)	188
C		189
C	TERM 2	190
	X2=X*V/D	191
	Y2=(X+V*T)/ALPHA	192
	CALL EXERFC(X2,Y2,Z2)	193
	CN=(Z1+Z2)/2.0D0	194
	RETURN	195
	END	196
	SUBROUTINE CNRML3(DK,T,X,D,V,CN)	197
	IMPLICIT DOUBLE PRECISION (A-H,O-Z)	198
C		199
C	SOLUTION FOR THE ONE-DIMENSIONAL SOLUTE TRANSPORT EQUATION	200
C	FOR A SEMI-INFINITE SYSTEM WITH A THIRD-TYPE SOURCE	201
C	BOUNDARY CONDITION. VALUE RETURNED IS THE NORMALIZED SOLUTE	202
C	CONCENTRATION AT A GIVEN X-COORDINATE AND TIME.	203
C	FOR NO SOLUTE DECAY, A SIMPLIFIED SOLUTION IS USED.	204
C		205
	ALPHA=2.0D0*DSQRT(D*T)	206
	U=DSQRT(V*V+4.0D0*D*DK)	207
	X2D=X/(2.0D0*D)	208
	VXD=V*X/D	209
C		210
C	SOLUTION WITH SOLUTE DECAY	211
	IF(DK.EQ.0.0) GO TO 10	212

C		213
C	TERM 1	214
	X1=VXD-DK*T	215
	Y1=(X+V*T)/ALPHA	216
	CALL EXERFC(X1, Y1, Z1)	217
	Z1=Z1*2.0D0	218
C		219
C	TERM 2	220
	X2=X2D*(V-U)	221
	Y2=(X-U*T)/ALPHA	222
	CALL EXERFC(X2, Y2, Z2)	223
	Z2=Z2*(U/V-1.0D0)	224
C		225
C	TERM 3	226
	X3=X2D*(V+U)	227
	Y3=(X+U*T)/ALPHA	228
	CALL EXERFC(X3, Y3, Z3)	229
	Z3=Z3*(U/V+1.0D0)	230
	CN=V*V*(Z1+Z2-Z3)/(4.0D0*D*DK)	231
	RETURN	232
C		233
C	SOLUTION FOR NO SOLUTE DECAY	234
10	PI=3.14159265358979D0	235
C		236
C	TERM 1	237
	Y1=(X-V*T)/ALPHA	238
	CALL EXERFC(0.0D0, Y1, Z1)	239
	Z1=0.50D0*Z1	240
C		241
C	TERM 2	242
	X2=VXD	243
	Y2=(X+V*T)/ALPHA	244
	CALL EXERFC(X2, Y2, Z2)	245
	Z2=Z2*0.50D0*(1.0D0+V*(X+V*T)/D)	246
C		247
C	TERM 3	248
	Z3=DEXP(-1.0D0*Y1*Y1)	249
	Z3=Z3*V*DSQRT(T/(PI*D))	250
	CN=Z1-Z2+Z3	251
	RETURN	252
	END	253

C		(UNITS MUST BE SAME AS DISPERSION COEFFICIENT)	54
C	POR	AQUIFER PORISITY [DIMENSIONLESS]	55
C			56
C	NX	NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED	57
C	NY	NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED	58
C	NT	NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED	59
C	NMAX	NUMBER OF TERMS USED IN GAUSS-LEGENDRE NUMERICAL	60
C		INTEGRATION TECHNIQUE (MUST EQUAL 4, 20, 60, 104 OR 256)	61
C			62
C	IPLT	PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED	63
C	XSCLP	SCALING FACTOR TO CONVERT X TO PLOTTER INCHES	64
C	YSCLP	SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES	65
C	DELTA	CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)	66
C			67
C		CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS	68
C	CUNITS	UNITS OF CONCENTRATION (M/L**3)	69
C	VUNITS	UNITS OF GROUND-WATER VELOCITY (L/T)	70
C	DUNITS	UNITS OF DISPERSION COEFFICIENT (L**2/T)	71
C	KUNITS	UNITS OF SOLUTE DECAY CONSTANT (1/T)	72
C	LUNITS	UNITS OF LENGTH (L)	73
C	TUNITS	UNITS OF TIME (T)	74
C			75
C		DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE	76
	CALL OFILE		77
	CALL TITLE		78
	WRITE(IO,201)		79
C			80
C		READ IN MODEL PARAMETERS	81
	READ(IN,101) NX,NY,NT,NMAX,IPLT		82
	WRITE(IO,205) NX,NY,NT,NMAX		83
	READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS		84
	READ(IN,110) C0,VX,DX,DY,DK		85
	WRITE(IO,210) C0,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DK,KUNITS		86
	READ(IN,110) XC,YC,QM,POR		87
	WRITE(IO,212) XC,LUNITS,YC,LUNITS,QM,DUNITS,POR		88
	READ(IN,110) (X(I),I=1,NX)		89
	WRITE(IO,215) LUNITS		90
	WRITE(IO,220) (X(I),I=1,NX)		91
	READ(IN,110) (Y(I),I=1,NY)		92
	WRITE(IO,216) LUNITS		93
	WRITE(IO,220) (Y(I),I=1,NY)		94
	READ(IN,110) (T(I),I=1,NT)		95
	WRITE(IO,225) TUNITS		96
	WRITE(IO,220) (T(I),I=1,NT)		97
	IF(IPLT.GT.0) READ(IN,110) XSCLP,YSCLP,DELTA		98
	IF(IPLT.GT.0) WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS		99
C			100
C		READ IN GAUSS-LEGENDRE POINTS AND WEIGHTING FACTORS	101
	CALL GLQPTS (NMAX)		102
C			103
C		BEGIN TIME LOOP	104
	DO 20 IT=1,NT		105
C			106

C	BEGIN X LOOP	107
	DO 40 IX=1,NX	108
	XX=X(IX)-XC	109
C		110
C	CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX)	111
	DO 50 IY=1,NY	112
	YY=Y(IY)-YC	113
	CALL CNRML2(QM,POR,DK,T(IT),XX,YY,DX,DY,VX,CN,NMAX)	114
	CXY(IX,IY)=C0*CN	115
50	CONTINUE	116
40	CONTINUE	117
C		118
C	PRINT OUT TABLES OF CONCENTRATION VALUES	119
	NPAGE=1+(NY-1)/9	120
	DO 60 NP=1,NPAGE	121
	IF(NP.EQ.1) WRITE(IO,230) T(IT),TUNITS,LUNITS	122
	IF(NP.NE.1) WRITE(IO,231) T(IT),TUNITS,LUNITS	123
	NP1=(NP-1)*9	124
	NP2=9	125
	IF((NP1+NP2).GT.NY) NP2=NY-NP1	126
	WRITE(IO,235) (Y(NP1+J),J=1,NP2)	127
	WRITE(IO,236) CUNITS,LUNITS	128
	DO 70 IX=1,NX	129
	WRITE(IO,240) X(IX),(CXY(IX,NP1+J),J=1,NP2)	130
	IF(MOD(IX,45).NE.0) GO TO 70	131
	WRITE(IO,231) T(IT),TUNITS,LUNITS	132
	WRITE(IO,235) (Y(NP1+J),J=1,NP2)	133
	WRITE(IO,236) CUNITS,LUNITS	134
70	IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(IO,241)	135
60	CONTINUE	136
C		137
C	CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE	138
C	PLOT SCALING FACTORS. CONVERT C(X,Y) AND DIVIDE BY C0 TO PLOT	139
C	CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH TIME VALUE.	140
	IF(IPLT.LT.1) GO TO 20	141
	NXY=NX*NY	142
	DO 80 I=1,NX	143
	IP=(I-1)*NY	144
	XP(I)=SNGL(X(I))	145
	DO 80 J=1,NY	146
	IF(I.EQ.1) YP(J)=SNGL(Y(J))	147
	CP(IP+J)=SNGL(CXY(I,J)/C0)	148
80	CONTINUE	149
	TP=SNGL(T(IT))	150
	NXY2=NXY*2	151
	CALL PLOT2D (XP,YP,CP,TP,DELTA,NX,NY,NXY,NXY2,IT,NT,IPLT,TUNITS,	152
	1 LUNITS,XSCLP,YSCLP,XPC,YPC,IFLAG)	153
20	CONTINUE	154
	CLOSE (IN)	155
	CLOSE (IO)	156
	STOP	157
C		158
C	FORMAT STATEMENTS	159

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101   FORMAT(20I4)                                     160
105   FORMAT(8A10)                                     161
110   FORMAT(8F10.0)                                   162
201   FORMAT(/////1H ,30X,'ANALYTICAL SOLUTION TO THE TWO-DIMENSIONAL'/ 163
      1 1H ,28X,'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION'/ 164
      2 1H ,28X,'FOR AN AQUIFER OF INFINITE AREAL EXTENT WITH A'/ 165
      3 1H ,31X,'CONTINUOUS POINT SOURCE AT X=0 AND Y=YC' 166
      4 ///1H0,40X,'INPUT DATA'/1H ,40X,10(1H-)) 167
205   FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',I4/1H ,25X, 168
      1 'NUMBER OF Y-COORDINATES (NY) = ',I4/1H ,25X, 169
      2 'NUMBER OF TIME VALUES (NT) = ',I4/1H ,25X, 170
      3 'NUMBER OF POINTS FOR NUMERICAL INTEGRATION (NMAX) = ',I4) 171
210   FORMAT(1H0,25X,'SOLUTE CONCENTRATION IN INJECTED FLUID (C0) =', 172
      1 1P1E13.6,1X,A10/1H ,25X, 173
      2 'GROUND-WATER VELOCITY IN X-DIRECTION (VX) =',1P1E13.6,1X,A10/ 174
      3 1H ,25X,'DISPERSION IN THE X-DIRECTION (DX) =',1P1E13.6,1X,A10/ 175
      4 1H ,25X,'DISPERSION IN THE Y-DIRECTION (DY) =',1P1E13.6,1X,A10/ 176
      5 1H ,25X,'FIRST-ORDER SOLUTE DECAY RATE (DK) =',1P1E13.6,1X,A10) 177
212   FORMAT(1H0,25X,'AQUIFER IS OF INFINITE AREAL EXTENT'/1H ,25X, 178
      1 'CONTINUOUS POINT SOURCE IS LOCATED AT X =',1P1E13.6,1X,A10/1H , 179
      1 63X,'Y =',1P1E13.6,1X,A10/1H ,25X, 180
      2 'FLUID INJECTION RATE PER UNIT THICKNESS OF AQUIFER (QM) =', 181
      3 1P1E13.6,1X,A10/1H ,25X,'AQUIFER POROSITY (POR) =',1P1E13.6) 182
215   FORMAT(1H0,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ', 183
      1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/) 184
216   FORMAT(1H0,25X,'Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ', 185
      1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/) 186
220   FORMAT(1H ,5X,8F12.4) 187
225   FORMAT(1H0,25X,'TIMES AT WHICH SOLUTE CONCENTRATIONS ' 188
      1 'WILL BE CALCULATED, IN ',A10/1H ,25X,70(1H-)/) 189
227   FORMAT(1H0,25X,'PLOT SCALING FACTOR FOR X (XSCLP) =',1P1E13.6/ 190
      1 1H ,25X,'PLOT SCALING FACTOR FOR Y (YSCLP) =',1P1E13.6/ 191
      2 1H ,25X,'CONTOUR INCREMENT (DELTA) =',1P1E13.6,1X,A10) 192
230   FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =', 193
      1 F12.4,1X,A10/ 194
      2 1H0,25X,'Y-COORDINATE, IN ',A10) 195
231   FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =', 196
      1 F12.4,1X,A10,5X,'(CONTINUED)'/ 197
      2 1H0,25X,'Y-COORDINATE, IN ',A10) 198
235   FORMAT(1H ,20X,9F12.4) 199
236   FORMAT(1H ,19X,'*',108(1H-)/ 200
      1 1H ,4X,'X-COORDINATE,',2X,'!',44X,'SOLUTE CONCENTRATION, IN ' 201
      2 A10/1H ,4X,'IN ',A10,2X,1H!/1H ,19X,'!') 202
240   FORMAT(1H ,5X,F12.4,2X,'!',9F12.5) 203
241   FORMAT(1H ,19X,'!') 204
      END 205
      SUBROUTINE CNRML2(QM,POR,DK,T,X,Y,DX,DY,VX,CN,NMAX) 206
      IMPLICIT DOUBLE PRECISION(A-H,O-Z) 207
      COMMON /IOUNIT/ IN,IO 208
      COMMON /GLPTS/ WN(256),ZN(256) 209
C 210
C THIS ROUTINE CALCULATES SOLUTE CONCENTRATION AT X,Y BASED ON 211
C THE ANALYTIC SOLUTION TO THE TWO-DIMENSIONAL ADVECTIVE- 212

