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Techniques of Water-Resources Investigations of the United States Geological Survey

Chapter B2

INTRODUCTION TO GROUND-WATER HYDRAULICS

A Programed Text for Self-Instruction

By Gordon D. Bennett

Book 3

APPLICATIONS OF HYDRAULICS

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PREFACE

The series of manuals on techniques describes procedures for planning and executing specialized work in water-resources investigations. The material is grouped under major subject headings called books and further subdivided into sections and chapters; Section B of Book 3 is on ground-water techniques.

This chapter is an introduction to the hydraulics of ground-water flow. With the exception of a few discussions in standard text format, the material is presented in programed form. In this form, a short section involving one or two concepts is followed by a question dealing with these concepts. If the correct answer to this question is chosen, the reader is directed to a new section, in which the theory is further developed or extended. If a wrong answer is chosen, the reader is directed to a section in which the earlier material is reviewed, and the reasons why the answer is wrong are discussed; the reader is then redirected to the earlier section, to choose another answer to the question. This approach allows students who are either partially familiar with the subject, or well prepared for its study, to proceed rapidly through the material, while those who require more explanation are provided it within the sections that deal with erroneous answers.

In the preparation of any text, difficult choices arise as to the material to be included. Because this text is an introduction to the subject, the discussion has been restricted, for the most part, to the flow of homogeneous fluid through an isotropic and homogeneous porous medium—that is, through a medium whose properties do not change from place to place or with direction. Emphasis has been placed upon theory rather than application. Basic principles of ground-water hydraulics are outlined, their uses in developing equations of flow are demonstrated, representative formal solutions are considered, and methods of approximate solution are described. At some points, rigorous mathematical derivation is employed; elsewhere, the development relies upon physical reasoning and plausibility argument.

The text has been prepared on the assumption that the reader has completed standard courses in calculus and college physics. Readers familiar with differential equations will find the material easier to follow than will readers who lack this advantage; and readers familiar with vector theory will notice that the materal could have been presented with greater economy using vector notation.

The material is presented in eight parts. Part I introduces some fundamental hydrologic concepts and definitions, such as porosity, specific discharge, head, and pressure. Part II discusses Darcy's law for unidirectional flow; a text-format discussion at the end of Part II deals with some generalizations of Darcy's law. Part III considers the application of Darcy's law to some simple field problems. The concept of ground-water storage is introduced in Part IV. A text-format discussion at the beginning of Part V deals with partial derivatives and their use in ground-water equations; the basic partial differential equation for unidirectional nonequilibrium flow is developed in the programed material of Part V. In Part VI, the partial differential equation for radial confined flow is derived and the "slug-test" solution, describing the effects of an instantaneous injection of fluid into a well, is presented and verified. A text-format discussion at the end of Part VI outlines the synthesis of additional solutions, including the Theis equation, from the "slug-test" solution. Part VII introduces the general concepts of finite-difference analysis, and a text format discussion at the end of Part VII outlines some widely used finite-difference techniques. Part VIII is concerned with electric-analog techniques. The material in Part VI is not prerequisite to that in Parts VII and VIII; readers who prefer may proceed directly from Part V to Part VII.

A program outline is presented in the table of contents of this report. This outline indicates the correct-answer sequence through each of the eight parts and describes briefly the material presented in each correct-answer section. Readers may find the outline useful in review or in locating discussions of particular topics, or may wish to consult it for an overview of the order of presentation.

It is impossible, in this or any other form of instruction, to cover every facet of each development, or to anticipate every difficulty which a reader may experience, particularly in a field such as ground water, where readers may vary widely in experience and mathematical background. An additional difficulty inherent in the programed text approach is that some continuity may be lost in the process of dividing the material into sections. For all these reasons, it is suggested that the programed instruction presented here be used in conjunction with one or more of the standard references on ground-water hydraulics.

This text is based on a set of notes used by the author in presenting the subject of ground-water hydraulics to engineers and university students in Lahore, West Pakistan, while on assignment with the U.S. Agency for International Development. The material has been drawn from a number of sources. The chapter by Ferris (1959) in the text by Wisler and Brater and that by Jacob (1950) in "Engineering Hydraulics" were both used extensively. Water-Supply Paper 1536–E (1962) by Ferris, Knowles, Brown, and Stallman was an important source, as was the paper by Hubbert (1940), "The Theory of Ground Water Motion." The text "The Flow of Homogeneous Fluids through Porous Media" by Muskat (1937) and the paper "Theoretical Investigation of the Motion of Ground Waters" by Slichter (1899) were both used as basic references. The development of the Theis equation from the "slug-test" solution follows the derivation given in the original reference by Theis (1935). The material on analog models is drawn largely from the book, "Analog Simulation," by Karplus (1958). In preparing the material on numerical methods, use was made of the book, "Finite-Difference Equations and Simulations," by Hildebrand (1968), and the paper "Selected Digital Computer Techniques for Groundwater Resource Evaluation," by Prickett and Lonnquist (1971). A number of additional references are mentioned in the text.

The author is indebted to Messrs. David W. Greenman and Maurice J. Mundorff, both formerly Project Advisors, U.S. Geological Survey–U.S.A.I.D., Lahore, for their support and encouragement during preparation of the original notes from which this text was developed. The author is grateful to Patricia Bennett for her careful reading and typing of the manuscript.

