Techniques of Water-Resources Investigations of the United States Geological Survey

Chapter B2

INTRODUCTION TO GROUND-WATER HYDRAULICS

A Programed Text for Self-Instruction

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Book 3

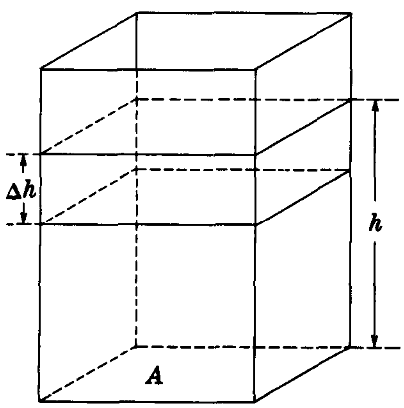
APPLICATIONS OF HYDRAULICS

Part IV. Ground-Water Storage

Introduction

In Parts II and III we dealt with aquifers and porous media only as conduits—that is, we discussed only their properties relating to the transmission of water in steady flow. Aquifers have another very important hydraulic property—that of water storage. In Part IV we will examine this property of ground-water storage and develop an equation to describe it. In Part V we will develop the differential equations for a simple case of nonequilibrium flow by combining the storage equation with Darcy's law, by means of the equation of continuity, which is simply a statement of the principle of conservation of mass. In Part VI, we will repeat this process for the case of nonequilibrium radial flow to a well and will obtain an important solution to the resulting differential equation.

1



The picture shows an open tank, having a square base of area A. If a volume of water, ΔV, is poured into this tank, the water level will rise by an increment, Δh, such that $\Delta V = A \cdot \Delta h$. The total volume, V, of water in storage in the tank at any time can be determined by measuring the depth, h, of water in the tank and multiplying this depth by A.

QUESTION

Suppose the total volume of water in storage is plotted as a function of the level of water in the tank, so that the volume associated with any water level can be read directly from the plot. The graph will be:

	Turn to Section:
(a) a parabola with slope $\dfrac{\Delta V}{\Delta h}$	10
(b) a straight line with slope $\dfrac{\Delta V}{\Delta h} = A$	11
(c) a logarithmic curve	9

2

Your answer in Section 26 is not correct. The volume of water present in the sand initially was hAn. A certain fraction, β, of this fluid volume was drained off by gravity, leaving the fraction $1-\beta$ still occupied by fluid. β thus represents the fraction of the total pore space, below the level h, which does does not already contain water, and which must be refilled in order to resaturate the sand to the level h. That is, in order to resaturate the sand to the level h, a volume of water equal to this unoccupied pore volume must be pumped into the tank.

Return to Section 26 and choose another answer.

3

Your answer in Section 21 is not correct. In the imaginary experiment described in Section 21, it was stated that doubling the base area of the prism had the effect of doubling the slope of the V,h plot—that is, of doubling the term dV/dh. Thus, dV/dh depends upon the size of the prism considered, as well as upon the type of aquifer material; it cannot be considered a constant representative of the aquifer material.

Return to Section 21 and choose another, answer.

4

Your answer in Section 16,

$$\frac{\Delta V}{\Delta h}=\frac{dV}{dh}=n\beta,$$

is not correct. It neglects the effect of the base area, A, of the tank.

We have seen that when the tank is drained by gravity and then resaturated to the level h, the relation between V and h is

$$V=hAn\beta$$

where n is the porosity of the sand and β the fraction of the water in the sand that can be drained out by gravity. Now if, instead of draining the sand to the bottom of the tank, we simply remove a small volume of water, ΔV, so that the water level in the tank falls by a small amount Δh, we should expect ΔV and Δh to be related in the same way as V and h in our previous experiment. If we are resaturating the sand by increments, when it has previously been saturated and then drained by gravity, the same relation should hold.

Return to Section 16 and choose another answer.