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```

C
C *****
C *
C *          **** STRIPF ****
C *
C * TWO-DIMENSIONAL GROUND-WATER SOLUTE-TRANSPORT MODEL
C *
C *   FOR A SEMI-INFINITE AQUIFER WITH A FINITE WIDTH
C *
C *   A STRIP SOURCE EXTENDS FROM Y1 TO Y2 AT X=0
C *
C *   GROUND-WATER FLOW IN X-DIRECTION ONLY
C *
C *   VERSION CURRENT AS OF 04/01/90
C *
C *****
C
C THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE
C GREATER THAN THOSE GIVEN HERE.
C   MAXX = MAXIMUM NUMBER OF X-VALUES
C   MAXY = MAXIMUM NUMBER OF Y-VALUES
C   MAXT = MAXIMUM NUMBER OF TIME VALUES
C   MAXXY = MAXX * MAXY
C   MAXXY2 = 2 * MAXX * MAXY
C PARAMETER MAXX=100,MAXY=50,MAXT=20,MAXXY=5000,MAXXY2=10000
C
C IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
C CHARACTER*1 IERR(MAXX,MAXY)
C REAL XP,YP,CP,TP,DELTA,XPC,YPC,XSCLP,YSCLP
C DIMENSION CXY(MAXX,MAXY),X(MAXX),Y(MAXY),T(MAXT)
C COMMON /PDAT/ XP(MAXX),YP(MAXY),CP(MAXXY),XPC(50),YPC(50),
1 IFLAG(MAXXY2)
C COMMON /IOUNIT/ IN,IO
C
C PROGRAM VARIABLES
C
C   NOTE: ANY CONSISTANT SET OF UNITS MAY BE USED IN THE
C   MODEL. NO FORMAT STATEMENTS NEED TO BE CHANGED AS
C   LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.
C
C CO      SOLUTE CONCENTRATION AT THE INFLOW BOUNDARY [M/L**3]
C DX      LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
C DY      TRANSVERSE DISPERSION COEFFICIENT [L**2/T]
C VX      GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
C DK      FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
C X       X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C Y       Y-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C T       TIME AT WHICH CONCENTRATION IS EVALUATED [T]
C CN      NORMALIZED CONCENTRATION C/CO [DIMENSIONLESS]
C CXY     SOLUTE CONCENTRATION C(X,Y,T) [M/L**3]
C W       AQUIFER WIDTH (AQUIFER EXTENDS FROM Y=0 TO Y=W) [L]
C Y1     Y-COORDINATE OF LOWER LIMIT OF STRIP SOLUTE SOURCE [L]

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C	Y2	Y-COORDINATE OF UPPER LIMIT OF STRIP SOLUTE SOURCE [L]	54
C			55
C	NX	NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED	56
C	NY	NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED	57
C	NT	NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED	58
C	NMAX	NUMBER OF TERMS USED IN INFINITE SERIES SUMMATION	59
C			60
C	IPLT	PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED	61
C	XSCLP	SCALING FACTOR TO CONVERT X TO PLOTTER INCHES	62
C	YSCLP	SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES	63
C	DELTA	CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)	64
C			65
C		CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS	66
C	CUNITS	UNITS OF CONCENTRATION (M/L**3)	67
C	VUNITS	UNITS OF GROUND-WATER VELOCITY (L/T)	68
C	DUNITS	UNITS OF DISPERSION COEFFICIENT (L**2/T)	69
C	KUNITS	UNITS OF SOLUTE DECAY CONSTANT (1/T)	70
C	LUNITS	UNITS OF LENGTH (L)	71
C	TUNITS	UNITS OF TIME (T)	72
C			73
C		DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE	74
	CALL OFILE		75
	CALL TITLE		76
	WRITE(IO,201)		77
C			78
C		READ IN MODEL PARAMETERS	79
	READ(IN,101)	NX,NY,NT,NMAX,IPLT	80
	WRITE(IO,205)	NX,NY,NT,NMAX	81
	READ(IN,105)	CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS	82
	READ(IN,110)	CO,VX,DX,DY,DK	83
	WRITE(IO,210)	CO,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DK,KUNITS	84
	READ(IN,110)	W,Y1,Y2	85
	WRITE(IO,212)	W,LUNITS,Y1,LUNITS,Y2,LUNITS	86
	READ(IN,110)	(X(I),I=1,NX)	87
	WRITE(IO,215)	LUNITS	88
	WRITE(IO,220)	(X(I),I=1,NX)	89
	READ(IN,110)	(Y(I),I=1,NY)	90
	WRITE(IO,216)	LUNITS	91
	WRITE(IO,220)	(Y(I),I=1,NY)	92
	READ(IN,110)	(T(I),I=1,NT)	93
	WRITE(IO,225)	TUNITS	94
	WRITE(IO,220)	(T(I),I=1,NT)	95
	IF(IPLT.GT.0)	READ(IN,110) XSCLP,YSCLP,DELTA	96
	IF(IPLT.GT.0)	WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS	97
C			98
C		BEGIN TIME LOOP	99
	DO 20	IT=1,NT	100
C			101
C		BEGIN X LOOP	102
	DO 40	IX=1,NX	103
C			104
C		CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX)	105
	DO 50	IY=1,NY	106

```

CALL CNRMLF(DK, T(IT), X(IX), Y(IY), W, Y1, Y2, DX, DY,          107
1 VX, CN, NMAX, IERR(IX, IY))                                     108
CXY(IX, IY)=C0*CN                                               109
50 CONTINUE                                                       110
40 CONTINUE                                                       111
C                                                                    112
C PRINT OUT TABLES OF CONCENTRATION VALUES                       113
NPAGE=1+(NY-1)/9                                                114
DO 60 NP=1, NPAGE                                               115
IF(NP.EQ.1) WRITE(IO, 230) T(IT), TUNITS, LUNITS               116
IF(NP.NE.1) WRITE(IO, 231) T(IT), TUNITS, LUNITS               117
NP1=(NP-1)*9                                                    118
NP2=9                                                            119
IF((NP1+NP2).GT.NY) NP2=NY-NP1                                  120
WRITE(IO, 235) (Y(NP1+J), J=1, NP2)                             121
WRITE(IO, 236) CUNITS, LUNITS                                    122
DO 70 IX=1, NX                                                  123
WRITE(IO, 240) X(IX), (CXY(IX, NP1+J), IERR(IX, NP1+J), J=1, NP2) 124
IF(MOD(IX, 45).NE.0) GO TO 70                                    125
WRITE(IO, 231) T(IT), TUNITS, LUNITS                             126
WRITE(IO, 235) (Y(NP1+J), J=1, NP2)                             127
WRITE(IO, 236) CUNITS, LUNITS                                    128
70 IF(MOD(IX, 5).EQ.0 .AND. MOD(IX, 45).NE.0) WRITE(IO, 241)   129
60 CONTINUE                                                       130
C                                                                    131
C CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE          132
C PLOT SCALING FACTORS. CONVERT C(X,Y) AND DIVIDE BY C0 TO PLOT 133
C CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH TIME VALUE. 134
IF(IPLT.LT.1) GO TO 20                                           135
NXY=NX*NY                                                        136
DO 80 I=1, NX                                                    137
IP=(I-1)*NY                                                      138
XP(I)=SNGL(X(I))                                                139
DO 80 J=1, NY                                                    140
IF(I.EQ.1) YP(J)=SNGL(Y(J))                                      141
CP(IP+J)=SNGL(CXY(I, J)/C0)                                     142
80 CONTINUE                                                       143
TP=SNGL(T(IT))                                                  144
NXY2=NXY*2                                                       145
CALL PLOT2D (XP, YP, CP, TP, DELTA, NX, NY, NXY, NXY2, IT, NT, IPLT, TUNITS, 146
1 LUNITS, XSCLP, YSCLP, XPC, YPC, IFLAG)                         147
20 CONTINUE                                                       148
CLOSE (IN)                                                        149
CLOSE (IO)                                                        150
STOP                                                              151
C                                                                    152
C FORMAT STATEMENTS                                              153
101 FORMAT(20I4)                                                 154
105 FORMAT(8A10)                                                 155
110 FORMAT(8F10.0)                                               156
201 FORMAT(/////1H ,30X, 'ANALYTICAL SOLUTION TO THE TWO-DIMENSIONAL'// 157
1 1H ,28X, 'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION'// 158
2 1H ,30X, 'FOR A SEMI-INFINITE AQUIFER OF FINITE WIDTH'// 159

```



```

IERR=' ' 213
C 214
C FOR T=0, ALL CONCENTRATIONS EQUAL 0.0 215
IF(T.LE.0.0D0) RETURN 216
C 217
C FOR X=0.0, CONCENTRATIONS ARE SPECIFIED BY BOUNDARY CONDITIONS 218
IF(X.GT.0.0D0) GO TO 10 219
IF(Y.GT.Y1 .AND. Y.LT.Y2) CN=1.0D0 220
IF(Y.EQ.Y1) CN=0.50D0 221
IF(Y.EQ.Y2) CN=0.50D0 222
RETURN 223
C 224
C BEGIN SUMMATION OF TERMS IN INFINITE SERIES 225
10 RTDXT=2.0D0*DSQRT(DX*T) 226
SIGMA=0.0D0 227
SUBTOT=0.0D0 228
NMAX1=NMAX+1 229
DO 20 NN=1,NMAX1 230
N=NN-1 231
ETA=N*PI/W 232
PN=(Y2-Y1)/(2.0D0*W) 233
IF(N.NE.0) PN=(DSIN(ETA*Y2)-DSIN(ETA*Y1))/(N*PI) 234
COSRY=DCOS(ETA*Y) 235
ALPHA=4.0D0*DX*(ETA*ETA*DY+DK) 236
BETA=DSQRT(VX*VX+ALPHA) 237
BETAT=BETA*T 238
C 239
C CALCULATE TERM 1 240
A1=X*(VX-BETA)/(2.0D0*DX) 241
B1=(X-BETAT)/RTDXT 242
CALL EXERFC(A1,B1,C1) 243
C 244
C CALCULATE TERM 2 245
A2=X*(VX+BETA)/(2.0D0*DX) 246
B2=(X+BETAT)/RTDXT 247
CALL EXERFC(A2,B2,C2) 248
C 249
C ADD TERMS TO SUMMATION 250
TERM=PN*COSRY*(C1+C2) 251
SIGMA=SIGMA+TERM 252
C 253
C CHECK FOR CONVERGENCE. BECAUSE SERIES OSCILLATES, CHECK 254
C SUBTOTAL OF LAST 10 TERMS. 255
SUBTOT=SUBTOT+TERM 256
IF(MOD(NN,10).NE.0) GO TO 20 257
IF(DABS(SUBTOT).LT.1.0D-12) GO TO 30 258
SUBTOT=0.0D0 259
20 CONTINUE 260
IERR='*' 261
30 CN=SIGMA 262
RETURN 263
END 264