CONTENTS

SYMBOLS

Symbol	Dimensions (*M*, mass; *L*, distance; *T*, time)	Explanation
A	L^2	face area of aquifer, gross cross-sectional area of flow
a	L	node spacing in finite-difference grid
b	L	aquifer thickness
C	farad (coulombs/volt)	electrical capacitance
e		base of natural logarithms
$\mathbf{F}_g$	MLT^{-2}	gravitational force
$\mathbf{f}_l$	MLT^{-2}	component of gravitational force parallel to conduit
$\mathbf{f}_n$	MLT^{-2}	component of gravitational force normal to conduit
g	LT^{-2}	gravitational acceleration
h	L	head; static head
h_p	L	pressure head

Symbol	Dimensions	Explanation
I	amperes (coulombs/ second)	electrical current
K	LT^{-1}	hydraulic conductivity
k	L^2	intrinsic permeability
l	L	length
n		porosity
p	$ML^{-1}T^{-2}$	pressure
Q	L^3T^{-1}	volumetric fluid discharge
q	LT^{-1}	specific discharge—discharge per unit face area of aquifer, Q/A
R	ohms (volts/ ampere)	electrical resistance
S		storage coefficient
S_s	L^{-1}	specific storage
S_y		specific yield
T	L^2T^{-1}	transmissivity (transmissibility)

Symbol	Dimensions	Explanation
u		$r^2S/4Tt$—argument of the well function
V	L^3	fluid volume
v	LT^{-1}	velocity
$W(u)$		well function
w	L	width
z	L	elevation above datum
β		fraction of the total water in storage that can be drained by gravity
$\Delta_{xx}h$	L^{-1}	finite-difference approximation to $\partial^2h/\partial x^2$
$\Delta_{yy}h$	L^{-1}	finite-difference approximation to $\partial^2h/\partial y^2$
ϵ	coulomb	electrical charge
μ	$ML^{-1}T^{-1}$	dynamic viscosity
ρ	ML^{-3}	fluid density
ρ_e	ohm-metres	electrical resistivity
σ	mhos/metre	electrical conductivity
ϕ	volts	voltage or electrical potential

UNIT CONVERSION

English	Factor for converting English units to international system of units	Metric SI
ft (foot)	3.048×10^{-1}	m (metre)
gal (gallon)	3.785	l (litre)
ft^3/s (cubic foot per second)	2.832×10^{-2}	m^3/s (cubic metre per second)

PROGRAM OUTLINE

This program outline is provided to assist the reader in review, and to facilitate the location of particular topics or discussions in the text. Hopefully, it may also provide some feeling for the organization of the material and the order of presentation, both of which tend to be obscured by the programed format.

The section numbers in the left margin correspond to correct answers in the programed instruction; they give the sequence of sections which will be followed if no errors are made in answering the questions. An outline of the content of each of the correct-answer sections is given to the right of the section number. Two numbers are listed beneath each of these section outlines. These numbers identify the wrong-answer sections for the question presented in the outlined correct-answer section. The correct answer to this question is indicated by the next entry in the left margin.

The discussions written in standard text format are also outlined. For these discussions, page numbers corresponding to the listed material are given in parentheses in the left margin.

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9 effective porosity; saturation
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3 tortuous flow path effects (review); problems in determining actual cross-sectional flow area; relation of discharge per unit face area to flow velocity
28; 10

14 relation of discharge per unit face area to flow velocity (review); definition of specific discharge or specific flux; definition of head
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24 omission of velocity head in ground water; relation between pressure and height of fluid column (Pascal's law)
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35 expression for component of gravitational force parallel to conduit (review); substitution of $\Delta z/\Delta l$ for cosine in this expression (question)
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(p. 32) flowlines and surfaces of equal head in the anisotropic case; solution by transformation of coordinates

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(p. 33) aquifer heterogeneity

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TECHNIQUES OF WATER-RESOURCES INVESTIGATIONS OF THE U.S. GEOLOGICAL SURVEY

The U.S. Geological Survey publishes a series of manuals describing procedures for planning and conducting specialized work in water-resources investigations. The manuals published to date are listed below and may be ordered by mail from the **U.S. Geological Survey, Books and Open-File Reports Section, Federal Center, Box 25425, Denver, Colorado 80225** (an authorized agent of the Superintendent of Documents, Government Printing Office).

Prepayment is required. Remittance should be sent by check or money order payable to U.S. Geological Survey. Prices are not included in the listing below as they are subject to change. **Current prices can be obtained** by writing to the USGS, Books and Open File Reports Section. Prices include cost of domestic surface transportation. For transmittal outside the U.S.A. (except to Canada and Mexico) a surcharge of 25 percent of the net bill should be included to cover surface transportation. When ordering any of these publications, please give the title, book number, chapter number, and "U.S. Geological Survey Techniques of Water-Resources Investigations."