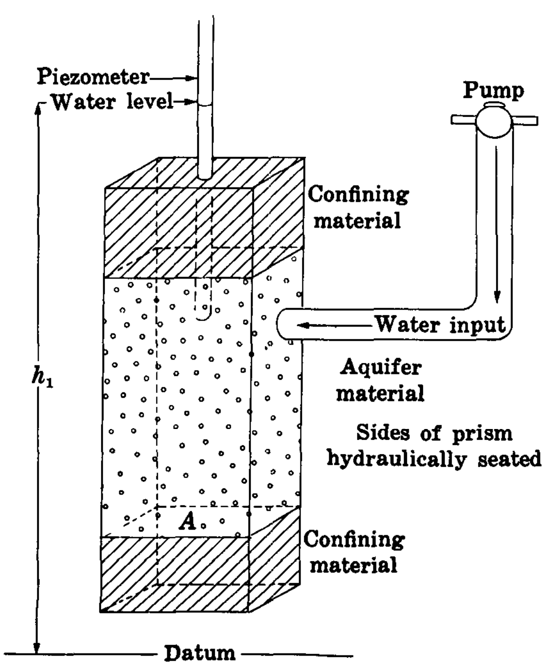
5

Your answer in Section 20 is not correct. If each well penetrated both aquifers, there would be no reason for the responses of the two wells to differ. The form of the response might be difficult to predict, but at least it should be roughly the same for each well. Keep in mind that the storage coefficient of the artesian zone will probably be smaller than the specific yield of the water-table aquifer by at least two orders of magnitude.

Return to Section 20 and choose another answer.

6

Your answer in Section 32 is correct. Specific yield figures for normal aquifer materials may range from 0.01 to 0.35. It is common to speak of the specific yield of an unconfined aquifer as a whole; but it should be noted that the process of release from unconfined storage really occurs at the water table. If the water table falls or rises within an aquifer, into layers or strata having different hydraulic properties, specific yield must change. In addition, of course, specific yield can vary with map location, in response to local geologic conditions.



Unconfined storage is probably the most important mechanism of ground-water storage from an economic point of view, but it is not the only such mechanism. Storage effects have also been observed in confined or artesian aquifers. The mechanism of confined storage depends, at least in part, upon compression and expansion of the water itself and of the porous framework of the aquifer; for this reason confined storage is sometimes referred to as compressive storage. In this outline we will not attempt an analysis of the mechanism of confined storage, but will concentrate instead on developing a mathematical description of its effects, suitable for hydrologic calculations. A discussion of the mechanism of confined storage is given by Jacob (1950, p. 328–334), and by Cooper (1966).

The diagram shows a vertical prism extending through a uniform confined aquifer. The base area of the prism is A. Although the prism remains structurally a part of the confined aquifer, we suppose it to be isolated hydraulically from the rest of the aquifer by imaginary hydraulic barriers, so that water added to the prism remains within it. We further imagine that we have some method of pumping water into the prism in measured increments, and that we have a piezometer, as shown in the diagram, through which we can measure the head within the prism.

(*continued on next page*)

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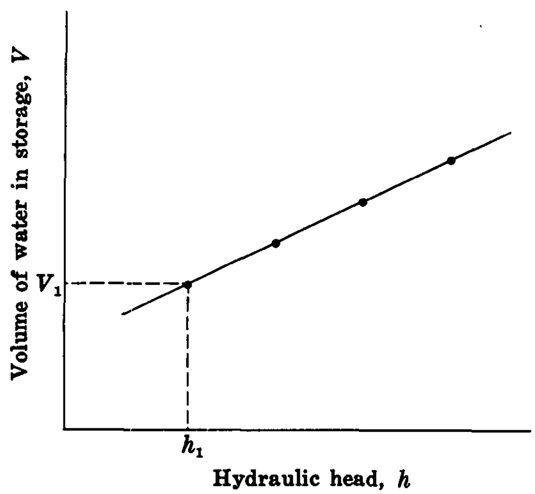
QUESTION

Suppose that head is initially at the level h_1, which is above the top of the aquifer, indicating that the prism is not only saturated, but under confined hydrostatic pressure. We designate the volume of water in storage in this initial condition as V_1. Now suppose more water is pumped into the prism by increments; and that the head is measured after each addition, and a graph of the volume of water in storage versus the hydraulic head in the prism is plotted. If the resulting plot had the form shown in the figure, which of the following statements would you accept as valid?



	Turn to Section:
(a) The rate of change of volume of water in confined storage, with respect to hydraulic head, h, is constant; that is $\frac{dV}{dh}$ = constant	21
(b) The rate of change of hydraulic head with respect to volume in storage, depends upon the volume in storage.	23
(c) The rate of change of volume in storage, with respect to the base area of the prism, is equal to Δh.	30

7

Your answer in Section 32 is not correct. One important concept which is missing from the definition you selected is that specific yield refers to a unit base area of the aquifer. The definition you selected talks about the volume of water which can be drained *from the aquifer*—this would vary with extent of the aquifer and would normally be a very large quantity. As we wish specific yield to represent a property of the aquifer material, we define it in terms of the volume that can be drained per unit map area of aquifer.