```

```

C
C *****
C *
C *          **** STRIPI ****
C *
C * TWO-DIMENSIONAL GROUND-WATER SOLUTE-TRANSPORT MODEL
C *
C *   FOR A SEMI-INFINITE AQUIFER OF INFINITE WIDTH
C *
C *   A STRIP SOURCE EXTENDS FROM Y1 TO Y2 AT X=0
C *
C *   GROUND-WATER FLOW IN X-DIRECTION ONLY
C *
C *   VERSION CURRENT AS OF 04/01/90
C *
C *****
C
C THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE
C GREATER THAN THOSE GIVEN HERE.
C   MAXX = MAXIMUM NUMBER OF X-VALUES
C   MAXY = MAXIMUM NUMBER OF Y-VALUES
C   MAXT = MAXIMUM NUMBER OF TIME VALUES
C   MAXXY = MAXX * MAXY
C   MAXXY2 = 2 * MAXX * MAXY
C   PARAMETER MAXX=100,MAXY=50,MAXT=20,MAXXY=5000,MAXXY2=10000
C
C   IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C   CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
C   REAL XP,YP,CP,TP,DELTA,XPC,YPC,XSCLP,YSCLP
C   DIMENSION CXY(MAXX,MAXY),X(MAXX),Y(MAXY),T(MAXT)
C   COMMON /PDAT/ XP(MAXX),YP(MAXY),CP(MAXXY),XPC(50),YPC(50),
1 IFLAG(MAXXY2)
C   COMMON /IOUNIT/ IN,IO
C
C   PROGRAM VARIABLES
C
C   NOTE: ANY CONSISTANT SET OF UNITS MAY BE USED IN THE
C   MODEL. NO FORMAT STATEMENTS NEED TO BE CHANGED AS
C   LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.
C
C   CO      SOLUTE CONCENTRATION AT THE INFLOW BOUNDARY [M/L**3]
C   DX      LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
C   DY      TRANSVERSE DISPERSION COEFFICIENT [L**2/T]
C   VX      GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
C   DK      FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
C   X       X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C   Y       Y-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C   T       TIME AT WHICH CONCENTRATION IS EVALUATED [T]
C   CN      NORMALIZED CONCENTRATION C/CO [DIMENSIONLESS]
C   CXY     SOLUTE CONCENTRATION C(X,Y,T) [M/L**3]
C   Y1     Y-COORDINATE OF LOWER LIMIT OF STRIP SOLUTE SOURCE [L]
C   Y2     Y-COORDINATE OF UPPER LIMIT OF STRIP SOLUTE SOURCE [L]
C   NX     NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED

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C	NY	NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED	54
C	NT	NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED	55
C	NMAX	NUMBER OF TERMS USED IN GAUSS-LEGENDRE NUMERICAL	56
C		INTEGRATION TECHNIQUE (MUST EQUAL 4, 20, 60, 104 OR 256)	57
C			58
C	IPLT	PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED	59
C	XSCLP	SCALING FACTOR TO CONVERT X TO PLOTTER INCHES	60
C	YSCLP	SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES	61
C	DELTA	CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)	62
C			63
C		CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS	64
C	CUNITS	UNITS OF CONCENTRATION (M/L**3)	65
C	VUNITS	UNITS OF GROUND-WATER VELOCITY (L/T)	66
C	DUNITS	UNITS OF DISPERSION COEFFICIENT (L**2/T)	67
C	KUNITS	UNITS OF SOLUTE DECAY CONSTANT (1/T)	68
C	LUNITS	UNITS OF LENGTH (L)	69
C	TUNITS	UNITS OF TIME (T)	70
C			71
C		DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE	72
	CALL OFILE		73
	CALL TITLE		74
	WRITE(IO,201)		75
C			76
C		READ IN MODEL PARAMETERS	77
	READ(IN,101) NX,NY,NT,NMAX,IPLT		78
	WRITE(IO,205) NX,NY,NT,NMAX		79
	READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS		80
	READ(IN,110) C0,VX,DX,DY,DK		81
	WRITE(IO,210) C0,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DK,KUNITS		82
	READ(IN,110) Y1,Y2		83
	WRITE(IO,212) Y1,LUNITS,Y2,LUNITS		84
	READ(IN,110) (X(I),I=1,NX)		85
	WRITE(IO,215) LUNITS		86
	WRITE(IO,220) (X(I),I=1,NX)		87
	READ(IN,110) (Y(I),I=1,NY)		88
	WRITE(IO,216) LUNITS		89
	WRITE(IO,220) (Y(I),I=1,NY)		90
	READ(IN,110) (T(I),I=1,NT)		91
	WRITE(IO,225) TUNITS		92
	WRITE(IO,220) (T(I),I=1,NT)		93
	IF(IPLT.GT.0) READ(IN,110) XSCLP,YSCLP,DELTA		94
	IF(IPLT.GT.0) WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS		95
C			96
C		READ IN GAUSS-LEGENDRE POINTS AND WEIGHTING FACTORS	97
	CALL GLQPTS (NMAX)		98
C			99
C		BEGIN TIME LOOP	100
	DO 20 IT=1,NT		101
C			102
C		BEGIN X LOOP	103
	DO 40 IX=1,NX		104
C			105
C		CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX)	106

```

DO 50 IY=1,NY
CALL CNRMLI(DK,T(IT),X(IX),Y(IY),Y1,Y2,DX,DY,VX,CN,NMAX)
CXY(IX,IY)=C0*CN
50 CONTINUE
40 CONTINUE
C
C PRINT OUT TABLES OF CONCENTRATION VALUES
NPAGE=1+(NY-1)/9
DO 60 NP=1,NPAGE
IF(NP.EQ.1) WRITE(IO,230) T(IT),TUNITS,LUNITS
IF(NP.NE.1) WRITE(IO,231) T(IT),TUNITS,LUNITS
NP1=(NP-1)*9
NP2=9
IF((NP1+NP2).GT.NY) NP2=NY-NP1
WRITE(IO,235) (Y(NP1+J),J=1,NP2)
WRITE(IO,236) CUNITS,LUNITS
DO 70 IX=1,NX
WRITE(IO,240) X(IX),(CXY(IX,NP1+J),J=1,NP2)
IF(MOD(IX,45).NE.0) GO TO 70
WRITE(IO,231) T(IT),TUNITS,LUNITS
WRITE(IO,235) (Y(NP1+J),J=1,NP2)
WRITE(IO,236) CUNITS,LUNITS
70 IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(IO,241)
60 CONTINUE
C
C CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE
C PLOT SCALING FACTORS. CONVERT C(X,Y) AND DIVIDE BY C0 TO PLOT
C CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH TIME VALUE.
IF(IPLT.LT.1) GO TO 20
NXY=NX*NY
DO 80 I=1,NX
IP=(I-1)*NY
XP(I)=SNGL(X(I))
DO 80 J=1,NY
IF(I.EQ.1) YP(J)=SNGL(Y(J))
CP(IP+J)=SNGL(CXY(I,J)/C0)
80 CONTINUE
TP=SNGL(T(IT))
NXY2=NXY*2
CALL PLOT2D (XP,YP,CP,TP,DELTA,NX,NY,NXY,NXY2,IT,NT,IPLT,TUNITS,
1 LUNITS,XSCLP,YSCLP,XPC,YPC,IFLAG)
20 CONTINUE
CLOSE (IN)
CLOSE (IO)
STOP
C
C FORMAT STATEMENTS
101 FORMAT(20I4)
105 FORMAT(8A10)
110 FORMAT(8F10.0)
201 FORMAT(//////1H ,30X,'ANALYTICAL SOLUTION TO THE TWO-DIMENSIONAL'/
1 1H ,28X,'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION'/
2 1H ,29X,'FOR A SEMI-INFINITE AQUIFER OF INFINITE WIDTH'/

```


C		213
C	FOR T=0, ALL CONCENTRATIONS EQUAL 0.0	214
	IF(T.LE.0.0D0) RETURN	215
C		216
C	FOR X=0.0, CONCENTRATIONS ARE SPECIFIED BY BOUNDARY CONDITIONS	217
	IF(X.GT.0.0D0) GO TO 10	218
	IF(Y.GT.Y1 .AND. Y.LT.Y2) CN=1.0D0	219
	IF(Y.EQ.Y1) CN=0.50D0	220
	IF(Y.EQ.Y2) CN=0.50D0	221
	RETURN	222
C		223
C	START NUMERICAL INTEGRATION LOOP	224
10	SUM=0.0D0	225
	DO 20 I=1,NMAX	226
C		227
C	SCALE THE GAUSS-LEGENDRE COEFFICIENTS TO ACCOUNT FOR THE	228
C	NON-NORMALIZED LIMITS OF INTEGRATION	229
	LIMITS OF INTEGRATION ARE FROM 0 TO T**0.25	230
	TT=T**0.250D0	231
	WI=WN(I)	232
	ZI=TT*(ZN(I)+1.0D0)/2.0D0	233
	ZSQ=ZI*ZI	234
	Z4=ZSQ*ZSQ	235
C		236
C	TERM 1	237
	XVT=X-VX*Z4	238
	EXP1=-XVT*XVT/(4.0D0*DX*Z4)-DK*Z4	239
	ERFC1=(Y1-Y)/(2.0D0*ZSQ*DSQRT(DY))	240
	CALL EXERFC(EXP1,ERFC1,Z1)	241
C		242
C	TERM 2	243
	ERFC2=(Y2-Y)/(2.0D0*ZSQ*DSQRT(DY))	244
	CALL EXERFC(EXP1,ERFC2,Z2)	245
	TERM=(Z1-Z2)*WI/(ZI*ZSQ)	246
	SUM=SUM+TERM	247
20	CONTINUE	248
	SUM=SUM*TT/2.0D0	249
	CN=SUM*X/DSQRT(PI*DX))	250
	RETURN	251
	END	252