TWI 1-D1. Water temperature—influential factors, field measurement, and data presentation, by H.H. Stevens, Jr., J.F. Ficke, and G.F. Smoot, 1975, 65 pages.
TWI 1-D2. Guidelines for collection and field analysis of ground-water samples for selected unstable constituents, by W.W. Wood. 1976. 24 pages.
TWI 2-D1. Application of surface geophysics to ground water investigations, by A.A.R. Zohdy, G.P. Eaton, and D.R. Mabey. 1974. 116 pages.
TWI 2-D2. Application of seismic-refraction techniques to hydrologic studies. F.P. Haeni. 1988. 86 pages.
TWI 2-E1. Application of borehole geophysics to water- resources investigations, by W.S. Keys and L.M. MacCary. 1971. 126 pages.
TWI 3-A1. General field and office procedures for indirect discharge measurements, by M.A. Benson and Tate Dalrymple. 1967. 30 pages.
TWI 3-A2. Measurement of peak discharge by the slope-area method, by Tate Dalrymple and M.A. Benson. 1967. 12 pages.
TWI 3-A3. Measurement of peak discharge at culverts by indirect methods, by G.L. Bodhaine. 1968. 60 pages.
TWI 3-A4. Measurement of peak discharge at width contractions by indirect methods, by H.F. Matthai. 1967. 44 pages.
TWI 3-A5. Measurement of peak discharge at dams by indirect methods, by Harry Hulsing. 1967. 29 pages.
TWI 3-A6. General procedure for gaging streams, by R.W. Carter and Jacob Davidian. 1968. 13 pages.
TWI 3-A7. Stage measurements at gaging stations, by T.J. Buchanan and W.P. Somers. 1968. 28 pages.
TWI 3-A8. Discharge measurements at gaging stations, by T.J. Buchanan and W.P. Somers. 1969. 65 pages.
TWI 3-A9. Measurement of time of in streams by dye tracings, by F.A. Kilpatrick and J.F. Wilson, Jr. 1989. 27 pages.
TWI 3-A10. Discharge ratings at gaging stations, by E.J. Kennedy. 1984. 59 pages.
TWI 3-A11. Measurement of discharge by moving-boat method, by G.F. Smoot and C.E. Novak. 1969. 22 pages.
TWI 3-A12. Fluorometric procedures for dye tracing, Revised, by James F. Wilson, Jr., Ernest D. Cobb, and Frederick A. Kilpatrick. 1986. 41 pages.
TWI 3-A13. Computation of continuous records of streamflow, by Edward J. Kennedy. 1983. 53 pages.
TWI 3-A14. Use of flumes in measuring discharge, by F.A. Kilpatrick, and V.R. Schneider. 1983. 46 pages.
TWI 3-A15. Computation of water-surface profiles in open channels, by Jacob Davidian. 1984. 48 pages.
TWI 3-A16. Measurement of discharge using tracers, by F.A. Kilpatrick and E.D. Cobb. 1985. 52 pages.
TWI 3-A17. Acoustic velocity meter systems, by Antonius Laenen. 1985. 38 pages.
TWI 3-A18. Determination of stream reaeration coefficients by use of tracers, by F.A. Kilpatrick, R.E. Rathbun, N. Yotsukura, G.W. Parker, and L.L. DeLong. 1989. 52 pages.
TWI 3-B1. Aquifer-test design, observation, and data analysis, by R.W. Stallman. 1971. 26 pages.
TWI 3-B2.[1] Introduction to ground-water hydraulics, a programmed text for self-instruction, by G.D. Bennett. 1976. 172 pages.
TWI 3-B3. Type curves for selected problems of flow to wells in confined aquifers, by J.E. Reed. 1980. 106 pages.
TWI 3-B5. Definition of boundary and initial conditions in the analysis of saturated ground-water flow systems—an introduction, by O. Lehn Franke, Thomas E. Reilly, and Gordon D. Bennett. 1987. 15 pages.

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[1]Spanish translation also available.

TWI 3-B6. The principle of superposition and its application in ground-water hydraulics, by Thomas E. Reilly, O. Lehn Franke, and Gordon D. Bennett. 1987. 28 pages.

TWI 3-C1. Fluvial sediment concepts, by H.P. Guy. 1970. 55 pages.

TWI 3-C2. Field methods of measurement of fluvial sediment, by H.P. Guy and V.W. Norman. 1970. 59 pages.

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INTRODUCTION TO GROUND-WATER HYDRAULICS—A PROGRAMED TEXT FOR SELF-INSTRUCTION

By Gordon D. Bennett

Instructions to the Reader

This programed text is designed to help you learn the theory of ground-water hydraulics through self-study. Programed instruction is an approach to a subject, a method of learning; it does not eliminate mental effort from the learning process. Some sections of this program need only be read; others must be worked through with pencil and paper. Some of the questions can be answered directly; others require some form of calculation. You may have frequent occasion, as you work through the text, to consult standard texts or references in mathematics, fluid mechanics, and hydrology.

In each of the eight parts of the text, begin the programed instruction by reading Section 1. Choose an answer to the question at the end of the section, and turn to the new section indicated beside the answer you have chosen. If your answer was correct, you will turn to a section containing new material and another question, and you may proceed again as in Section 1. If your answer was not correct, you will turn to a section which contains some further explanation of the earlier material, and which directs you to go back for another try at the question. Usually, in this event, it will be worthwhile to reread the material of the earlier section. Continue in this way through the program until you reach a section indicating the end of the part. Note that although the sections are arranged in numerical order within each of the eight parts, you would not normally proceed in numerical sequence (Section 1 to Section 2 and so on) through the instruction.