Return to Section 32 and choose another answer.

8

Your answer in Section 25 is not correct. The relation given in Section 25 for the rate of release of water from storage was

$$\frac{dV}{dt} = SA\frac{dh}{dt}$$

where S is the storage coefficient, A the area of aquifer under study, and dh/dt the rate of change of head with time within that area of aquifer. In the question of Section 25, the the specific yield of the water-table aquifer was given as 0.20, and the rate of decline of water level in the shallow well was given as 0.5 foot per day. The surface area of a section of the aquifer within a 10 foot radius of the well would be $\pi \times 10^2$, or 314 square feet. The

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rate of release from storage in this section would therefore be

$$\frac{dV}{dt}=SA\frac{dh}{dt}=0.2\times 314\times 0.5$$

$$=31.4 \text{ cubic feet per day.}$$

Return to Section 25 and choose another answer.

9

Your answer in Section 1 is not correct. Whenever we add a fixed volume of water—say 10 cubic feet—to the tank, the water level must rise by a corresponding fixed amount. If the base area of the tank is 5 square feet, the addition of 10 cubic feet of water must always produce an increase of 2 feet in h; the addition of 15 cubic feet of water must produce an increase of 3 feet in h; and so on. The ratio $\Delta V/\Delta h$ in this case must always be 5. In other words, the ratio $\Delta V/\Delta h$ is constant and is equal to the base area, A, of the tank.

Now if we plot V versus h, the slope of this plot will be $\Delta V/\Delta h$, by definition. This slope, as we have seen above, must be a constant. A logarithmic curve does not exhibit a constant slope.

Return to Section 1 and choose another answer.

10

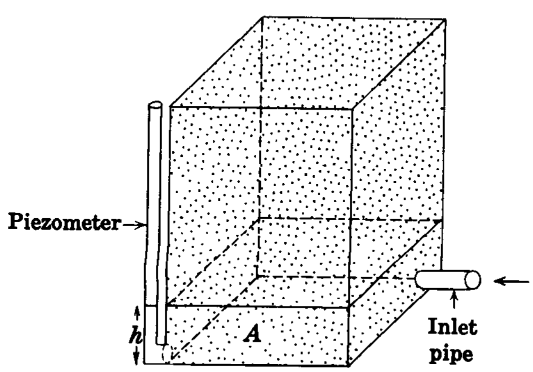
Your answer in Section 1 is not correct. The increment in the volume of water within the tank, resulting from an increase in water level of Δh, is given by $\Delta V=A\Delta h$. Thus,

$$\frac{\Delta V}{\Delta h}=A$$

where A, the base area of the tank, is a constant. If we construct a plot of V, the volume of water in the tank, versus h, the level in the tank, the slope of the plot will by definition be $\Delta V/\Delta h$; but since $\Delta V/\Delta h$ is a constant, the plot cannot be a parabola. The slope of a parabola changes continuously along the graph.

Return to Section 1 and choose another answer.

11



Your answer in Section 1 is correct. The slope of the graph, $\Delta V/\Delta h$ or dV/dh, is constant and equal to A. Thus the volume of water in storage per foot of head (water level) in the tank is A.

Now consider the tank shown in the sketch. It is similar to the one we just dealt with, except that it is packed with dry sand having an interconnected (effective) porosity denoted by n. The tank is open at the top and has a base of area A. Water can be

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pumped into the tank through a pipe connected at its base, and the water level within the tank—that is, the level of saturation in the sand—can be measured by means of a piezometer, also connected at the base of the tank.

QUESTION

Suppose we pump a small volume of water, V, into the tank and observe the level, h, to which water rises in the piezometer. Neglecting all capillary effects, which of the following expressions would constitute a valid relation between the volume of water pumped into the tank and the rise in water level above the base of the tank?

	Turn to Section:
$V=Ah$	31
$h=VAn$	12
$V=hAn$	14

12

Your answer in Section 11 is not correct. If the water rises to a level h above the base of the tank, the bulk volume of saturated sand (neglecting capillary effects) will be hA. This bulk volume must be multiplied by the porosity to obtain the total volume of saturated pore space. A review of the definition of porosity as given in Part I may help to clarify this.