```

C
C ***** 1
C * 2
C * 3
C *      **** GAUSS **** 4
C * 5
C * TWO-DIMENSIONAL GROUND-WATER SOLUTE-TRANSPORT MODEL * 6
C * 7
C *   FOR A SEMI-INFINITE AQUIFER OF INFINITE WIDTH. A * 8
C * 9
C *   SOURCE HAVING A GAUSSIAN-SHAPED CONCENTRATION DIS- * 10
C * 11
C *   TRIBUTION IS LOCATED AT X=0 AND CENTERED ABOUT Y=YC * 12
C * 13
C *       GROUND-WATER FLOW IN X-DIRECTION ONLY * 14
C * 15
C *       VERSION CURRENT AS OF 04/01/90 * 16
C * 17
C ***** 18
C 19
C   THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE 20
C   GREATER THAN THOSE GIVEN HERE. 21
C   MAXX = MAXIMUM NUMBER OF X-VALUES 22
C   MAXY = MAXIMUM NUMBER OF Y-VALUES 23
C   MAXT = MAXIMUM NUMBER OF TIME VALUES 24
C   MAXXY = MAXX * MAXY 25
C   MAXXY2 = 2 * MAXX * MAXY 26
C   PARAMETER MAXX=100,MAXY=50,MAXT=20,MAXXY=5000,MAXXY2=10000 27
C 28
C   IMPLICIT DOUBLE PRECISION (A-H,O-Z) 29
C   CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS 30
C   REAL XP,YP,CP,TP,DELTA,XPC,YPC,XSCLP,YSCLP 31
C   DIMENSION CXY(MAXX,MAXY),X(MAXX),Y(MAXY),T(MAXT) 32
C   COMMON /PDAT/ XP(MAXX),YP(MAXY),CP(MAXXY),XPC(50),YPC(50), 33
1 IFLAG(MAXXY2) 34
C   COMMON /IOUNIT/ IN,IO 35
C 36
C   PROGRAM VARIABLES 37
C 38
C   NOTE: ANY CONSISTANT SET OF UNITS MAY BE USED IN THE 39
C   MODEL. NO FORMAT STATEMENTS NEED TO BE CHANGED AS 40
C   LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT. 41
C 42
C   CM      MAXIMUM SOLUTE CONCENTRATION AT THE INFLOW BOUNDARY [M/L* 43
C   DX      LONGITUDINAL DISPERSION COEFFICIENT [L**2/T] 44
C   DY      TRANSVERSE DISPERSION COEFFICIENT [L**2/T] 45
C   VX      GROUND-WATER VELOCITY IN X-DIRECTION [L/T] 46
C   DK      FIRST-ORDER SOLUTE DECAY CONSTANT [1/T] 47
C   X       X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L] 48
C   Y       Y-POSITION AT WHICH CONCENTRATION IS EVALUATED [L] 49
C   T       TIME AT WHICH CONCENTRATION IS EVALUATED [T] 50
C   CN      NORMALIZED CONCENTRATION C/CM [DIMENSIONLESS] 51
C   CXY     SOLUTE CONCENTRATION C(X,Y,T) [M/L**3] 52
C   YC     Y-COORDINATE OF THE CENTER OF SOLUTE SOURCE AT X=0 [L] 53

```


C	SIGMA	STANDARD DEVIATION OF GAUSSIAN CONCENTRATION DISTRIBUTION	54
C		FOR THE SOLUTE SOURCE [L]	55
C			56
C	NX	NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED	57
C	NY	NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED	58
C	NT	NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED	59
C	NMAX	NUMBER OF TERMS USED IN GAUSS-LEGENDRE NUMERICAL	60
C		INTEGRATION TECHNIQUE (MUST EQUAL 4, 20, 60, 104 OR 256)	61
C			62
C	IPLT	PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED	63
C	XSCLP	SCALING FACTOR TO CONVERT X TO PLOTTER INCHES	64
C	YSCLP	SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES	65
C	DELTA	CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)	66
C			67
C		CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS	68
C	CUNITS	UNITS OF CONCENTRATION (M/L**3)	69
C	VUNITS	UNITS OF GROUND-WATER VELOCITY (L/T)	70
C	DUNITS	UNITS OF DISPERSION COEFFICIENT (L**2/T)	71
C	KUNITS	UNITS OF SOLUTE DECAY CONSTANT (1/T)	72
C	LUNITS	UNITS OF LENGTH (L)	73
C	TUNITS	UNITS OF TIME (T)	74
C			75
C		DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE	76
	CALL OFILE		77
	CALL TITLE		78
	WRITE(IO,201)		79
C			80
C		READ IN MODEL PARAMETERS	81
	READ(IN,101)	NX,NY,NT,NMAX,IPLT	82
	WRITE(IO,205)	NX,NY,NT,NMAX	83
	READ(IN,105)	CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS	84
	READ(IN,110)	CM,VX,DX,DY,DK	85
	WRITE(IO,210)	CM,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DK,KUNITS	86
	READ(IN,110)	YC,SIGMA	87
	WRITE(IO,212)	YC,LUNITS,SIGMA,LUNITS	88
	READ(IN,110)	(X(I),I=1,NX)	89
	WRITE(IO,215)	LUNITS	90
	WRITE(IO,220)	(X(I),I=1,NX)	91
	READ(IN,110)	(Y(I),I=1,NY)	92
	WRITE(IO,216)	LUNITS	93
	WRITE(IO,220)	(Y(I),I=1,NY)	94
	READ(IN,110)	(T(I),I=1,NT)	95
	WRITE(IO,225)	TUNITS	96
	WRITE(IO,220)	(T(I),I=1,NT)	97
	IF(IPLT.GT.0)	READ(IN,110) XSCLP,YSCLP,DELTA	98
	IF(IPLT.GT.0)	WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS	99
C			100
C		READ IN GAUSS-LEGENDRE POINTS AND WEIGHTING FACTORS	101
	CALL GLQPTS (NMAX)		102
C			103
C		BEGIN TIME LOOP	104
	DO 20 IT=1,NT		105
C			106

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C      BEGIN X LOOP                                     107
DO 40 IX=1,NX                                         108
C                                                     109
C      CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX) 110
DO 50 IY=1,NY                                         111
CALL CNRMLG(DK,T(IT),X(IX),Y(IY),YC,SIGMA,DX,DY,VX,CN,NMAX) 112
CXY(IX,IY)=CM*CN                                     113
50 CONTINUE                                           114
40 CONTINUE                                           115
C                                                     116
C      PRINT OUT TABLES OF CONCENTRATION VALUES          117
NPAGE=1+(NY-1)/9                                     118
DO 60 NP=1,NPAGE                                     119
IF(NP.EQ.1) WRITE(IO,230) T(IT),TUNITS,LUNITS        120
IF(NP.NE.1) WRITE(IO,231) T(IT),TUNITS,LUNITS        121
NP1=(NP-1)*9                                         122
NP2=9                                                 123
IF((NP1+NP2).GT.NY) NP2=NY-NP1                       124
WRITE(IO,235) (Y(NP1+J),J=1,NP2)                     125
WRITE(IO,236) CUNITS,LUNITS                          126
DO 70 IX=1,NX                                         127
WRITE(IO,240) X(IX),(CXY(IX,NP1+J),J=1,NP2)          128
IF(MOD(IX,45).NE.0) GO TO 70                          129
WRITE(IO,231) T(IT),TUNITS,LUNITS                    130
WRITE(IO,235) (Y(NP1+J),J=1,NP2)                     131
WRITE(IO,236) CUNITS,LUNITS                          132
70 IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(IO,241) 133
60 CONTINUE                                           134
C                                                     135
C      CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE 136
C      PLOT SCALING FACTORS. CONVERT C(X,Y) AND DIVIDE BY CM TO PLOT 137
C      CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH TIME VALUE. 138
IF(IPLT.LT.1) GO TO 20                               139
NXY=NX*NY                                            140
DO 80 I=1,NX                                         141
IP=(I-1)*NY                                          142
XP(I)=SNGL(X(I))                                     143
DO 80 J=1,NY                                         144
IF(I.EQ.1) YP(J)=SNGL(Y(J))                          145
CP(IP+J)=SNGL(CXY(I,J)/CM)                          146
80 CONTINUE                                           147
TP=SNGL(T(IT))                                       148
NXY2=NXY*2                                           149
CALL PLOT2D (XP,YP,CP,TP,DELTA,NX,NY,NXY,NXY2,IT,NT,IPLT,TUNITS, 150
1 LUNITS,XSCLP,YSCLP,XPC,YPC,IFLAG)                   151
20 CONTINUE                                           152
CLOSE (IN)                                           153
CLOSE (IO)                                           154
STOP                                                 155
C                                                     156
C      FORMAT STATEMENTS                                  157
101 FORMAT(20I4)                                       158
105 FORMAT(8A10)                                       159

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110  FORMAT(8F10.0) 160
201  FORMAT(/////1H ,30X,'ANALYTICAL SOLUTION TO THE TWO-DIMENSIONAL'/ 161
1 1H ,28X,'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION'/ 162
2 1H ,29X,'FOR A SEMI-INFINITE AQUIFER OF INFINITE WIDTH'/ 163
3 1H ,25X,'WITH A SOLUTE SOURCE HAVING A GAUSSIAN CONCENTRATION'/ 164
4 1H ,24X,'DISTRIBUTION LOCATED AT X=0.0 AND CENTERED ABOUT Y=YC' 165
5 ///1H0,40X,'INPUT DATA'/1H ,40X,10(1H-) 166
205  FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',I4/1H ,25X, 167
1 'NUMBER OF Y-COORDINATES (NY) = ',I4/1H ,25X, 168
2 'NUMBER OF TIME VALUES (NT) = ',I4/1H ,25X, 169
3 'NUMBER OF POINTS FOR NUMERICAL INTEGRATION (NMAX) = ',I4) 170
210  FORMAT(1H0,25X,'MAXIMUM SOLUTE CONCENTRATION AT THE BOUNDARY', 171
1 ' CM) =',1P1E13.6,1X,A10/1H ,25X, 172
2 'GROUND-WATER VELOCITY IN X-DIRECTION (VX) =',1P1E13.6,1X,A10/ 173
3 1H ,25X,'DISPERSION IN THE X-DIRECTION (DX) =',1P1E13.6,1X,A10/ 174
4 1H ,25X,'DISPERSION IN THE Y-DIRECTION (DY) =',1P1E13.6,1X,A10/ 175
5 1H ,25X,'FIRST-ORDER SOLUTE DECAY RATE (DK) =',1P1E13.6,1X,A10) 176
212  FORMAT(1H0,25X,'AQUIFER WIDTH (W) IS INFINITE'/1H ,25X, 177
1 'SOLUTE SOURCE IS CENTERED AT Y =',1P1E13.6,1X,A10/1H ,25X, 178
2 'STANDARD DEVIATION OF GAUSSIAN DISTRIBUTION (SIGMA) =', 179
3 1P1E13.6,1X,A10) 180
215  FORMAT(1H0,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ', 181
1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/) 182
216  FORMAT(1H0,25X,'Y COORDINATES AT WHICH SOLUTE CONCENTRATIONS ', 183
1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/) 184
220  FORMAT(1H ,5X,8F12.4) 185
225  FORMAT(1H0,25X,'TIMES AT WHICH SOLUTE CONCENTRATIONS ' 186
1 'WILL BE CALCULATED, IN ',A10/1H ,25X,70(1H-)/) 187
227  FORMAT(1H0,25X,'PLOT SCALING FACTOR FOR X (XSCLP) =',1P1E13.6/ 188
1 1H ,25X,'PLOT SCALING FACTOR FOR Y (YSCLP) =',1P1E13.6/ 189
2 1H ,25X,'CONTOUR INCREMENT (DELTA) =',1P1E13.6,1X,A10) 190
230  FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =', 191
1 F12.4,1X,A10/ 192
2 1H0,25X,'Y-COORDINATE, IN ',A10) 193
231  FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =', 194
1 F12.4,1X,A10,5X,'(CONTINUED)'/ 195
2 1H0,25X,'Y-COORDINATE, IN ',A10) 196
235  FORMAT(1H ,20X,9F12.4) 197
236  FORMAT(1H ,19X,'*',108(1H-)/ 198
1 1H ,4X,'X-COORDINATE,',2X,'!',44X,'SOLUTE CONCENTRATION, IN ' 199
2 A10/1H ,4X,'IN ',A10,2X,1H!/1H ,19X,'!') 200
240  FORMAT(1H ,5X,F12.4,2X,'!',9F12.6) 201
241  FORMAT(1H ,19X,'!') 202
      END 203
      SUBROUTINE CNRMLG(DK,T,X,Y,YC,SIGMA,DX,DY,VX,CN,NMAX) 204
      IMPLICIT DOUBLE PRECISION(A-H,O-Z) 205
      COMMON /IOUNIT/ IN,IO 206
      COMMON /GLPTS/ WN(256),ZN(256) 207
C 208
C 209
C 210
C 211
C 212
      THIS ROUTINE CALCULATES THE NORMALIZED CONCENTRATION AT X,Y 209
      BASED ON THE ANALYTIC SOLUTION TO THE TWO-DIMENSIONAL 210
      ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION FOR A SEMI- 211
      INFINITE AQUIFER OF INFINITE WIDTH. THE SOLUTE SOURCE, LOCATED 212

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```

C
C *****
C *
C *          **** POINT3 ****
C *
C *   THREE-DIMENSIONAL GROUND-WATER SOLUTE-TRANSPORT
C *
C *   MODEL FOR AN AQUIFER OF INFINITE EXTENT WITH A
C *
C *   CONTINUOUS POINT SOURCE AT X=XC, Y=YC, AND Z=ZC
C *
C *          GROUND-WATER FLOW IN X-DIRECTION ONLY
C *
C *          VERSION CURRENT AS OF 04/01/90
C *
C *****
C
C   THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE
C   GREATER THAN THOSE GIVEN HERE.
C   MAXX = MAXIMUM NUMBER OF X-VALUES
C   MAXY = MAXIMUM NUMBER OF Y-VALUES
C   MAXZ = MAXIMUM NUMBER OF Z-VALUES
C   MAXT = MAXIMUM NUMBER OF TIME VALUES
C   MAXXY = MAXX * MAXY
C   MAXXY2 = 2 * MAXX * MAXY
C   PARAMETER MAXX=100,MAXY=50,MAXZ=30,MAXT=20,MAXXY=5000,MAXXY2=10000
C
C   IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C   CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,QUNITS,TUNITS
C   REAL XP,YP,ZP,CP,TP,DELTA,XPC,YPC,XSCLP,YSCLP
C   DIMENSION CXY(MAXX,MAXY),X(MAXX),Y(MAXY),Z(MAXZ),T(MAXT)
C   COMMON /PDAT/ XP(MAXX),YP(MAXY),CP(MAXXY),XPC(50),YPC(50),
1 IFLAG(MAXXY2)
C   COMMON /IOUNIT/ IN,IO
C
C   PROGRAM VARIABLES
C
C   NOTE: ANY CONSISTANT SET OF UNITS MAY BE USED IN THE
C   MODEL. NO FORMAT STATEMENTS NEED TO BE CHANGED AS
C   LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.
C
C   CO      SOLUTE CONCENTRATION IN INJECTED FLUID [M/L**3]
C   DX      LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
C   DY      TRANSVERSE (Y-DIRECTION) DISPERSION COEFFICIENT [L**2/T]
C   DZ      TRANSVERSE (Z-DIRECTION) DISPERSION COEFFICIENT [L**2/T]
C   VX      GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
C   DK      FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