Part I. Definitions and General Concepts

Introduction

In Part I, certain concepts which are frequently used in ground-water hydraulics are introduced. Among these are porosity, specific discharge, hydraulic head, and fluid pressure. Rigorous development of theorems relating to these terms is not attempted. The material is intended only to introduce and define these terms and to provide an indication of their physical significance.

The porosity of a specimen of porous material is defined as the ratio of the volume of open pore space in the specimen to the bulk volume of the specimen.

QUESTION

What volume of solid material is present in 1 cubic foot of sandstone, if the porosity of the sandstone is 0.20?

	Turn to Section:
0.5 cubic feet	13
0.2 cubic feet	18
0.8 cubic feet	9

1.

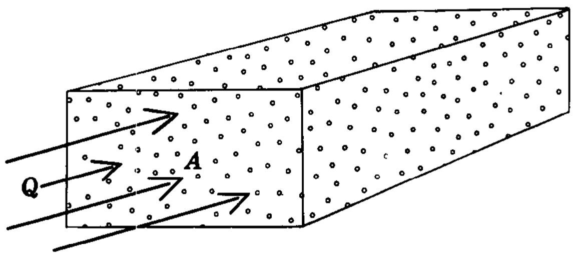
Nowhere in Part I is there an instruction to turn to Section 2. Perhaps you have just read Section 1 and have turned to Section 2 without considering the question in Section 1. If so, return to Section 1, choose an answer to the question, and turn to the section indicated opposite the answer you select.

2.

Your answer in Section 6 is correct. Any flow path between A and B will be longer than the linear distance AB; it is generally impossible to know the actual distance that a particle of fluid travels in moving through a section of porous material.

In the same way, it is difficult to know the actual cross-sectional area of the flow, when dealing with flow in a porous medium. Any cross-sectional area selected will be occupied partly by grains of solid material and partly by pores containing the fluid. For this reason, a problem may arise if we attempt to define average fluid velocity as a ratio of discharge to cross-sectional area, as is customarily done in open-flow hydraulics.

Con.— ***3.***



3. —Con.

QUESTION

In the block of saturated porous material in the figure, a fluid discharge, Q, is crossing the area, A, at right angles. A represents the gross area of the block face, including both solid particles and fluid-filled pore space. The quotient Q/A would be:

	Turn to Section:
less than	14
equal to	28
greater than	10

the average velocity of the fluid particles

4.

Your answer in Section 6 is not correct. The particle would move a distance equal to the linear interval AB if the two points were connected by a straight capillary tube, but the probability of such a connection is essentially zero in a normal porous medium. In general, the possible paths of flow between any two points will be tortuous in character.

Return to Section 6 and select another answer.

5.

Your answer in Section 22 is not correct. Pressure does represent potential energy per unit volume due to the forces transmitted through the surrounding fluid, but z represents potential energy per unit *weight* due to elevation. The question asked for total potential energy per unit volume.

Return to Section 22 and select another answer.

---

6. —Con.

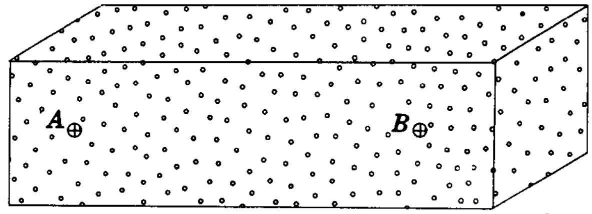
Your answer in Section 9 is correct. Thirty percent of the interconnected pore space in a porous medium whose effective porosity is 0.20 is 6 percent of the bulk volume, or 0.06 cubic feet. In the remainder of this program, fully saturated conditions will be assumed unless unsaturated flow is specifically mentioned.

Variation in the flow velocity of an individual fluid particle is inherent in the nature of flow through porous media. Within an individual pore, boundary resistance causes the velocity to decrease from a maximum along the centerline to essentially zero at the pore wall. Another form of velocity variation is imposed by the tortuous character of the flow—that is, the repeated branching and reconnecting of flow paths, as the particles of fluid make their way around the individual grains of solid. This anastomizing or braided pattern causes the velocity of a fluid particle to vary from point to point in both magnitude and direction, even if its motion occurs along the centerline of the pore space. However, if we view a small segment of the medium but one which is still large enough to contain a great number of pores, we find that the microscopic components of motion cancel in all except one resultant direction of flow.

QUESTION

In the porous block in the figure, a particle of fluid moving from point A to point B would travel a distance:

	Turn to Section:
greater than the linear distance AB	3
equal to the linear distance AB	4
less than the linear distance AB	21



Con.— **6.**

Your answer in Section 16 is not correct. If we were considering the height of a static column of water above a point, which as we have seen is given by $p/\rho g$, we would be dealing with dimensions of potential energy per unit weight. The question in Section 16, however, relates to the units of pressure alone. These units are force per unit area—for example, pounds of force per square foot of area, which can be written in the form pounds/ft^2. Now we may "multiply" these units by the term ft/ft to obtain an equivalent set of units applicable to pressure.

Return to Section 16 and choose another answer.

7.