Return to Section 11 and choose another answer.

13

Your answer in Section 25 is correct. The release from storage in a given area in the water-table aquifer is given by

$$\frac{dV}{dt}=S_yA\frac{dh}{dt}=0.2\times A\times 0.5=0.1A.$$

The release from storage in an equal area in the artesian aquifer would be

$$\frac{dV}{dt}=SA\frac{dh}{dt}=2\times 10^{-4}\times A\times 5=0.001A.$$

Thus the water-table contribution exceeds the artesian release by a factor of 100.

This completes our introductory discussion of aquifer storage. You may go on to Part V, in which we will combine the concept of aquifer storage with Darcy's law, using the equation of continuity, to develop the differential equation for a simple problem in nonequilibrium ground-water flow.

14

Your answer, $V=hAn$, in Section 11 is correct. Now suppose water is added to the tank in increments, and h is measured after the addition of each increment; and suppose a graph of V versus h is plotted, where V is the total or cumulative volume which has been added, and h is the water level in the tank.

QUESTION

Again neglecting all capillary effects, the resulting graph would be:

	Turn to Section:
(a) a straight line with slope $\frac{\Delta V}{\Delta h}=\frac{1}{An}$	17
(b) a straight line with slope $\frac{\Delta V}{\Delta h}=An$	26
(c) a logarithmic curve with slope depending on h	22

15

Your answer in Section 20 is not correct. The specific yield of the water-table aquifer would normally be greater than the storage coefficient of the artesian zone by at least two orders of magnitude. A seasonal fluctuation in pumpage would usually involve a brief withdrawal from storage, or a brief period of accumulation in storage. The two aquifers are pumped at about the same rate, so presumably seasonal adjustments in the pumpage will be of the same order of magnitude for each. However, the response of the two aquifers to withdrawal (or accumulation) of a similar volume of water would be completely different, and would be governed by their storage coefficients. The aquifer with the higher storage coefficient could sustain the withdrawal with less drawdown of water level than could the aquifer with the lower storage coefficient.

Return to Section 20 and choose another answer.

16

Your answer, $V=hAn\beta$, in Section 26 is correct. This expression gives the volume of water withdrawn in draining the tank by gravity, and the volume which must be added to resaturate the sand to the original level, under our assumption that the fraction held by capillary forces is constant.

QUESTION

Suppose, subject to the same assumption, that the tank is drained by removing increments of water (or resaturated by adding increments of water) and a graph of the volume of water in storage, V, versus the level of saturation, h, is plotted from the results of the experiment. Which of the following expressions would describe the slope of the resulting graph?

	Turn to Section:
$\frac{\Delta V}{\Delta h}=\frac{dV}{dh}=n\beta$	4
$\frac{\Delta V}{\Delta h}=\frac{dV}{dh}=An\beta$	33
$\frac{\Delta V}{\Delta h}=\frac{dV}{dh}=hAn\beta$	29

17

Your answer in Section 14 is not correct. We have seen that if a volume of water, V, is pumped into the tank when it is initially dry, the equation

$$V=h\cdot A\cdot n$$

describes the relation between V and h, the level of water in the sand. If the sand is already saturated to some level, and an additional volume of water, ΔV, is pumped in, the water level will rise by an increment Δh, such that

$$\Delta V=\Delta h\cdot A\cdot n.$$

Return to Section 14 and use this relation in choosing another answer.

18

Your answer in Section 26 is not correct. $h \cdot A \cdot n$ would represent the volume of water required to raise the water level to a distance h above the base of the tank, *if the sand were initially dry.* In this case, however, the sand is not initially dry. Some of the pore space is already occupied by water at the beginning of the experiment, since after drainage by gravity, capillary effects cause some water to be held in permanent retention. The volume of water which must be added to resaturate the sand to the level h is equal to the volume of pore space below the level h which does not already contain water. The total volume of pore space below the level h is $h \cdot A \cdot n$; when the sand was initially saturated, this entire volume contained water. When the sand was drained, a certain fraction of this water, which we designate β, was removed. The remaining fraction, $1-\beta$, was held by capillary retention in the sand. Thus β represents the *fraction of the pore space* which is empty when we begin to refill the tank.

Return to Section 26 and choose another answer.