C   X       X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C   Y       Y-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C   Z       Z-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C   T       TIME AT WHICH CONCENTRATION IS EVALUATED [T]
C   CN      NORMALIZED CONCENTRATION C/CO [DIMENSIONLESS]
C   CXY     SOLUTE CONCENTRATION C(X,Y,Z,T) [M/L**3]

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C   XC      X-COORDINATE OF CONTINUOUS POINT SOURCE [L]          54
C   YC      Y-COORDINATE OF CONTINUOUS POINT SOURCE [L]          55
C   ZC      Z-COORDINATE OF CONTINUOUS POINT SOURCE [L]          56
C   QM      FLUID INJECTION RATE [L**3/T]                        57
C   POR     AQUIFER POROSITY [DIMENSIONLESS]                     58
C                                                  59
C   NX      NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED 60
C   NY      NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED 61
C   NZ      NUMBER OF Z-POSITIONS AT WHICH SOLUTION IS EVALUATED 62
C   NT      NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED 63
C                                                  64
C   IPLT    PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED    65
C   XSCLP   SCALING FACTOR TO CONVERT X TO PLOTTER INCHES        66
C   YSCLP   SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES        67
C   DELTA   CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0) 68
C                                                  69
C   CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS 70
C   CUNITS  UNITS OF CONCENTRATION (M/L**3)                       71
C   VUNITS  UNITS OF GROUND-WATER VELOCITY (L/T)                 72
C   DUNITS  UNITS OF DISPERSION COEFFICIENT (L**2/T)             73
C   KUNITS  UNITS OF SOLUTE DECAY CONSTANT (1/T)                 74
C   LUNITS  UNITS OF LENGTH (L)                                   75
C   QUNITS  UNITS OF FLUID INJECTION RATE (L**3/T)               76
C   TUNITS  UNITS OF TIME (T)                                     77
C                                                  78
C   DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE                79
C   CALL OFILE                                                    80
C   CALL TITLE                                                    81
C   WRITE(IO,201)                                                 82
C                                                  83
C   READ IN MODEL PARAMETERS                                       84
C   READ(IN,101) NX,NY,NZ,NT,IPLT                                  85
C   WRITE(IO,205) NX,NY,NZ,NT                                     86
C   READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,QUNITS,TUNITS 87
C   READ(IN,110) C0,VX,DX,DY,DZ,DK                               88
C   WRITE(IO,210) C0,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DZ,DUNITS, 89
1  DK,KUNITS                                                    90
C   READ(IN,110) XC,YC,ZC,QM,POR                                  91
C   WRITE(IO,212) XC,LUNITS,YC,LUNITS,ZC,LUNITS,QM,QUNITS,POR    92
C   READ(IN,110) (X(I),I=1,NX)                                    93
C   WRITE(IO,215) LUNITS                                          94
C   WRITE(IO,220) (X(I),I=1,NX)                                    95
C   READ(IN,110) (Y(I),I=1,NY)                                    96
C   WRITE(IO,216) LUNITS                                          97
C   WRITE(IO,220) (Y(I),I=1,NY)                                    98
C   READ(IN,110) (Z(I),I=1,NZ)                                    99
C   WRITE(IO,217) LUNITS                                          100
C   WRITE(IO,220) (Z(I),I=1,NZ)                                    101
C   READ(IN,110) (T(I),I=1,NT)                                    102
C   WRITE(IO,225) TUNITS                                          103
C   WRITE(IO,220) (T(I),I=1,NT)                                    104
C   IF(IPLT.GT.0) READ(IN,110) XSCLP,YSCLP,DELTA                105
C   IF(IPLT.GT.0) WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS        106

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C		107
C	BEGIN TIME LOOP	108
	DO 20 IT=1,NT	109
C		110
C	BEGIN Z LOOP	111
	DO 30 IZ=1,NZ	112
	ZZ=Z(IZ)-ZC	113
C		114
C	BEGIN X LOOP	115
	DO 40 IX=1,NX	116
	XX=X(IX)-XC	117
C		118
C	CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX) AND Z=Z(IZ)	119
	DO 50 IY=1,NY	120
	YY=Y(IY)-YC	121
	CALL CNRML3(QM,POR,DK,T(IT),XX,YY,ZZ,DX,DY,DZ,VX,CN)	122
	CXY(IX,IY)=C0*CN	123
50	CONTINUE	124
40	CONTINUE	125
C		126
C	PRINT OUT TABLES OF CONCENTRATION VALUES	127
	NPAGE=1+(NY-1)/9	128
	DO 60 NP=1,NPAGE	129
	IF(NP.EQ.1) WRITE(IO,230) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS	130
	IF(NP.NE.1) WRITE(IO,231) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS	131
	NP1=(NP-1)*9	132
	NP2=9	133
	IF((NP1+NP2).GT.NY) NP2=NY-NP1	134
	WRITE(IO,235) (Y(NP1+J),J=1,NP2)	135
	WRITE(IO,236) CUNITS,LUNITS	136
	DO 70 IX=1,NX	137
	WRITE(IO,240) X(IX),(CXY(IX,NP1+J),J=1,NP2)	138
	IF(MOD(IX,45).NE.0) GO TO 70	139
	WRITE(IO,231) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS	140
	WRITE(IO,235) (Y(NP1+J),J=1,NP2)	141
	WRITE(IO,236) CUNITS,LUNITS	142
70	IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(IO,241)	143
60	CONTINUE	144
C		145
C	CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE	146
C	PLOT SCALING FACTORS. CONVERT C(X,Y) AND DIVIDE BY C0 TO PLOT	147
C	CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH TIME VALUE.	148
	IF(IPLT.LT.1) GO TO 30	149
	NXY=NX*NY	150
	DO 80 I=1,NX	151
	IP=(I-1)*NY	152
	XP(I)=SNGL(X(I))	153
	DO 80 J=1,NY	154
	IF(I.EQ.1) YP(J)=SNGL(Y(J))	155
	CP(IP+J)=SNGL(CXY(I,J)/C0)	156
80	CONTINUE	157
	TP=SNGL(T(IT))	158
	ZP=SNGL(Z(IZ))	159

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      NXY2=NXY*2                                     160
      CALL PLOT3D (XP,YP,ZP,CP,TP,DELTA,NX,NY,NXY,NXY2,IZ,NZ,IPLT, 161
1 TUNITS,LUNITS,XSCLP,YSCLP,XPC,YPC,IFLAG)         162
30 CONTINUE                                         163
20 CONTINUE                                         164
      CLOSE (IN)                                     165
      CLOSE (IO)                                     166
      STOP                                           167
C                                                    168
C      FORMAT STATEMENTS                             169
101 FORMAT(20I4)                                     170
105 FORMAT(8A10)                                     171
110 FORMAT(8F10.0)                                   172
201 FORMAT(/////1H ,29X,'ANALYTICAL SOLUTION TO THE THREE-DIMENSIONAL' 173
1 /1H ,28X,'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION'/ 174
2 1H ,34X,'FOR AN AQUIFER OF INFINITE EXTENT'/ 175
3 1H ,30X,'WITH A CONTINUOUS POINT SOURCE AT XC,YC,ZC' 176
4 ///1H0,40X,'INPUT DATA'/1H ,40X,10(1H-)) 177
205 FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',I4/1H ,25X, 178
1 'NUMBER OF Y-COORDINATES (NY) = ',I4/1H ,25X, 179
2 'NUMBER OF Z-COORDINATES (NZ) = ',I4/1H ,25X, 180
3 'NUMBER OF TIME VALUES (NT) = ',I4) 181
210 FORMAT(1H0,25X,'SOLUTE CONCENTRATION IN INJECTED FLUID (C0) = ', 182
1 1P1E13.6,1X,A10/1H ,25X, 183
2 'GROUND-WATER VELOCITY IN X-DIRECTION (VX) = ',1P1E13.6,1X,A10/ 184
3 1H ,25X,'DISPERSION IN THE X-DIRECTION (DX) = ',1P1E13.6,1X,A10/ 185
4 1H ,25X,'DISPERSION IN THE Y-DIRECTION (DY) = ',1P1E13.6,1X,A10/ 186
5 1H ,25X,'DISPERSION IN THE Z-DIRECTION (DZ) = ',1P1E13.6,1X,A10/ 187
6 1H ,25X,'FIRST-ORDER SOLUTE DECAY RATE (DK) = ',1P1E13.6,1X,A10) 188
212 FORMAT(1H0,25X,'AQUIFER IS OF INFINITE EXTENT' 189
2 /1H0,25X,'CONTINUOUS POINT SOURCE IS AT X = ',1P1E13.6,1X,A10/ 190
3 1H ,55X,'Y = ',1P1E13.6,1X,A10/1H ,55X,'Z = ',1P1E13.6,1X,A10/ 191
5 1H ,25X,'FLUID INJECTION RATE (QM) = ',1P1E13.6,1X,A10/ 192
6 1H ,25X,'AQUIFER POROSITY (POR) = ',1P1E13.6) 193
215 FORMAT(1H0,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ', 194
1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/) 195
216 FORMAT(1H0,25X,'Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ', 196
1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/) 197
217 FORMAT(1H0,25X,'Z-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ', 198
1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/) 199
220 FORMAT(1H ,5X,8F12.4) 200
225 FORMAT(1H0,25X,'TIMES AT WHICH SOLUTE CONCENTRATIONS ' 201
1 'WILL BE CALCULATED, IN ',A10/1H ,25X,70(1H-)/) 202
227 FORMAT(1H0,25X,'PLOT SCALING FACTOR FOR X (XSCLP) = ',1P1E13.6/ 203
1 1H ,25X,'PLOT SCALING FACTOR FOR Y (YSCLP) = ',1P1E13.6/ 204
2 1H ,25X,'CONTOUR INCREMENT (DELTA) = ',1P1E13.6,1X,A10) 205
230 FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME = ', 206
1 F12.4,1X,A10/1H ,35X,'AND AT Z = ',F12.4,1X,A10/ 207
2 1H0,25X,'Y-COORDINATE, IN ',A10) 208
231 FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME = ', 209
1 F12.4,1X,A10,5X,'(CONTINUED)'/1H ,35X,'AND AT Z = ',F12.4,1X,A10/ 210
2 1H0,25X,'Y-COORDINATE, IN ',A10) 211
235 FORMAT(1H ,20X,9F12.4) 212

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236  FORMAT(1H ,19X,'*',108(1H-)/                213
      1 1H ,4X,'X-COORDINATE,',2X,'!',44X,'SOLUTE CONCENTRATION, IN ' 214
      2 A10/1H ,4X,'IN ',A10,2X,1H!/1H ,19X,'!') 215
240  FORMAT(1H ,5X,F12.4,2X,'!',9F12.6)          216
241  FORMAT(1H ,19X,'!')                          217
      END                                          218
      SUBROUTINE CNRML3(QM,POR,DK,T,X,Y,Z,DX,DY,DZ,VX,CN) 219
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)         220
      COMMON /IOUNIT/ IN,IO                      221
C                                          222
C      THIS ROUTINE CALCULATES SOLUTE CONCENTRATION AT X,Y,Z BASED ON 223
C      THE ANALYTIC SOLUTION TO THE THREE-DIMENSIONAL ADVECTIVE-    224
C      DISPERSIVE SOLUTE TRANSPORT EQUATION FOR AN AQUIFER OF        225
C      INFINITE EXTENT WITH A CONTINUOUS POINT SOURCE LOCATED AT    226
C      X=XC, Y=YC, AND Z=ZC. A CLOSED FORM SOLUTION WAS OBTAINED.  227
C                                          228
      PI=3.14159265358979D0                       229
      CN=0.0D0                                     230
C                                          231
C      FOR T=0, ALL CONCENTRATIONS EQUAL 0.0                232
C      IF(T.LE.0.0D0) RETURN                             233
C                                          234
C      CHECK FOR X=Y=Z=0                                    235
C      IF(X.EQ.0.0D0 .AND. Y.EQ.0.0D0 .AND. Z.EQ.0.0D0) THEN 236
      WRITE(IO,200)                                 237
      RETURN                                         238
      END IF                                          239
C                                          240
      BETA=DSQRT(VX*VX+4.0D0*DX*DK)                 241
      GAMMA=DSQRT(X*X+Y*Y*DX/DY+Z*Z*DX/DZ)          242
      RTDXT=2.0D0*DSQRT(DX*T)                       243
C                                          244
C      TERM 1                                               245
C      X1=(VX*X-GAMMA*BETA)/(2.0D0*DX)               246
      Y1=(GAMMA-BETA*T)/RTDXT                       247
      CALL EXERFC(X1,Y1,Z1)                          248
C                                          249
C      TERM 2                                               250
C      X2=(VX*X+GAMMA*BETA)/(2.0D0*DX)               251
      Y2=(GAMMA+BETA*T)/RTDXT                       252
      CALL EXERFC(X2,Y2,Z2)                          253
C                                          254
C      TERM 3                                               255
C      Z3=Z1+Z2                                         256
      Z4=DSQRT(DY*DZ)                                257
      CN=QM*Z3/(8.0D0*POR*PI*GAMMA*Z4)             258
      RETURN                                         259
C                                          260
C      FORMAT STATEMENTS                                    261
200  FORMAT (1H0,5X,'**** WARNING **** A SOLUTION CAN NOT BE COMPUTED' 262
      1 ' FOR X=XC,Y=YC,Z=ZC' /)                    263
      END                                          264