Your answer, $p+\rho gz$, in Section 22 is correct. We have seen that pressure is equivalent to potential energy per unit volume attributable to forces transmitted through the surrounding fluid. Potential energy per unit volume due to elevation is obtained by multiplying the potential energy per unit weight due to elevation—that is, z—by the weight per unit volume, ρg. The total potential energy per unit volume is then the sum of these two terms, that is, $p+\rho gz$.

No discussion of flow energy would be complete without mention of kinetic energy. In the mechanics of solid particles, the kinetic energy, KE, of a mass, m, moving with a velocity v, is given by

$$\text{KE} = mv^2/2.$$

Now suppose we are dealing with a fluid of mass density ρ. We wish to know the kinetic energy of a volume V of this fluid which is moving at a velocity v. The mass of the volume is ρV, and the kinetic energy is thus

$$\rho V v^2/2.$$

If we divide by the volume, V, we obtain

$$\rho v^2/2$$

as the kinetic energy per unit volume of fluid; and dividing this in turn by the weight per unit volume, ρg, gives $v^2/2g$ as the kinetic energy per unit weight of fluid. Each of these kinetic energy expressions is proportional to the square of the velocity. The velocities of flow in ground-water movement are almost always extremely low, and therefore the kinetic energy terms are extremely small compared to the potential energy terms. Consequently, in dealing with ground-water problems we can generally neglect the kinetic energy altogether and take into account only the potential energy of the system and the losses in potential energy due to friction. This is an important respect in which ground-water hydraulics differs from the hydraulics of open flow.

This discussion concludes Part I. In Part II we will consider Darcy's law, which relates the specific discharge, q, to the gradient of hydraulic head, in flow through porous media.

8.

Your answer in Section 1 is correct; if 0.20 of the cube is occupied by pore space, 0.80 of its volume must be solid matter. In ground-water studies we are normally interested in the interconnected, or effective, porosity, which is the ratio of the volume of interconnected pore space—excluding completely isolated pores—to the bulk volume. As used in this text the term "porosity" will always refer to the interconnected or effective porosity. Ground water is said to occur under saturated conditions when all interconnected pore space is completely filled with water, and it occurs under unsaturated conditions when part of the pores contain water and part contain air. In problems of unsaturated flow, the degree of saturation is often expressed as a percentage of the interconnected pore space.

9.

QUESTION

What volume of water is contained in 1 cubic foot of porous material, if the effective porosity is 0.20 and saturation expressed as a percentage of the interconnected pore space is 30 percent?

	Turn to Section:
0.30 cubic feet	12
0.06 cubic feet	6
0.20 cubic feet	29

Your answer in Section 3 is not correct. The area A represents the gross cross-sectional area of the porous block, normal to the direction of flow. A part of this area is occupied by grains of solid, and a part by open pore space. Let us say that 20 percent of the area A represents pore space; the actual cross-sectional area available for the flow is thus 0.2 A. If we were willing to take the ratio of discharge to *flow* area as equal to the average velocity, without considering any other factor, we would have to use the ratio $Q/0.2A$. The actual average particle velocity would presumably exceed even this figure, because of the excess distance traveled in tortuous flow.

Return to Section 3 and choose another answer.

10.

Your answer in Section 14 is not correct. The column of water in the piezometer is static, but h_p is the elevation of the top of this column above the point of measurement, 0 (h_p is sometimes referred to as the pressure head at point 0). We have defined *head* as the elevation *above datum* of the top of a static column of water that can be supported at the point.

Return to Section 14 and choose another answer.

11.

Your answer in Section 9 is not correct. Saturation is expressed here as a percentage of the interconnected pore space, not as a percentage of the sample volume; that is, 30 percent of the interconnected pore space is occupied by water. Since the effective porosity was given as 0.20, and the sample volume as 1 cubic foot, the volume of interconnected pore space is 0.20 cubic feet.

Return to Section 9 and choose another answer.

12.

Your answer in Section 1 is not correct. Porosity is defined by the equation

$$n=\frac{V_p}{V_g}=\frac{V_p}{V_s+V_p}$$

where V_p is the volume of pore space in the specimen, V_g is the gross volume of the specimen, and V_s is the volume of solid material in the specimen (note that $V_g=V_s+V_p$). The question in Section 1 asked for the volume of solid material, V_s, in a specimen for which the gross volume, V_g, is 1 cubic foot and the porosity, n, is 0.20.

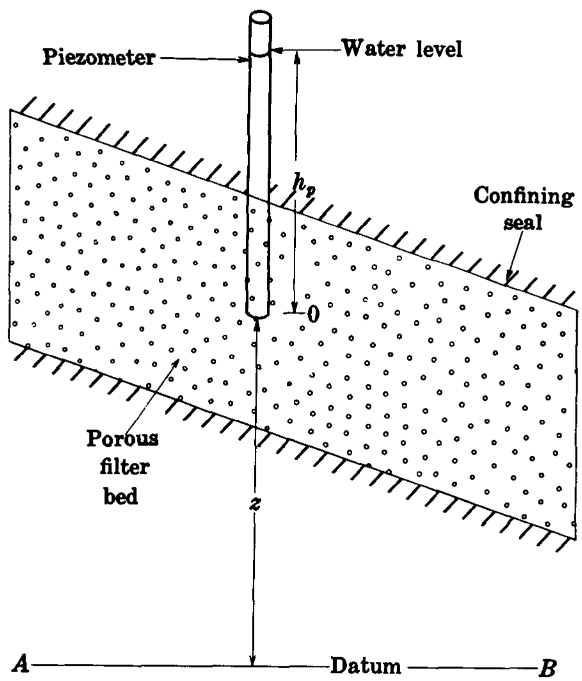
Return to Section 1 and choose another answer.