19

Your answer in Section 33 is not correct. Because the aquifer material is identical to the sand of our tank experiments and because the base area of our prism of aquifer is equal to the base area of our tank, we should expect the relation between volume released from storage and decline in water level within the prism to be identical to that obtained for the tank. In the answer which you selected, however, there is no description of the effect of capillary retention. Remember that the factor β, which was used in the tank experiment to describe the fraction of the water which could be drained by gravity, as opposed to that held in capillary retention, must appear in your answer.

Return to Section 33 and choose another answer.

20

Your answer in Section 21 is correct. The results of the imaginary experiment suggest that the term

$$\frac{1}{A}\frac{dV}{dh}$$

is a constant for the aquifer material.

In practice, in dealing with the confined or compressive storage of an aquifer, it is usually *assumed* that the quantity $(1/A)(dV/dh)$ is a constant for the aquifer, or is at least a constant for any given location in the aquifer. This quantity, $(1/A)(dV/dh)$, is denoted S and is called the confined or compressive storage coefficient, or simply the storage coefficient, of the aquifer.

It would of course be difficult or impossible to perform the experiment described in Section 6. However, if storage coefficient is defined by the equation

$$S=\frac{1}{A}\frac{dV}{dh},$$

a nonequilibrium theory can be developed from this definition which explains many of the observed phenomena of confined flow.

The following points should be noted regarding confined storage coefficient:

(1) The storage coefficient is the volume of water released from storage in a prism of unit area, extending through the full thickness of the aquifer, in

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response to a unit decline in head. This statement can be appreciated by a review of the hypothetical experiment described earlier, or by letting $A=1$ in the finite-difference form of the definition, $S=(1/A)(\Delta V/\Delta h)$.

(2) The definition of storage coefficient is similar to that of specific yield, in the sense that each is defined as the term $(1/A)(dV/dh)$, for a prism extending through an aquifer. Thus in many applications, the two terms occupy the the same position in the theory. In the case of an unconfined aquifer the specific yield is often referred to as the storage coefficient.

(3) It should be noted, however, that the processes involved in the two types of storage are completely different. Withdrawal from or addition to unconfined storage takes place at the water table; it is spoken of as occurring in a prism of aquifer because it is usually the only significant form of storage within such a prism in most water-table situations. Confined storage effects, on the other hand, are distributed throughout the vertical thickness of an aquifer.

(4) Confined storage coefficient values are generally several orders of magnitude less than specific yield values. Specific yields range typically from 0.01 to 0.35, whereas confined storage values usually range from 10^{-5} to 10^{-3}.

The definition of confined storage in terms of a prism extending through the aquifer is adequate where the flow is entirely horizontal—that is, where no differences in head or in lithology occur along a vertical within the aquifer. Where vertical differences do occur, one must allow for the possibility of different patterns of storage release at different points along the vertical, and a storage definition based on a prism is no longer adequate. Use is therefore made of the *specific storage*, S_s, which is defined as the volume of water released from confined storage in a unit *volume* of aquifer, per unit decline in head. In a homogeneous aquifer, S_s would be equal to S divided by the thickness of the aquifer.

QUESTION

Consider a small ground-water basin that has both an artesian aquifer and a water-table aquifer. Regional withdrawal from the artesian aquifer is about equal to that from the water-table aquifer, and seasonal fluctuations in pumpage are similar. Records are kept on two observation wells, neither of which is in the immediate vicinity of a discharging well. One well shows very little fluctuation in water level in response to seasonal variations in pumpage, while the other shows great fluctuation. Which of the following statements would more probably be true?

Turn to Section:

(a) The well showing little fluctuation taps the water-table aquifer, while that showing great fluctuation taps the artesian zone. 25

(b) Each well penetrates both aquifers. 5

(c) The well showing great fluctuation taps the water-table aquifer, while that showing little fluctuation taps the artesian zone. 15

21

Your answer in Section 6 is correct. The plot is a straight line, so the slope, dV/dh, is a constant. Now suppose the prism is expanded to twice its original base area, and our imaginary experiment is repeated; and suppose we observe that, as a result of the increase in base area, the slope of our V, h plot is twice its original value.

QUESTION

Let A now represent the base area of any general (vertical) prism through the aquifer; or in general, let A represent the surface area of the section of the aquifer we are isolating for discussion. On the basis of the evidence described, which of the following statements would you be inclined to accept?