```

C		1
C	*****	2
C	*	3
C	**** PATCHF ****	4
C	*	5
C	* THREE-DIMENSIONAL GROUND-WATER SOLUTE-TRANSPORT	6
C	*	7
C	* MODEL FOR A SEMI-INFINITE AQUIFER WITH A FINITE	8
C	*	9
C	* WIDTH AND HEIGHT. A PATCH SOURCE EXTENDS FROM	10
C	*	11
C	* Y1 TO Y2 AND Z1 TO Z2 AT X=0	12
C	*	13
C	* GROUND-WATER FLOW IN X-DIRECTION ONLY	14
C	*	15
C	* VERSION CURRENT AS OF 04/01/90	16
C	*	17
C	*****	18
C		19
C	THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE	20
C	GREATER THAN THOSE GIVEN HERE.	21
C	MAXX = MAXIMUM NUMBER OF X-VALUES	22
C	MAXY = MAXIMUM NUMBER OF Y-VALUES	23
C	MAXZ = MAXIMUM NUMBER OF Z-VALUES	24
C	MAXT = MAXIMUM NUMBER OF TIME VALUES	25
C	MAXXY = MAXX * MAXY	26
C	MAXXY2 = 2 * MAXX * MAXY	27
C	PARAMETER MAXX=100,MAXY=50,MAXZ=30,MAXT=20,MAXXY=5000,MAXXY2=10000	28
C		29
C	IMPLICIT DOUBLE PRECISION (A-H,O-Z)	30
C	CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS	31
C	CHARACTER*1 IERR(MAXX,MAXY)	32
C	REAL XP,YP,ZP,CP,TP,DELTA,XPC,YPC,XSCLP,YSCLP	33
C	DIMENSION CXY(MAXX,MAXY),X(MAXX),Y(MAXY),Z(MAXZ),T(MAXT)	34
C	COMMON /PDAT/ XP(MAXX),YP(MAXY),CP(MAXXY),XPC(50),YPC(50),	35
C	1 IFLAG(MAXXY2)	36
C	COMMON /IUNIT/ IN,IO	37
C		38
C	PROGRAM VARIABLES	39
C		40
C	NOTE: ANY CONSISTANT SET OF UNITS MAY BE USED IN THE	41
C	MODEL. NO FORMAT STATEMENTS NEED TO BE CHANGED AS	42
C	LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.	43
C		44
C	C0 SOLUTE CONCENTRATION AT THE INFLOW BOUNDARY [M/L**3]	45
C	DX LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]	46
C	DY TRANSVERSE (Y-DIRECTION) DISPERSION COEFFICIENT [L**2/T]	47
C	DZ TRANSVERSE (Z-DIRECTION) DISPERSION COEFFICIENT [L**2/T]	48
C	VX GROUND-WATER VELOCITY IN X-DIRECTION [L/T]	49
C	DK FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]	50
C	X X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]	51
C	Y Y-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]	52
C	Z Z-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]	53

C	T	TIME AT WHICH CONCENTRATION IS EVALUATED [T]	54
C	CN	NORMALIZED CONCENTRATION C/CO [DIMENSIONLESS]	55
C	CXY	SOLUTE CONCENTRATION C(X,Y,Z,T) [M/L**3]	56
C	W	AQUIFER WIDTH (AQUIFER EXTENDS FROM Y=0 TO Y=W) [L]	57
C	H	AQUIFER HEIGHT (AQUIFER EXTENDS FROM Z=0 TO Z=H) [L]	58
C	Y1	Y-COORDINATE OF LOWER LIMIT OF PATCH SOLUTE SOURCE [L]	59
C	Y2	Y-COORDINATE OF UPPER LIMIT OF PATCH SOLUTE SOURCE [L]	60
C	Z1	Z-COORDINATE OF LOWER LIMIT OF PATCH SOLUTE SOURCE [L]	61
C	Z2	Z-COORDINATE OF UPPER LIMIT OF PATCH SOLUTE SOURCE [L]	62
C			63
C	NX	NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED	64
C	NY	NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED	65
C	NZ	NUMBER OF Z-POSITIONS AT WHICH SOLUTION IS EVALUATED	66
C	NT	NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED	67
C	NMAX	NUMBER OF TERMS USED IN INNER INFINITE SERIES SUMMATION	68
C	MMAX	NUMBER OF TERMS USED IN OUTER INFINITE SERIES SUMMATION	69
C			70
C	IPLT	PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED	71
C	XSCLP	SCALING FACTOR TO CONVERT X TO PLOTTER INCHES	72
C	YSCLP	SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES	73
C	DELTA	CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)	74
C			75
C		CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS	76
C	CUNITS	UNITS OF CONCENTRATION (M/L**3)	77
C	VUNITS	UNITS OF GROUND-WATER VELOCITY (L/T)	78
C	DUNITS	UNITS OF DISPERSION COEFFICIENT (L**2/T)	79
C	KUNITS	UNITS OF SOLUTE DECAY CONSTANT (1/T)	80
C	LUNITS	UNITS OF LENGTH (L)	81
C	TUNITS	UNITS OF TIME (T)	82
C			83
C		DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE	84
	CALL OFILE		85
	CALL TITLE		86
	WRITE(IO,201)		87
C			88
C		READ IN MODEL PARAMETERS	89
	READ(IN,101) NX,NY,NZ,NT,NMAX,MMAX,IPLT		90
	WRITE(IO,205) NX,NY,NZ,NT,NMAX,MMAX		91
	READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS		92
	READ(IN,110) CO,VX,DX,DY,DZ,DK		93
	WRITE(IO,210) CO,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DZ,DUNITS,		94
1	DK,KUNITS		95
	READ(IN,110) W,H,Y1,Y2,Z1,Z2		96
	WRITE(IO,212) W,LUNITS,H,LUNITS,Y1,LUNITS,Y2,LUNITS,Z1,LUNITS,		97
1	Z2,LUNITS		98
	READ(IN,110) (X(I),I=1,NX)		99
	WRITE(IO,215) LUNITS		100
	WRITE(IO,220) (X(I),I=1,NX)		101
	READ(IN,110) (Y(I),I=1,NY)		102
	WRITE(IO,216) LUNITS		103
	WRITE(IO,220) (Y(I),I=1,NY)		104
	READ(IN,110) (Z(I),I=1,NZ)		105
	WRITE(IO,217) LUNITS		106

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WRITE(IO,220) (Z(I),I=1,NZ) 107
READ(IN,110) (T(I),I=1,NT) 108
WRITE(IO,225) TUNITS 109
WRITE(IO,220) (T(I),I=1,NT) 110
IF(IPLT.GT.0) READ(IN,110) XSCLP,YSCLP,DELTA 111
IF(IPLT.GT.0) WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS 112
C 113
C BEGIN TIME LOOP 114
DO 20 IT=1,NT 115
C 116
C BEGIN Z LOOP 117
DO 30 IZ=1,NZ 118
C 119
C BEGIN X LOOP 120
DO 40 IX=1,NX 121
C 122
C CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX) AND Z=Z(IZ) 123
DO 50 IY=1,NY 124
CALL CNRMLP(DK,T(IT),X(IX),Y(IY),Z(IZ),W,H,Y1,Y2,Z1,Z2,DX,
1 DY,DZ,VX,CN,NMAX,MMAX,IERR(IX,IY)) 125
CXY(IX,IY)=C0*CN 126
50 CONTINUE 127
40 CONTINUE 128
C 129
C PRINT OUT TABLES OF CONCENTRATION VALUES 130
C NPAGE=1+(NY-1)/9 131
DO 60 NP=1,NPAGE 132
IF(NP.EQ.1) WRITE(IO,230) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS 134
IF(NP.NE.1) WRITE(IO,231) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS 135
NP1=(NP-1)*9 136
NP2=9 137
IF((NP1+NP2).GT.NY) NP2=NY-NP1 138
WRITE(IO,235) (Y(NP1+J),J=1,NP2) 139
WRITE(IO,236) CUNITS,LUNITS 140
DO 70 IX=1,NX 141
WRITE(IO,240) X(IX),(CXY(IX,NP1+J),IERR(IX,NP1+J),J=1,NP2) 142
IF(MOD(IX,45).NE.0) GO TO 70 143
WRITE(IO,231) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS 144
WRITE(IO,235) (Y(NP1+J),J=1,NP2) 145
WRITE(IO,236) CUNITS,LUNITS 146
70 IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(IO,241) 147
60 CONTINUE 148
C 149
C CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE 150
C PLOT SCALING FACTORS. CONVERT C(X,Y) AND DIVIDE BY C0 TO PLOT 151
C CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH TIME VALUE. 152
IF(IPLT.LT.1) GO TO 30 153
NXY=NX*NY 154
DO 80 I=1,NX 155
IP=(I-1)*NY 156
XP(I)=SNGL(X(I)) 157
DO 80 J=1,NY 158
IF(I.EQ.1) YP(J)=SNGL(Y(J)) 159

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      CP(IP+J)=SNGL(CXY(I,J)/CO)
80    CONTINUE
      TP=SNGL(T(IT))
      ZP=SNGL(Z(IZ))
      NXY2=NXY*2
      CALL PLOT3D (XP,YP,ZP,CP,TP,DELTA,NX,NY,NXY,NXY2,IZ,NZ,IPLT,
1     TUNITS,LUNITS,XSCLP,YSCLP,XPC,YPC,IFLAG)
30    CONTINUE
20    CONTINUE
      CLOSE (IN)
      CLOSE (IO)
      STOP
C
C     FORMAT STATEMENTS
101   FORMAT(20I4)
105   FORMAT(8A10)
110   FORMAT(8F10.0)
201   FORMAT(/////1H ,29X,'ANALYTICAL SOLUTION TO THE THREE-DIMENSIONAL'
1     /1H ,28X,'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION'/
2     1H ,30X,'FOR A SEMI-INFINITE AQUIFER OF FINITE WIDTH'/
3     1H ,28X,'AND HEIGHT WITH A PATCH SOLUTE SOURCE AT X=0.0'
4     ///1H0,40X,'INPUT DATA'/1H ,40X,10(1H-))
205   FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',I4/1H ,25X,
1     'NUMBER OF Y-COORDINATES (NY) = ',I4/1H ,25X,
2     'NUMBER OF Z-COORDINATES (NZ) = ',I4/1H ,25X,
3     'NUMBER OF TIME VALUES (NT) = ',I4/1H ,25X,
4     'NUMBER OF TERMS IN INNER INFINITE SERIES SUMMATION (NMAX) = ',
5     I4/1H ,25X,
6     'NUMBER OF TERMS IN OUTER INFINITE SERIES SUMMATION (MMAX) = ',I4)
210   FORMAT(1H0,25X,'SOLUTE CONCENTRATION ON MODEL BOUNDARY (CO) =',
1     1P1E13.6,1X,A10/1H ,25X,
2     'GROUND-WATER VELOCITY IN X-DIRECTION (VX) =',1P1E13.6,1X,A10/
3     1H ,25X,'DISPERSION IN THE X-DIRECTION (DX) =',1P1E13.6,1X,A10/
4     1H ,25X,'DISPERSION IN THE Y-DIRECTION (DY) =',1P1E13.6,1X,A10/
5     1H ,25X,'DISPERSION IN THE Z-DIRECTION (DZ) =',1P1E13.6,1X,A10/
6     1H ,25X,'FIRST-ORDER SOLUTE DECAY RATE (DK) =',1P1E13.6,1X,A10)
212   FORMAT(1H0,25X,'AQUIFER WIDTH (W) =',1P1E13.6,1X,A10/1H ,25X,
1     'AQUIFER HEIGHT (H) =',1P1E13.6,1X,A10/1H ,25X,
2     'SOLUTE SOURCE IS LOCATED BETWEEN Y1 =',1P1E13.6,1X,A10/1H ,58X,
3     'Y2 =',1P1E13.6,1X,A10/1H ,58X,
4     'Z1 =',1P1E13.6,1X,A10/1H ,54X,
5     'AND Z2 =',1P1E13.6,1X,A10)
215   FORMAT(1H0,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
1     'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
216   FORMAT(1H0,25X,'Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
1     'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
217   FORMAT(1H0,25X,'Z-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
1     'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
220   FORMAT(1H ,5X,8F12.4)
225   FORMAT(1H0,25X,'TIMES AT WHICH SOLUTE CONCENTRATIONS '
1     'WILL BE CALCULATED, IN ',A10/1H ,25X,70(1H-)/)
227   FORMAT(1H0,25X,'PLOT SCALING FACTOR FOR X (XSCLP) =',1P1E13.6/
1     1H ,25X,'PLOT SCALING FACTOR FOR Y (YSCLP) =',1P1E13.6/