13.

Your answer in Section 3 is correct. Q/A will be less than the average velocity of fluid motion since the gross cross-sectional area, A, will be greater than the actual cross-sectional area of flow. In many porous media, the ratio of actual area of flow to gross cross-sectional area can be taken as equal to the interconnected porosity of the material.

We have seen that it is generally difficult or impossible to know or measure the actual velocity of fluid motion or the actual cross-sectional area of flow in a porous medium. For this reason, we usually work in terms of discharge and gross cross-sectional area. That is, we use the quantity Q/A, where Q is the discharge through a segment of porous material, and A is the gross cross-sectional area of the segment. This quantity is referred to as the specific discharge, or specific flux, and is designated by the symbol q.

Another quantity we will use frequently is the static head, or simply the head. In ground-water problems, the head at a point is taken as the elevation, above an arbitrary datum, of the top of a static column of water that can be supported above the point. In using this definition, we assume that the density of the water in the measuring column is equal to that of the ground water, and that the density of the ground water is uniform.



QUESTION

The diagram represents an enclosed porous filter bed; the plane AB is taken as the datum and a piezometer is inserted to the point 0. What is the head at point 0?

	Turn to Section:
The distance h_p	11
The distance z	17
The distance h_p+z	24

14.

Your answer in Section 16 is not correct. Pressure is usually expressed as force per unit area—for example, as pounds per square foot, which may be written pounds/ft^2. A term having units of work or energy per unit area, such as ft-pounds/ft^2, would represent the product of pressure and a term having units of distance, feet. We are interested here in an equivalent set of units for pressure alone. Now note that if a pressure term were multiplied by a dimensionless factor having "units" of ft/ft, we would obtain a result still having the units of pressure.

Return to Section 16 and select another answer.

15.

Your answer, $p/\rho g$, in Section 24 is correct. The column of water inside the pipe is static and must obey the laws of hydrostatics. Thus the pressure at the bottom of the pipe is related to the height of the column of water in the pipe by Pascal's law, which here takes the form

$$p=\rho g h_p,$$

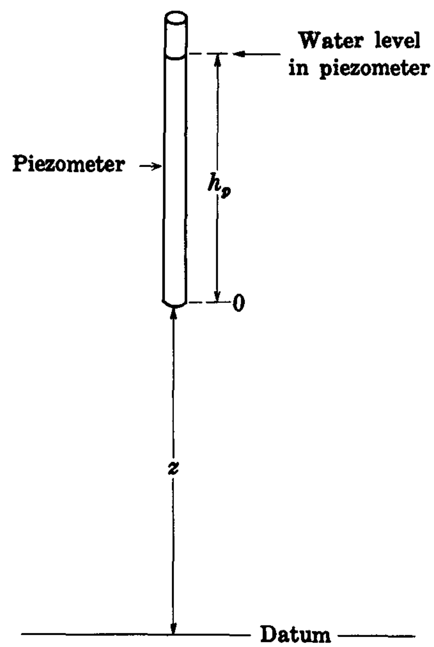
or

$$h_p=p/\rho g$$

h_p thus actually serves as a measure of the pressure at the point occupied by the end of the pipe and, for this reason, is termed the pressure head at that point. It is added to the elevation of the point to yield the head at the point.

Head in ground water is actually a measure of the potential energy per unit weight of water. This is an important concept.

The elevation term, z, in the diagram represents the potential energy of a unit weight of water at point 0 that accrues from the position of the point above the datum. For example, if z is 10 feet, 10 pounds of water in the vicinity of point 0 could accomplish 100 foot-pounds of work in falling to the datum; the potential energy per unit weight of water at point 0 due to the elevation of the point alone would thus be 10 feet. Similarly, the pressure term, h_p, represents the potential energy of a unit weight of water at point 0 originating from the forces exerted on the point through the surrounding fluid. This concept is considered further in the following sections.



(Point 0 represents a general point in a fluid system)

QUESTION

Pressure is normally thought of as force per unit area. Dimensionally this is equivalent to:

	Turn to Section:
energy per unit weight	7
energy per unit volume	26
work per unit area	15

16.

Your answer in Section 14 is not correct. z is the elevation of the point above the datum; we defined head as the elevation, above datum, of the top of a static column of water that can be supported at the point. The column of water in the piezometer is static when conditions in the porous medium are at equilibrium.

Return to section 14 and choose another answer.

17.

Your answer in Section 1 is not correct. If the porosity is 0.20, there will be 0.20 cubic foot of pore space in a specimen of 1-cubic-foot volume. The question asked for the volume of solid material in the specimen.

Return to Section 1 and choose another answer.

18.