Turn to Section:

(a)
$$\frac{dV}{dh}$$
is a constant for the aquifer material 3

(b) The term
$$\frac{1}{A}\frac{dV}{dh}$$
is a constant for the aquifer material 20

(c) The term
$$A\frac{dV}{dh}$$
is a constant for the aquifer material 34

22

Your answer in Section 14 is not correct. We have seen that, neglecting capillary effects, there is a linear relationship between the volume of water, V, pumped into the tank when it is initially dry, and the level of water, h, above the base of the tank. That is, a constant coefficient, An, relates these two quantities: $V=h\cdot A\cdot n$. This linearity holds as well if the water is added to the tank in increments. Each incremental volume of water, ΔV, pumped into the tank produces an increment in head, Δh, such that

$$\Delta V=\Delta h\cdot A\cdot n.$$

Return to Section 14 and choose another answer.

23

Your answer in Section 6 is not correct. The ratio of the change of volume of water in storage, to the change in hydraulic head is by definition the slope, $\Delta V/\Delta h$ or dV/dh, of a plot of V versus h. If this rate of change of V with h were to depend upon V, the plot of V versus h would show a different slope at different values of V. The plot, in other words, would be some sort of curve. The plot shown in Section 6, however, is a straight line—it has a constant slope, the same for any value of V.

Return to Section 6 and choose another answer.

24

Your answer in Section 25 is not correct. The relation given in Section 25 for the rate of release of water from storage was

$$\frac{dV}{dt}=SA\frac{dh}{dt}$$

where S is the storage coefficient, A the area of aquifer under study, and dh/dt the rate of change of head with time within that area of aquifer. In the question of Section 25, S was given as 2×10^{-4} for the artesian aquifer, and dh/dt, as measured in the deep well, was 5 feet per day. A section of the aquifer within a 10 foot radius of the observation well would have a surface area of $\pi\times10^2$, or 314 square feet. The rate of release of water from storage in this section would therefore be

$$\frac{dV}{dt}=SA\frac{dh}{dt}=2\times10^{-4}\times314\times5$$
$$=0.314 \text{ cubic feet per day.}$$

Return to Section 25 and choose another answer.

25

Your answer in Section 20 is correct. Because of the higher storage coefficient of the water-table aquifer, release or accumulation of a comparable volume of water will cause a much smaller fluctuation of water level in the water-table aquifer than in the artesian aquifer. In effect, we have introduced time variation into the problem here, since we are discussing changes in head with time. To bring time into the equations, we may proceed as follows.

Let S represent either specific yield or storage coefficient. Then according to our definitions we may write, using the finite-difference form,

$$S=\frac{1}{A}\frac{\Delta V}{\Delta h}.$$

The relation between the volume of water taken into or released from aquifer storage in a prism of base area A and the accompanying change in head, is therefore:

$$\Delta V=SA\Delta h.$$

Now let us divide both sides of this equation by Δt, the time interval over which the decline in head was observed. We then have:

$$\frac{\Delta V}{\Delta t}=SA\frac{\Delta h}{\Delta t}$$

or, if we are talking about a vanishingly small time interval,

$$\frac{dV}{dt}=SA\frac{dh}{dt}$$

Here dV/dt is the time rate of accumulation of water in storage, expressed, for example, in cubic feet per day; and dh/dt is the rate of increase in head, expressed, for example, in feet per day. If we are dealing with release from storage, head will decline, and both dV/dt and dh/dt will be negative. The partial derivative notation, $\partial h/\partial t$, is usually used instead of dh/dt, because head may vary with distance in the aquifer as well as with time. This equation is frequently referred to as the storage equation.

The equation can also be obtained using the rules of differentiation. For the case we are considering we have

$$\frac{dV}{dt}=\frac{dV}{dh}\cdot\frac{dh}{dt},$$

but from the definition of storage coefficient, $dV/dh=SA$, so that by substitution

$$\frac{dV}{dt}=SA\frac{dh}{dt}.$$

(*continued on next page*)

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QUESTION

Suppose we record the water levels in a deep observation well, penetrating a confined aquifer which has a storage coefficient of 2×10^{-4}, and a shallow observation well, tapping a water-table aquifer which has a specific yield of 0.20. The water level in the deep well falls at a rate of 5 feet per day, while that in the shallow well falls at a rate of 0.5 foot per day. Considering the release of water from storage in each aquifer within a radius of 10 feet of the observation well, which of the following statements would be most accurate?