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230 2 1H ,25X,'CONTOUR INCREMENT (DELTA) =',1P1E13.6,1X,A10) 213
    FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =', 214
1 F12.4,1X,A10/1H ,35X,'AND AT Z =',F12.4,1X,A10, 215
1 15X,'* INDICATES SOLUTION DID NOT CONVERGE'/ 216
2 1H0,25X,'Y-COORDINATE, IN ',A10) 217
231 FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =', 218
1 F12.4,1X,A10,5X,'(CONTINUED)'/1H ,35X,'AND AT Z =',F12.4,1X, 219
2 A10/1H0,25X,'Y-COORDINATE, IN ',A10) 220
235 FORMAT(1H ,20X,9F12.4) 221
236 FORMAT(1H ,19X,'*',108(1H-)/ 222
1 1H ,4X,'X-COORDINATE,',2X,'!',44X,'SOLUTE CONCENTRATION, IN ' 223
2 A10/1H ,4X,'IN ',A10,2X,1H!/1H ,19X,'!') 224
240 FORMAT(1H ,5X,F12.4,2X,'! ',9(F11.5,A1)) 225
241 FORMAT(1H ,19X,'!') 226
    END 227
    SUBROUTINE CNRMLP(DK,T,X,Y,Z,W,H,Y1,Y2,Z1,Z2,DX,DY,DZ,VX,CN,NMAX, 228
1 MMAX,IERR) 229
    IMPLICIT DOUBLE PRECISION(A-H,O-Z) 230
    CHARACTER*1 IERR 231
    COMMON /IOUNIT/ IN,IO 232
C 233
C THIS ROUTINE CALCULATES THE NORMALIZED CONCENTRATION AT X,Y,Z 234
C BASED ON THE ANALYTIC SOLUTION TO THE THREE-DIMENSIONAL 235
C ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION FOR A SEMI- 236
C INFINITE AQUIFER WITH A FINITE WIDTH AND HEIGHT. THE SOLUTE 237
C SOURCE HAS A FINITE WIDTH AND HEIGHT, EXTENDING FROM Y=Y1 TO 238
C Y=Y2 AND Z=Z1 TO Z=Z2. SOLUTE MAY BE SUBJECT TO FIRST-ORDER 239
C CHEMICAL TRANSFORMATION. THE SOLUTION CONTAINS TWO INFINITE 240
C SERIES SUMMATIONS WHICH MAY CONVERGE SLOWLY. 241
C 242
    PI=3.14159265358979D0 243
    CN=0.0D0 244
    IERR=' ' 245
C 246
C FOR T=0, ALL CONCENTRATIONS EQUAL 0.0 247
    IF(T.LE.0.0D0) RETURN 248
C 249
C FOR X=0.0, CONCENTRATIONS ARE SPECIFIED BY BOUNDARY CONDITIONS 250
    IF(X.GT.0.0D0) GO TO 10 251
    IF(Y.EQ.Y1.OR.Y.EQ.Y2) THEN 252
        IF(Z.GT.Z1.AND.Z.LT.Z2) CN=0.50D0 253
        IF(Z.EQ.Z1.OR.Z.EQ.Z2) CN=0.25D0 254
    END IF 255
    IF(Z.EQ.Z1.OR.Z.EQ.Z2) THEN 256
        IF(Y.GT.Y1.AND.Y.LT.Y2) CN=0.50D0 257
    END IF 258
    IF(Y.GT.Y1.AND.Y.LT.Y2.AND.Z.GT.Z1.AND.Z.LT.Z2) CN=1.0D0 259
    RETURN 260
10 RTDXT=2.0D0*DSQRT(DX*T) 261
C 262
C BEGIN SUMMATION OF TERMS IN INFINITE SERIES (OUTER SERIES) 263
    NMAX1=NMAX+1 264
    MMAX1=MMAX+1 265

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SIGMAM=0.0DO                                266
SUBTM=0.0DO                                  267
DO 20 MM=1,MMAX1                             268
M=MM-1                                        269
ZETA=M*PI/H                                  270
OM=(Z2-Z1)/H                                  271
IF(M.NE.0) OM=(DSIN(ZETA*Z2)-DSIN(ZETA*Z1))/(M*PI) 272
COSSZ=DCOS(ZETA*Z)                            273
C                                              274
C      BEGIN SUMMATION OF TERMS IN INFINITE SERIES (INNER SERIES) 275
SIGMAM=0.0DO                                  276
SUBTN=0.0DO                                  277
DO 30 NN=1,NMAX1                             278
N=NN-1                                        279
ETA=N*PI/W                                    280
PN=(Y2-Y1)/W                                  281
IF(N.NE.0) PN=(DSIN(ETA*Y2)-DSIN(ETA*Y1))/(N*PI) 282
COSRY=DCOS(ETA*Y)                             283
ALPHA=4.0DO*DX*(ETA*ETA*DY+ZETA*ZETA*DZ+DK) 284
BETA=DSQRT(VX*VX+ALPHA)                       285
BETAT=BETA*T                                  286
C                                              287
C      IF M>0 AND N>0, USE GENERAL FORM                288
C      TERM 1                                           289
A1=X*(VX-BETA)/(2.0DO*DX)                     290
B1=(X-BETAT)/RTDXT                             291
CALL EXERFC(A1,B1,C1)                           292
A2=X*(VX+BETA)/(2.0DO*DX)                     293
B2=(X+BETAT)/RTDXT                             294
CALL EXERFC(A2,B2,C2)                           295
TERM1=COSRY*PN*(C1+C2)                         296
C                                              297
C      MULTIPLY TERM BY L(MN)                            298
IF(M.EQ.0 .AND. N.EQ.0) TERM1=TERM1*0.50DO 299
IF(M.GT.0 .AND. N.GT.0) TERM1=TERM1*2.0DO 300
C                                              301
C      ADD TERM TO SUMMATION                             302
SIGMAM=SIGMAM+TERM1                            303
C                                              304
C      CHECK FOR CONVERGENCE OF INNER SERIES. BECAUSE SERIES 305
C      OSCILLATES, CHECK SUBTOTAL OF LAST 10 TERMS.    316
SUBTN=SUBTN+TERM1                              307
IF(MOD(NN,10).NE.0) GO TO 30                   308
IF(DABS(SUBTN).LT.1.0D-12) GO TO 25           309
SUBTN=0.0DO                                    310
30 CONTINUE                                    311
IERR='*'                                       312
25 SIGMAM=SIGMAM+SIGMAN*COSSZ*OM               313
C                                              314
C      CHECK FOR CONVERGENCE OF OUTER SERIES. BECAUSE SERIES 315
C      OSCILLATES, CHECK SUBTOTAL OF LAST 10 TERMS.    316
SUBTM=SUBTM+SIGMAN*OM*COSSZ                   317
IF(MOD(MM,10).NE.0) GO TO 20                  318

