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Techniques of Water-Resources Investigations of the United States Geological Survey

Chapter A18

DETERMINATION OF STREAM REAERATION COEFFICIENTS BY USE OF TRACERS

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Book 3
APPLICATIONS OF HYDRAULICS

Table 5.—*Locations of sampling points based on cumulative discharge*

Number of sampling points	Percent of total discharge sampled at each point	Locations of sampling points 1	2	3	4	5	6	7	8	9	10
		Cumulative discharge in percent									
3	33.3	16.7	50	83.3							
5	20.0	10.0	30.0	50.0	70.0	90.0					
7	14.3	7.1	21.4	35.7	50	64.3	78.6	92.9			
10	10.0	5.0	15.0	25.0	35.0	45.0	55.0	65.0	75.0	85.0	95.0

40-mL glass bottle. The bottle can be obtained from the Geological Survey Central Laboratory. Important features are as follows:

1. The sample bottle is flushed many times in the process of filling.
2. The sample bottle is kept submerged while the formalin is injected and the Teflon-lined cap is placed on the bottle.
3. The container is large enough to permit capping of the bottle with the fingers and easy removal of the bottle.

Samplers similar to that sketched in figure 17 are suggested, as they are inexpensive and effective in obtaining representative gas samples.

Figure 18 shows the 40-mL glass bottle in place; the intake tube must project into the bottle when the pipe cap is threaded in place. Figure 19 shows the sampler being lowered into the flow. In figure 20, 1 ml of a 37-percent formalin stock solution is being injected into the submerged sample bottle with a syringe just prior to capping. The sample bottle is then capped (fig. 21) while still covered with water, with the Teflon surface *facing the inside of the sample bottle.* Gas sample bottles are then placed in an insulated cooler and their caps checked for tightness. The samples should be neither excessively cooled nor warmed, the objective being to keep them at about the same temperature as collected. As soon as possible, the gas samples should be inspected for the presence of bubbles and if found, discarded. For this reason, extra samples should be collected and only selected ones shipped to the laboratory. The laboratory should be notified by phone when and how many samples are being shipped.

Performance of Slug-Injection Reaeration Measurements

This section illustrates by example the techniques and analysis procedures suggested for performing an SI reaeration measurement. The example involves a State Environmental Control Board's concerns with the waste loading entering Rath Creek just upstream from Highway 1 and its need to know the reaeration capabilities of the downstream reach (see fig. 22). Of particular concern was the extent of natural purification and the level of dissolved oxygen at a water supply withdrawal point 4 mi (miles) downstream. Data were obtained to allow computation of reaeration coefficients using both the peak and area methods.

Planning

Careful planning of the field test is as important as its execution. Planning begins with examination of maps, reports, and any existing hydrologic data for the stream to be tested.

Selection of test reach

Figure 22 shows a sketch of Rath Creek and provides data on its properties useful in planning and selecting a test reach. The initial plan was to inject dye and gas tracers at an access area just below the Highway 1 bridge in an area where depths were in excess of 2 ft; mixing of the tracers would be accomplished in the 12,000 ft down to the Highway 2 bridge crossing. The measurement reach would be the 8,000 ft of channel down to the State park. The downstream section was to be located just upstream from a significant inflow from Bun Creek, as it was recognized that poor mixing conditions would exist downstream from this inflow. The motel and the covered pavilion would

Figure 18.—Placement of 40-milliliter sample bottle in holder.

Figure 19.—Collecting gas sample at selected point in measuring section.

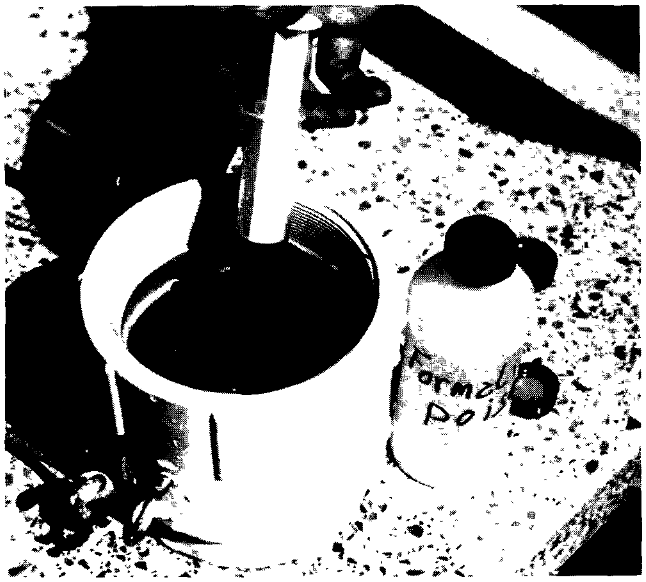


Figure 20.—Injection of 1 milliliter of formalin solution into submerged sample just prior to capping.

serve as sites for the fluorometer when sampling at the upstream and downstream sections, respectively.

Reach slope

A plot of contour elevations versus stream centerline distance through the proposed test reach, as well as upstream and downstream from the proposed reach, as picked from a topographic map, yielded an average slope of 0.001 ft/ft.