Turn to Section:

(a) within a radius of 10 feet of the shallow well, water is being released from storage in the water-table aquifer at a rate of 5 cubic feet per day. 8

(b) the rate of release of water from storage in the water-table aquifer, within 10 feet of the shallow well, is 100 times as great as that in the artesian aquifer, within 10 feet of the deep well. 13

(c) within a radius of 10 feet of the deep well, water is being released from storage in the artesian aquifer at a rate of 1 cubic foot per day. 24

26

Your answer in Section 14 is correct. If there were no capillary effects, the result of filling the tank with sand would simply be to take up some of the volume available for storage of water. Thus the slope of the plot of V versus h for the sand-filled tank would differ from that for the open tank (Section 1) only by the factor n, which is the ratio of the storage volume available in the sand-filled tank to that available in the open tank.

In practice, of course, capillary effects cannot be neglected. In this development we will take a simplified view of these effects, as a detailed examination of capillary phenomena is beyond the scope of our discussion. Let us assume that due to capillary forces, a certain constant fraction of the water in the sand is permanently retained. That is, we assume that following the initial saturation of the sand, we can never drain off by gravity the full volume of water which was added during the initial saturation. A part of this initially added water remains permanently held in the pore spaces by capillary attraction; thus the amount of water which can be alternately stored and recovered is reduced.

QUESTION

Suppose the tank is initially saturated to a level h and is then drained by gravity. Suppose further that the ratio of the volume of water drained to that initially added is observed to be β; that is, the fraction *of the added water* which can be drained is β, while the fraction retained in the sand by capillary forces is $(1-\beta)$. Subject to our assumption that the fraction retained is a constant, which of the following expressions gives the volume of water which would have to be restored to the tank, after draining, in order to resaturate the sand to the same level, h, as before?

	Turn to Section:
$V=hAn$	18
$V=hA\dfrac{n}{\beta}$	2
$V=hAn\beta$	16

27

Your answer in Section 32 is not correct. Your answer defines specific yield as the quantity (presumably the total quantity) of water which can be drained by gravity from a unit area of the aquifer. In the preceding analysis, we developed the concept of specific yield in terms of the quantity of water which can be drained *per unit decline in water level.* A verbal definition of specific yield must therefore include this latter concept in some manner—that is, it must indicate that we are referring to the quantity released from storage per unit decline in head.

Return to Section 32 and choose another answer.

28

Your answer in Section 33 is not correct. The aquifer material was given as identical to the sand of the tank experiments described previously, and the base area of the prism was taken as equal to the base area of the tank. We are considering only storage within the prism itself, in relation to water level in the prism, and are not concerned with what goes on in the aquifer beyond the boundaries of the prism. At this rate, we should expect the relation between the volume of water drained from storage and the accompanying decline in water level to be the same for our prism of aquifer as for the tank of the earlier experiments.

Return to Section 33 and choose another answer.

29

Your answer in Section 16,

$$\frac{\Delta V}{\Delta h}=\frac{dV}{dh}=hAn\beta$$

is not correct. This answer would indicate that the relation between V and h—that is, the slope of a plot of V versus h—is a function of h. However, we have already seen that if we refill the tank after it has been drained by gravity, we will find V and h to be related by a constant $An\beta$. That is, we will find that $V=hAn\beta$ or that the ratio of V to h is the constant $An\beta$. If the tank is drained by increments, or refilled by increments after draining, we would expect the same relationship to hold between the increments of fluid volume, ΔV, and the increments of head, Δh, as was observed between V and h in the initial problem. That is, we would expect to find that $\Delta V=\Delta h\cdot An\beta$.

Return to Section 16 and choose another answer.

30

Your answer in Section 6 is not correct. Δh represents a simple change in the hydraulic head, h. It does not represent any form of rate of change; when we describe a rate of change, we always require two variables, since we always consider the ratio of change of one variable to that of another. At this point of our discussion, moreover, we are considering the relation between the volume of water in storage and the hydraulic head. We have not yet taken into consideration the effect of varying the base area of our prism of aquifer.

Return to Section 6 and choose another answer.