Your answer in Section 24 is not correct. The column of water inside the pipe is static and must obey the laws of hydrostatics. The pressure at a depth d beneath the water surface, in a body of static water, is given by Pascal's law as

$$p = \rho g d$$

where again ρ is the mass density of the water, g is the acceleration due to gravity, and the pressure at the water surface is taken as zero. This relation may be applied to the water inside the pipe in the question of Section 24. If you are not familiar with Pascal's law it would be useful to read through a discussion of hydrostatics, as given in any standard physics text, before proceeding further in the program.

Return to Section 24 and choose another answer.

19.

Your answer in Section 26 is not correct. Potential energy is a scalar term; when it consists of contributions from different sources, these are simply added to obtain the total potential energy. The potential energy of the unit weight of water due to its elevation is z, while that due to the forces exerted on it through the surrounding water is h_p.

Return to Section 26 and choose another answer.

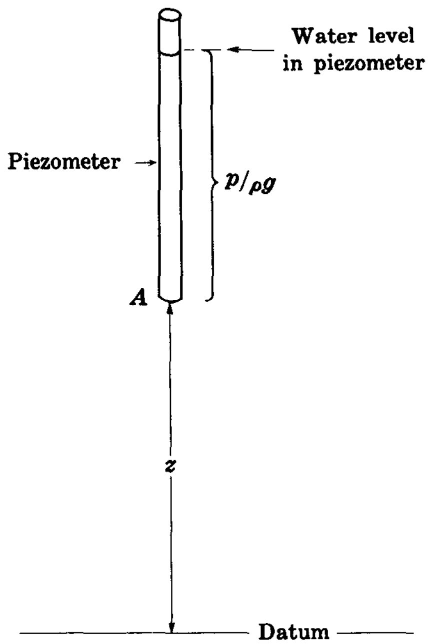
20.

Your answer in Section 6 is not correct. The line AB is, of course, the shortest distance between the two points, and no flow path could be any shorter than this.

Return to Section 6 and select another answer.

21.

Your answer in Section 26 is correct. The unit weight of water has hydraulic potential energy due to its elevation and due to the forces exerted on it by the surrounding fluid. The potential energy due to its elevation is z, and the potential energy due to the forces exerted on it through the surrounding fluid is $p/\rho g$ or h_p. The sum of z and h_p is of course the head, h, (as used in ground-water hydraulics) at the point in question. The two terms making up the head at a point—the elevation of the point itself above datum and the elevation of the top of a static column of water that can be supported above the point—measure respectively the two forms of hydraulic potential energy per unit weight. Their sum indicates the total hydraulic potential energy per unit weight of fluid at the point.



QUESTION

Which of the following expressions would indicate total hydraulic potential energy of a unit *volume* of fluid in the vicinity of point *A* in the diagram?

	Turn to Section:
$p+\rho gz$	8
$p+z$	5
$p/\rho g+z$	27

22.

Your answer in Section 26 is not correct. z represents the potential energy of a unit weight of water in the vicinity of point 0, due to its elevation above the datum. A unit weight of water in this vicinity will also possess potential energy because of the forces exerted upon it through the surrounding water. The question asked for total hydraulic potential energy.

Return to Section 26 and select another answer.

23.

Your answer in Section 14 is correct. Head consists of two terms in ground-water systems: the elevation of the point itself above datum, and the height of a static column of water that can be supported above the point. In this case, the column of water in the piezometer is the static column above the point.

The height of the column of water above the point is a measure of the pressure at the point and is sometimes termed the pressure

24.—Con.

head. Readers familiar with open flow hydraulics may recognize that the head we have defined here differs from the total head used in open flow hydraulics in that the velocity term, $v^2/2g$, is missing. Velocities of flow are usually small in ground-water systems, and the term $v^2/2g$ is almost always negligible in comparison to the elevation and pressure terms.

QUESTION

Suppose a pipe, open only at the top and bottom, is driven into the ground. The bottom of the pipe comes to rest at a point below the water table where the pressure is p. Water rises inside the pipe to a height h_p above the lower end of the pipe. The pressure on the water surface within the pipe (which is actually the atmospheric pressure) is here taken as zero. The height of the column of water inside the pipe, above the bottom of the pipe, will be given by:

	Turn to Section:
$h_p = p/\rho g$	16
$h_p = g/\rho p$	25
$h_p = p\rho g$	19

where ρ is the water density, or mass per unit volume, and g is the gravitational constant.

Con.— 24.

Your answer in Section 24 is not correct. Pressure within a body of static water varies in accordance with Pascal's law, which may be stated

$$p = \rho g d$$

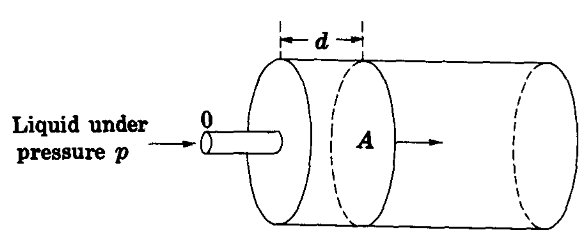
where ρ is the mass density of water, g is the acceleration due to gravity, and d is the depth below the surface at which the pressure is measured. The pressure on the upper surface of the water (sometimes denoted p_0 in textbooks of hydraulics) is here considered to be zero. If you are not familiar with this relation, it would be a good idea to read through a discussion of hydrostatics, as presented in any standard physics text, before proceeding further with the program.