Traveltimes

This is a pool-and-riffle type of stream, so equation 24 was used to compute a peak velocity:

$$V_p=0.38\ (40)^{0.4}\ (0.001)^{0.2}=0.42\ \text{ft/s}$$

This is consistent with a mean velocity of 0.5 ft/s estimated from current-meter measurements made at Highway 1 and 2 bridges several years earlier as part of a low-flow study. The slower velocity was thought

Figure 21.—Sealing sample with Teflon-lined cap just prior to removal and storage in ice chest.

to be more accurate because current-meter measurements are usually made in sections having greater than average velocities.

Using 0.4 ft/s, the peak traveltimes to the upstream and downstream sections were estimated as

$$\text{Upstream: } \frac{12{,}000\text{ ft}}{0.4\text{ ft/s}}\times\frac{1}{3{,}600\text{ s/h}}=8.33\text{ h}$$

$$\text{Downstream: } \frac{20{,}000\text{ ft}}{0.4\text{ ft/s}}\times\frac{1}{3{,}600\text{ s/h}}=13.89\text{ h}$$

Therefore, the traveltime of a peak, t_p, through the test reach is 13.89–8.33=5.56 h.

Mixing length

Using equation 23 and table 1, an optimum mixing length *from the point of injection* was computed as

$$L_o=0.1\frac{(0.4)(40)^2}{0.1}=640\text{ ft}$$

While this was just an estimate, 12,000 ft would be available from Highway 1 to Highway 2 and, hence, mixing was expected to be more than adequate.

Residence time

The decision was made to use propane as the gas tracer in the test. It is desirable that the product of its desorption coefficient, K_p, and the peak time of travel, t_p, be 1.00 or greater. Equation 29 was used to estimate the desorption coefficient for propane:

$$K_p=0.651(0.4)^{0.67}/(2)^{1.85}=0.098/\text{h}$$

thus, $K_pt_p=0.098/\text{h}\times5.56\text{ h}=0.54$, which is significantly less than 1.00. A longer test reach was needed.

Inspection of figure 22 revealed that mixing should be more than adequate at an old road crossing upstream, at a distance 4,000 ft downstream from the injection point. The traveltime to this section was computed as before and was approximately 2.78 h; thus, the time of travel of a peak in this longer test reach was 13.89–2.78=11.1 h and $K_pt_p=0.098/\text{h}\times11.1\text{ h}=1.09$. While this evaluation was based on numerous estimates, the actual field test was expected to result in a product close to 1.00, to ensure accuracy in the computation.

Injection rates

Gas injection

Gas injection should be reasonably efficient if depths at the injection site are in excess of 2 ft. Based on a downstream discharge of 42 ft^3/s and table 2, two diffusers, supplied by one gas tank, were used; gas injection for 1 h should be sufficient.

Using the curves in figure 11 (or eqs. 32, 35), $Q_me^{T_pK_p}$ was computed for the stream conditions described in figure 22 and previously estimated, applicable to the most downstream section:

$$Q_me^{T_pK_p}=(42\text{ ft}^3/\text{s})e^{(13.89\text{ h})(0.098/\text{h})}=164$$