31

Your answer in Section 11 is not correct. The sand-filled tank of Section 11 differs from the open tank of Section 1, in that any quantity of water pumped into the sand-filled tank can utilize only the interconnected pore volume as its storage space; in the open tank of Section 1 the full capacity of the tank was available. If the sand-filled tank is initially empty and a volume of water, V, is pumped in, this water will occupy the total volume of interconnected space between the base of the tank and the height to which the sand is saturated (neglecting capillary effects). If the water level in the sand is a distance h above the base of the tank, the bulk volume of the saturated part of the sand will be $h \cdot A$, where A is the base area of the tank. However, the volume of injected water will not equal this bulk saturated volume, but rather the interconnected pore volume within the saturated region. A review of the definition of porosity as given in Part I may help to clarify this.

Return to Section 11 and choose another answer.

32

Your answer in Section 33,

$$\frac{dV}{dh} = An\beta,$$

is correct. The aquifer material is assumed to be identical to the sand in the tank experiments; if the area of the prism is equal to that of the tank, the two plots of storage versus water level should be identical. Note, however, that area is a factor in the expression for dv/dh; if we were to choose a prismatic section of larger area, it would provide more storage, per foot of head change, than one of smaller area, just as a tank of larger base area would provide more storage, per foot of water-level change, than a tank of smaller area. If the base of our prism of aquifer were unity, the expression for dV/dh would be simply $n\beta$; and in general, an expression could be written for the change in storage volume per unit head change, *per unit area of aquifer,* as

$$\frac{1}{A} \cdot \frac{dV}{dh} = n\beta.$$

The term $n\beta$ is referred to as the specific yield of an aquifer, and is usually designated S_y. Because we have assumed $(1-\beta)$, the fraction of water retained by capillary forces, to be constant, we obtain the result that S_y is a constant; and for many engineering applications, this is a satisfactory approximation. It should be noted, however, that it is only an approximation; the fraction of water held in capillary retention may change with time, for various reasons, leading to apparent variations in S_y with time.

Specific yield describes the properties of an aquifer to store and release water (through unconfined storage) just as permeability describes its properties of transmitting water. Mathematically, specific yield is equivalent to the term $(1/A)(dV/dh)$ for an unconfined aquifer.

(continued on next page)

32—Con.

QUESTION

On the basis of the above discussion, which of the following statements would you select as the best verbal definition of specific yield?

Turn to Section:

(a) The specific yield of an unconfined aquifer is the volume of water which can be drained by gravity from the aquifer in response to a unit decline in head. 7

(b) The specific yield of a horizontal unconfined aquifer is the volume of water which is drained by gravity from a vertical prism of unit base area extending through the aquifer, in response to a unit lowering of the saturated level. 6

(c) The specific yield of an unconfined aquifer is the quantity of water which can be drained from a unit area of the aquifer. 27

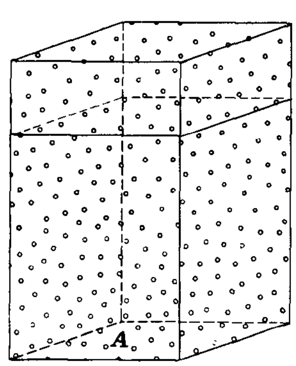
33

Your answer in Section 16,

$$\frac{\Delta V}{\Delta h}=\frac{dV}{dh}=An\beta,$$

is correct. The slope of the graph of volume of water in storage versus water level—or in other words, the derivative of V with respect to h—would be constant and equal to $An\beta$.

Now suppose that we are dealing with a prismatic section taken vertically through a uniform unconfined aquifer as shown in the figure. The base area of the prism is again denoted A. Suppose the aquifer material is identical in its hydraulic properties to the sand of our tank experiments. We wish to construct a graph of the water in recoverable storage within the prism versus the level of saturation, or water-table level, in the aquifer in the vicinity of the prism. We are interested only in water which can be drained by gravity; water in permanent capillary retention will not be considered part of the storage.



QUESTION

Which of the following expressions would describe the slope of this graph?

Turn to Section:

$$\frac{dV}{dh}=Ahn\beta$$ 28

$$\frac{dV}{dh}=An$$ 19

$$\frac{dV}{dh}=An\beta$$ 32

34

Your answer in Section 21 is not correct. In the imaginary experiment described in Section 21, it was stated that doubling the base area, A, of the prism had the effect of doubling the slope, dV/dh, of the V, h plot. Thus the term $A(dV/dh)$ would be four times as great for the prism of doubled area, as for the original prism. That is, the term $A(dV/dh)$ would depend upon the size of the prism considered, as well as upon the type of aquifer material, and could not be considered a constant representative of the aquifer material.

Return to Section 21 and choose another answer.