In the problem of Section 24, the column of water in the pipe is static, and Pascal's law may be used to give the pressure at any point within this column—even at its base, where it joins the ground-water system.

Return to Section 24 and choose another answer.

25.

Your answer in Section 16 is correct. Pressure may in fact be thought of as potential energy per unit volume of liquid. Physically, this concept is perhaps most easily appreciated using the example of a simple hydraulic cylinder, or hydraulic press, shown schematically in the diagram. Liquid under a pressure p is fed in through the port at 0. As the liquid enters, the piston is displaced to the right. Pressure is a measure of force per unit area, and it follows that the total force on the piston is given by the product of the pressure, p, and the face area of the



Con.— 26.

piston, which we designate A. Thus, $F=p\times A$, where F is the force on the piston.

The work accomplished in moving the piston is given as the product of the force and the distance through which it acts. If the piston moves a distance d, the work done is given by

$$W=F\times d=p\times A\times d$$

where W is the work accomplished in moving the piston. The product $A\times d$ is the volume of fluid in the cylinder at the completion of the work; and we could say that this volume of liquid is capable of doing the work W, provided the liquid is at the pressure p.

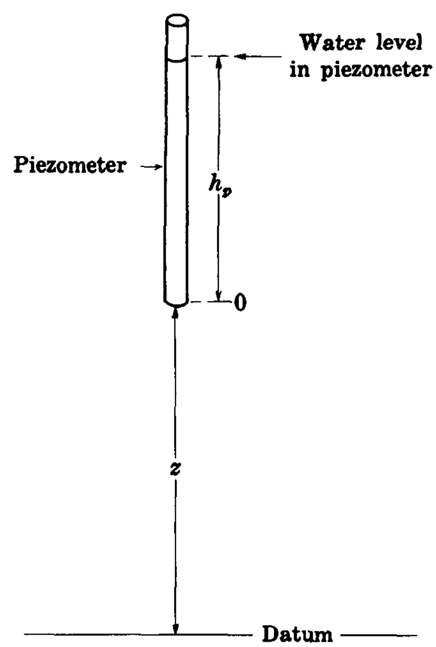
Potential energy is often termed the ability to do work. That is, if a system is capable of doing 10 foot-pounds of work, we say that it possesses a potential energy of 10 foot-pounds. In the case of our cylinder, the potential energy we assign depends upon how far we are willing to let the piston travel. If the piston is allowed to travel a distance $d=5$, the work that can be done is $p\times 5A$; if the piston is allowed to travel a distance $d=10$, the work that can be done is $p\times 10A$. Thus the assignment of a potential energy in this case is not altogether straightforward, since the distance which the piston will travel—or, equivalently, the volume of fluid which will be admitted to the cylinder under the pressure p—must be specified before the potential energy can be assigned. In this case, therefore, it is more convenient to talk about a potential energy per unit volume of liquid. For example, if we are told that the potential energy is 10 foot-pounds per cubic foot of water in the cylinder, we can calculate the particular potential energy associated with the admission of any specified volume of fluid to the cylinder. The work which can be done if a volume $A\times d$ of liquid is admitted is $p\times A\times d$; dividing this by the volume $A\times d$ gives the work which can be done per unit volume of liquid—that is, the potential energy per unit volume of liquid. This potential energy per unit volume turns out to be the pressure, p, under which the fluid is admitted to the cylinder.

This concept of pressure as potential energy per unit volume can be extended to general systems of flow, provided that we understand this potential energy to be only that due to forces exerted on a fluid element by the surrounding fluid. To obtain total potential energy, we would have to add the potential energy due to the force of gravity acting directly on the fluid element.

If pressure, representing potential energy per unit volume, is in turn divided by ρg, weight per unit volume, we obtain $p/\rho g$—or simply h_p, the height of a static column of water above the point—as the potential energy per unit weight that is due to the forces transmitted through the surrounding fluid.

QUESTION

Referring to the diagram, which of the following expressions will give the total hydraulic potential energy of a unit weight of water located in the vicinity of point 0?



	Turn to Section:
z	23
h_p+z	22
h_p-z	20

26.—Con.

Your answer in Section 22 is not correct. We have already seen that $p/\rho g + z$ was equal to the total potential energy per unit weight of water. To obtain potential energy per unit volume, we must multiply by weight per unit volume.

Return to Section 22 and choose another answer.

27.

Your answer in Section 3 is not correct. The quotient, Q/A, would yield an average velocity if we were dealing with an open flow. Here, however, A is not the cross-sectional area of flow; it is, rather, the cross-sectional area of the porous block normal to the flow. Only that fraction of this area which consists of open pore space can be considered the cross-sectional area of flow. Suppose, for example, that this pore area represents 20 percent of the total face area, A. The flow area would then be 0.2 A.

Return to Section 3 and choose another answer.

28.

Your answer in Section 9 is not correct. The volume of interconnected pore space is 0.20 cubic feet, but since saturation is less than 100 percent, the volume of water in the specimen cannot equal the volume of interconnected pore space. Keep in mind that we are expressing saturation as a percentage of the interconnected pore space.

Return to Section 9 and choose another answer.

29.