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	IF(DABS(SUBTM).LT.1.0D-12) GO TO 35	319
	SUBTM=0.0D0	320
20	CONTINUE	321
	IERR='*'	322
35	CN=SIGMAM	323
	RETURN	324
	END	325


```

C
C *****
C *
C *          **** PATCHI ****
C *
C * THREE-DIMENSIONAL GROUND-WATER SOLUTE-TRANSPORT
C *
C * MODEL FOR A SEMI-INFINITE AQUIFER OF INFINITE
C *
C * WIDTH AND HEIGHT. PATCH SOURCE EXTENDING FROM
C *
C *          Y1 TO Y2 AND Z1 TO Z2 LOCATED AT X=0
C *
C *          GROUND-WATER FLOW IN X-DIRECTION ONLY
C *
C *          VERSION CURRENT AS OF 04/01/90
C *
C *****
C
C THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE
C GREATER THAN THOSE GIVEN HERE.
C MAXX = MAXIMUM NUMBER OF X-VALUES
C MAXY = MAXIMUM NUMBER OF Y-VALUES
C MAXZ = MAXIMUM NUMBER OF Z-VALUES
C MAXT = MAXIMUM NUMBER OF TIME VALUES
C MAXXY = MAXX * MAXY
C MAXXY2 = 2 * MAXX * MAXY
C PARAMETER MAXX=100,MAXY=50,MAXZ=30,MAXT=20,MAXXY=5000,MAXXY2=10000
C
C IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,TUNITS
C REAL XP,YP,ZP,CP,TP,DELTA,XPC,YPC,XSCLP,YSCLP
C DIMENSION CDY(MAXX,MAXY),X(MAXX),Y(MAXY),Z(MAXZ),T(MAXT)
C COMMON /PDAT/ XP(MAXX),YP(MAXY),CP(MAXXY),XPC(50),YPC(50),
1 IFLAG(MAXXY2)
C COMMON /IOUNIT/ IN,IO
C
C PROGRAM VARIABLES
C
C NOTE: ANY CONSISTANT SET OF UNITS MAY BE USED IN THE
C MODEL. NO FORMAT STATEMENTS NEED TO BE CHANGED AS
C LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.
C
C CO SOLUTE CONCENTRATION AT THE INFLOW BOUNDARY [M/L**3]
C DX LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
C DY TRANSVERSE (Y-DIRECTION) DISPERSION COEFFICIENT [L**2/T]
C DZ TRANSVERSE (Z-DIRECTION) DISPERSION COEFFICIENT [L**2/T]
C VX GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
C DK FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
C X X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C Y Y-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C Z Z-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C T TIME AT WHICH CONCENTRATION IS EVALUATED [T]

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C	CN	NORMALIZED CONCENTRATION C/C0 [DIMENSIONLESS]	54
C	CXY	SOLUTE CONCENTRATION C(X,Y,Z,T) [M/L**3]	55
C	Y1	Y-COORDINATE OF LOWER LIMIT OF PATCH SOLUTE SOURCE [L]	56
C	Y2	Y-COORDINATE OF UPPER LIMIT OF PATCH SOLUTE SOURCE [L]	57
C	Z1	Z-COORDINATE OF LOWER LIMIT OF PATCH SOLUTE SOURCE [L]	58
C	Z2	Z-COORDINATE OF UPPER LIMIT OF PATCH SOLUTE SOURCE [L]	59
C			60
C	NX	NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED	61
C	NY	NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED	62
C	NZ	NUMBER OF Z-POSITIONS AT WHICH SOLUTION IS EVALUATED	63
C	NT	NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED	64
C	NMAX	NUMBER OF TERMS USED IN GAUSS-LEGENDRE NUMERICAL	65
C		INTEGRATION TECHNIQUE (MUST EQUAL 4, 20, 60, 104 OR 256)	66
C			67
C	IPLT	PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED	68
C	XSCLP	SCALING FACTOR TO CONVERT X TO PLOTTER INCHES	69
C	YSCLP	SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES	70
C	DELTA	CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)	71
C			72
C		CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS	73
C	CUNITS	UNITS OF CONCENTRATION (M/L**3)	74
C	VUNITS	UNITS OF GROUND-WATER VELOCITY (L/T)	75
C	DUNITS	UNITS OF DISPERSION COEFFICIENT (L**2/T)	76
C	KUNITS	UNITS OF SOLUTE DECAY CONSTANT (1/T)	77
C	LUNITS	UNITS OF LENGTH (L)	78
C	TUNITS	UNITS OF TIME (T)	79
C			80
C		DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE	81
	CALL OFILE		82
	CALL TITLE		83
	WRITE(IO,201)		84
C			85
C		READ IN MODEL PARAMETERS	86
	READ(IN,101)	NX,NY,NZ,NT,NMAX,IPLT	87
	WRITE(IO,205)	NX,NY,NZ,NT,NMAX	88
	READ(IN,105)	CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS	89
	READ(IN,110)	C0,VX,DX,DY,DZ,DK	90
	WRITE(IO,210)	C0,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DZ,DUNITS,	91
	1 DK,KUNITS		92
	READ(IN,110)	Y1,Y2,Z1,Z2	93
	WRITE(IO,212)	Y1,LUNITS,Y2,LUNITS,Z1,LUNITS,Z2,LUNITS	94
	READ(IN,110)	(X(I),I=1,NX)	95
	WRITE(IO,215)	LUNITS	96
	WRITE(IO,220)	(X(I),I=1,NX)	97
	READ(IN,110)	(Y(I),I=1,NY)	98
	WRITE(IO,216)	LUNITS	99
	WRITE(IO,220)	(Y(I),I=1,NY)	100
	READ(IN,110)	(Z(I),I=1,NZ)	101
	WRITE(IO,217)	LUNITS	102
	WRITE(IO,220)	(Z(I),I=1,NZ)	103
	READ(IN,110)	(T(I),I=1,NT)	104
	WRITE(IO,225)	TUNITS	105
	WRITE(IO,220)	(T(I),I=1,NT)	106

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IF(IPLT.GT.0) READ(IN,110) XSCLP,YSCLP,DELTA      107
IF(IPLT.GT.0) WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS 108
C                                                    109
C   READ IN GAUSS-LEGENDRE POINTS AND WEIGHTING FACTORS 110
CALL GLQPTS (NMAX)                                111
C                                                    112
C   BEGIN TIME LOOP                                  113
DO 20 IT=1,NT                                     114
C                                                    115
C   BEGIN Z LOOP                                      116
DO 30 IZ=1,NZ                                     117
C                                                    118
C   BEGIN X LOOP                                      119
DO 40 IX=1,NX                                     120
C                                                    121
C   CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX) AND Z=Z(IZ) 122
DO 50 IY=1,NY                                     123
CALL CNRMLP(DK,T(IT),X(IX),Y(IY),Z(IZ),Y1,Y2,Z1,Z2,DX,
1 DY,DZ,VX,CN,NMAX)                               124
CXIY(IX,IY)=CO*CN                                 125
50 CONTINUE                                        126
40 CONTINUE                                        127
C                                                    128
C   PRINT OUT TABLES OF CONCENTRATION VALUES        129
C   NPAGE=1+(NY-1)/9                                  130
DO 60 NP=1,NPAGE                                  131
IF(NP.EQ.1) WRITE(IO,230) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS 132
IF(NP.NE.1) WRITE(IO,231) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS 133
NP1=(NP-1)*9                                       134
NP2=9                                               135
IF((NP1+NP2).GT.NY) NP2=NY-NP1                     136
WRITE(IO,235) (Y(NP1+J),J=1,NP2)                   137
WRITE(IO,236) CUNITS,LUNITS                         138
DO 70 IX=1,NX                                       139
WRITE(IO,240) X(IX),(CXIY(IX,NP1+J),J=1,NP2)       140
IF(MOD(IX,45).NE.0) GO TO 70                        141
WRITE(IO,231) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS     142
WRITE(IO,235) (Y(NP1+J),J=1,NP2)                   143
WRITE(IO,236) CUNITS,LUNITS                         144
70 IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(IO,241) 145
60 CONTINUE                                        146
C                                                    147
C   CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE 148
C   PLOT SCALING FACTORS. CONVERT C(X,Y) AND DIVIDE BY CO TO PLOT 149
C   CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH TIME VALUE. 150
IF(IPLT.LT.1) GO TO 30                              151
NXY=NX*NY                                           152
DO 80 I=1,NX                                         153
IP=(I-1)*NY                                          154
XP(I)=SNGL(X(I))                                    155
DO 80 J=1,NY                                         156
IF(I.EQ.1) YP(J)=SNGL(Y(J))                         157
CP(IP+J)=SNGL(CXY(I,J)/CO)                         158

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80  CONTINUE 160
    TP=SNGL(T(IT)) 161
    ZP=SNGL(Z(IZ)) 162
    NXY2=NXY*2 163
    CALL PLOT3D (XP,YP,ZP,CP,TP,DELTA,NX,NY,NXY,NXY2,IZ,NZ,IPLT,
1  TUNITS,LUNITS,XSCLP,YSCLP,XPC,YPC,IFLAG) 164
30  CONTINUE 166
20  CONTINUE 167
    CLOSE (IN) 168
    CLOSE (IO) 169
    STOP 170
C 171
C  FORMAT STATEMENTS 172
101  FORMAT(20I4) 173
105  FORMAT(8A10) 174
110  FORMAT(8F10.0) 175
201  FORMAT(/////1H ,29X,'ANALYTICAL SOLUTION TO THE THREE-DIMENSIONAL' 176
1  /1H ,28X,'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION' / 177
2  1H ,30X,'FOR A SEMI-INFINITE AQUIFER OF INFINITE WIDTH' / 178
3  1H ,28X,'AND HEIGHT WITH A PATCH SOLUTE SOURCE AT X=0.0' 179
4  ///1H0,40X,'INPUT DATA'/1H ,40X,10(1H-)) 180
205  FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',I4/1H ,25X, 181
1  'NUMBER OF Y-COORDINATES (NY) = ',I4/1H ,25X, 182
2  'NUMBER OF Z-COORDINATES (NZ) = ',I4/1H ,25X, 183
3  'NUMBER OF TIME VALUES (NT) = ',I4/1H ,25X, 184
4  'NUMBER OF POINTS FOR NUMERICAL INTEGRATION (NMAX) = ',I4) 185
210  FORMAT(1H0,25X,'SOLUTE CONCENTRATION ON MODEL BOUNDARY (C0) = ', 186
1  1P1E13.6,1X,A10/1H ,25X, 187
2  'GROUND-WATER VELOCITY IN X-DIRECTION (VX) = ',1P1E13.6,1X,A10/ 188
3  1H ,25X,'DISPERSION IN THE X-DIRECTION (DX) = ',1P1E13.6,1X,A10/ 189
4  1H ,25X,'DISPERSION IN THE Y-DIRECTION (DY) = ',1P1E13.6,1X,A10/ 190
5  1H ,25X,'DISPERSION IN THE Z-DIRECTION (DZ) = ',1P1E13.6,1X,A10/ 191
6  1H ,25X,'FIRST-ORDER SOLUTE DECAY RATE (DK) = ',1P1E13.6,1X,A10) 192
212  FORMAT(1H0,25X,'AQUIFER WIDTH (W) AND HEIGHT (H) ARE INFINITE' 193
2  /1H ,25X,'SOLUTE SOURCE IS LOCATED BETWEEN Y1 = ',1P1E13.6,1X,A10/ 194
3  1H ,58X,'Y2 = ',1P1E13.6,1X,A10/1H ,58X, 195
4  'Z1 = ',1P1E13.6,1X,A10/1H ,54X, 196
5  'AND Z2 = ',1P1E13.6,1X,A10) 197
215  FORMAT(1H0,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ', 198
1  'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/) 199
216  FORMAT(1H0,25X,'Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ', 200
1  'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/) 201
217  FORMAT(1H0,25X,'Z-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ', 202
1  'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/) 203
220  FORMAT(1H ,5X,8F12.4) 204
225  FORMAT(1H0,25X,'TIMES AT WHICH SOLUTE CONCENTRATIONS ' 205
1  'WILL BE CALCULATED, IN ',A10/1H ,25X,70(1H-)/) 206
227  FORMAT(1H0,25X,'PLOT SCALING FACTOR FOR X (XSCLP) = ',1P1E13.6/ 207
1  1H ,25X,'PLOT SCALING FACTOR FOR Y (YSCLP) = ',1P1E13.6/ 208
2  1H ,25X,'CONTOUR INCREMENT (DELTA) = ',1P1E13.6,1X,A10) 209
230  FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME = ', 210
1  F12.4,1X,A10/1H ,35X,'AND AT Z = ',F12.4,1X,A10/ 211
2  1H0,25X,'Y-COORDINATE, IN ',A10) 212

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231  FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =',          213
1  F12.4,1X,A10,5X,'(CONTINUED)'/1H ,35X,'AND AT Z =',F12.4,1X,A10/ 214
2  1H0,25X,'Y-COORDINATE, IN ',A10)                                215
235  FORMAT(1H ,20X,9F12.4)                                         216
236  FORMAT(1H ,19X,'*',108(1H-)/                                    217
1  1H ,4X,'X-COORDINATE,',2X,'!',44X,'SOLUTE CONCENTRATION, IN ' 218
2  A10/1H ,4X,'IN ',A10,2X,1H!/1H ,19X,'!')                       219
240  FORMAT(1H ,5X,F12.4,2X,'!',9F12.6)                             220
241  FORMAT(1H ,19X,'!')                                           221
      END                                                            222
      SUBROUTINE CNRMLP(DK,T,X,Y,Z,Y1,Y2,Z1,Z2,DX,DY,DZ,VX,CN,NMAX) 223
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)                             224
      COMMON /IOUNIT/ IN,IO                                          225
      COMMON /GLPTS/ WN(256),ZN(256)                                 226
C
C
C      THIS ROUTINE CALCULATES THE NORMALIZED CONCENTRATION AT X,Y,Z 227
C      BASED ON THE ANALYTIC SOLUTION TO THE THREE-DIMENSIONAL      228
C      ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION FOR A SEMI-   229
C      INFINITE AQUIFER WITH INFINITE WIDTH AND HEIGHT. THE SOLUTE  230
C      SOURCE HAS A FINITE WIDTH AND HEIGHT, EXTENDING FROM Y=Y1 TO 231
C      Y=Y2 AND Z=Z1 TO Z=Z2. THE SOLUTE MAY BE SUBJECT TO FIRST-ORDER 232
C      CHEMICAL TRANSFORMATION. THE SOLUTION CONTAINS AN INTEGRAL  233
C      FROM 0 TO T**0.25 WHICH IS EVALUATED NUMERICALLY USING A GAUSS- 234
C      LEGENDRE QUADRATURE TECHNIQUE.                                235
C
C
C      PI=3.14159265358979D0                                         236
C      CN=0.0D0                                                       237
C
C
C      FOR T=0, ALL CONCENTRATIONS EQUAL 0.0                         238
C      IF(T.LE.0.0D0) RETURN                                         239
C
C
C      FOR X=0.0, CONCENTRATIONS ARE SPECIFIED BY BOUNDARY CONDITIONS 240
C      IF(X.GT.0.0D0) GO TO 10                                       241
C      IF(Y.EQ.Y1.OR.Y.EQ.Y2) THEN                                    242
C          IF(Z.GT.Z1.AND.Z.LT.Z2) CN=0.50D0                         243
C          IF(Z.EQ.Z1.OR.Z.EQ.Z2) CN=0.25D0                         244
C      END IF                                                         245
C      IF(Z.EQ.Z1.OR.Z.EQ.Z2) THEN                                    246
C          IF(Y.GT.Y1.AND.Y.LT.Y2) CN=0.50D0                         247
C      END IF                                                         248
C      IF(Y.GT.Y1.AND.Y.LT.Y2.AND.Z.GT.Z1.AND.Z.LT.Z2) CN=1.0D0   249
C      RETURN                                                         250
C
C
C      START NUMERICAL INTEGRATION LOOP                               251
10  SUM=0.0D0                                                         252
      DO 20 I=1,NMAX                                                  253
C
C          SCALE THE GAUSS-LEGENDRE COEFFICIENTS TO ACCOUNT FOR THE  254
C          NON-NORMALIZED LIMITS OF INTEGRATION                       255
C          LIMITS OF INTEGRATION ARE FROM 0 TO T**0.25              256
      TT=T**0.25D0                                                    257
      WI=WN(I)                                                         258
      ZI=TT*(ZN(I)+1.0D0)/2.0D0                                       259

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	ZSQ=ZI*ZI	266
	Z4=ZSQ*ZSQ	267
C		268
C	TERM 1	269
	XVT=X-VX*Z4	270
	EXP1=-XVT*XVT/(4.0D0*DX*Z4)-DK*Z4	271
	ERFC1=(Y1-Y)/(2.0D0*ZSQ*DSQRT(DY))	272
	CALL EXERFC(EXP1,ERFC1,Q1)	273
C		274
C	TERM 2	275
	ERFC2=(Y2-Y)/(2.0D0*ZSQ*DSQRT(DY))	276
	CALL EXERFC(EXP1,ERFC2,Q2)	277
C		278
C	TERM 3	279
	EXP2=0.0D0	280
	ERFC1=(Z1-Z)/(2.0D0*ZSQ*DSQRT(DZ))	281
	CALL EXERFC(EXP2,ERFC1,Q3)	282
	ERFC2=(Z2-Z)/(2.0D0*ZSQ*DSQRT(DZ))	283
	CALL EXERFC(EXP2,ERFC2,Q4)	284
	TERM=(Q1-Q2)*(Q3-Q4)*WI/(ZI*ZSQ)	285
	SUM=SUM+TERM	286
20	CONTINUE	287
	SUM=SUM*TT/2.0D0	288
	CN=SUM*X/(2.0D0*DSQRT(PI*DX))	289
	RETURN	290
	END	291