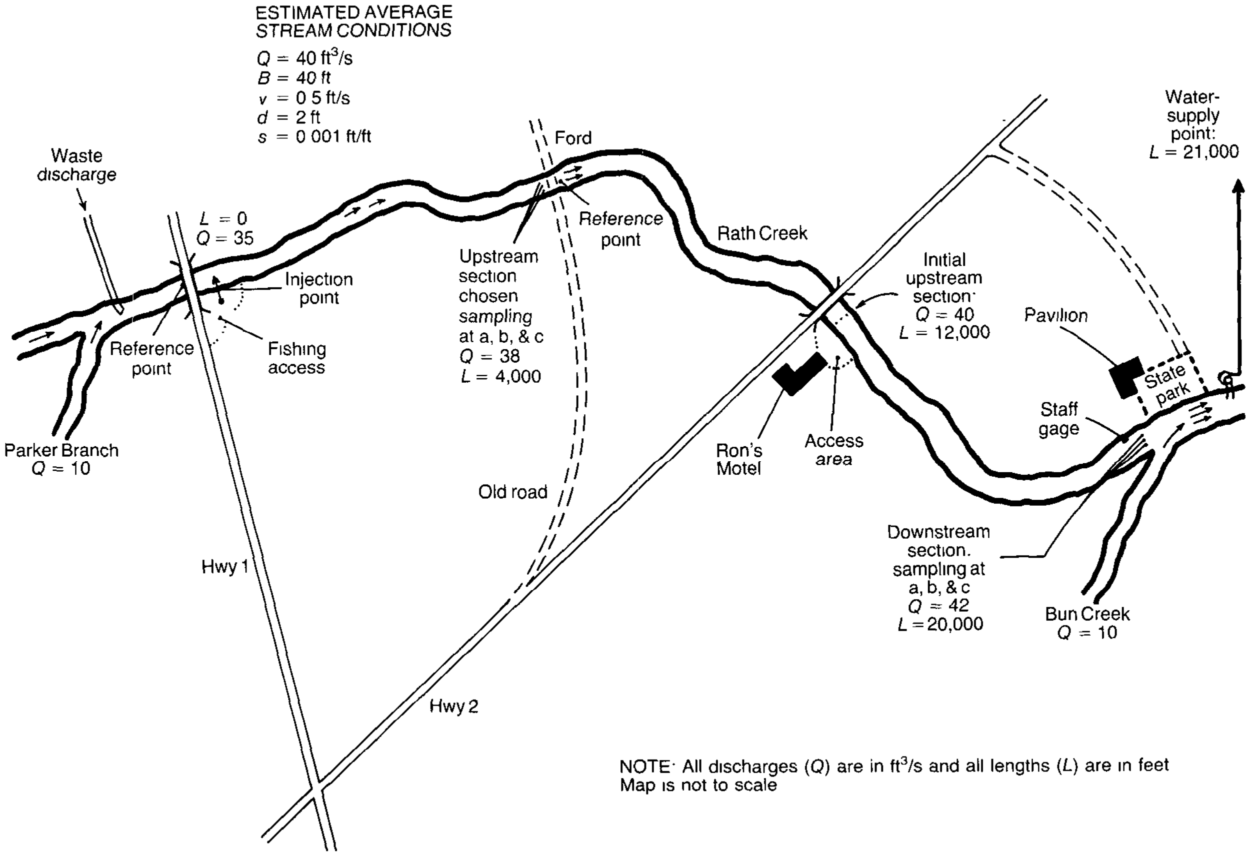
From figure 11, it was determined that a constant gas injection rate of 3.23 ft^3/h was needed for a plateau of 1 µg/L, assuming an absorption efficiency of 10 percent. Since this was to be a 1-h-long "slug injection," a plateau of 5 µg/L or more was planned for, knowing that lesser concentrations would actually result at the most downstream section. Thus, 5×3.23=16.15 ft^3/h. An injection rate of approximately 16 ft^3/h, which from figure 11 is equivalent to less than 2 lb of propane for the 1-h test, was planned for. A 20-lb propane tank was more than sufficient for the test.

Previous tests had shown that metering temperatures and pressures are typically about 32 °F and 55 psig, respectively. Thus, based on equation 34, the air injection rate to set was

$$q_a=(0.44)(16\text{ ft}^3/\text{h})\approx7\text{ ft}^3/\text{h}$$

Dye Injection

The dye injection was to be for 1 h duration at the same location as the gas injection. Examination of figure 15 indicated that for a stream discharge of 42 ft^3/s, a dye solution having a concentration, C, of 2×10^7 µg/L would yield a 10 µg/L plateau concentra-



NOTE: All discharges (Q) are in ft^3/s and all lengths (L) are in feet
Map is not to scale

Figure 22.—Sketch of stream reach used in examples of reaeration measurements.

tion if injected at a rate of 34 mL/min. While a 10 μg/L concentration might not exist at the downstream section, dye concentrations as low as 1 μg/L would be acceptable; thus a considerable margin existed for the dye plateau to be reduced by longitudinal dispersion and still be adequate for accurate fluorometric measurement. It was important, too, that dye concentrations be less than 10 μg/L at the water supply intake (Hubbard and others, 1982).

An injection rate of 34 mL/min for 1 h would require a total dye solution volume of approximately 2 L or 1/2 gal. Table 4 provided the volumes of dye and water to mix together for preparing approximately 5 gal of solution. Additional tests on other streams were planned in which stream discharges were to range from 20 to 50 ft^3/s, so a large volume of 2×10^7 μg/L solution was deemed useful. Using table 4, approximately 2.5 gal of 2×10^7 μg/L solution was prepared by adding 750 mL of 20 percent rhodamine WT dye to 8,175 mL of tap water. Note that it is satisfactory to use chlorinated tap water for preparing concentrated solutions of dye, *but not for preparing standards*. This is preferable to using cold river water, which may release air bubbles and cause problems with the dye-injection equipment.

After it was thoroughly mixed, the dye solution was retained and a 100-mL bottle of the solution was stored for preparing fluorometer standards later.

Preliminary test schedule

The elapsed times to the peaks of the dye cloud were estimated previously as approximately 2.8 h and 13.9 h for the upstream and downstream sections, respectively. Figure 12 was used to estimate other properties of the dye-response curves at the upstream and downstream sections to provide a means of scheduling tracer injection and sampling. These estimates were tabulated (table 6) and depicted visually (fig. 23) to aid planning. As can be seen, the first arrival of the dye was estimated to be at 2.2 and 11.8 h at the upstream and downstream sections, respectively, from the time injection started. The last dye was estimated to depart the upstream and downstream sections at 9.4 and 29.8 h elapsed time, respectively, allowing for the 1 h of injection.

A sketch such as figure 23 also aids in deciding when to make the tracer injections to best acquire the necessary samples during daylight hours. As can be seen, if tracer injection is started at dawn, the peak of the downstream tracer response curve may, it is

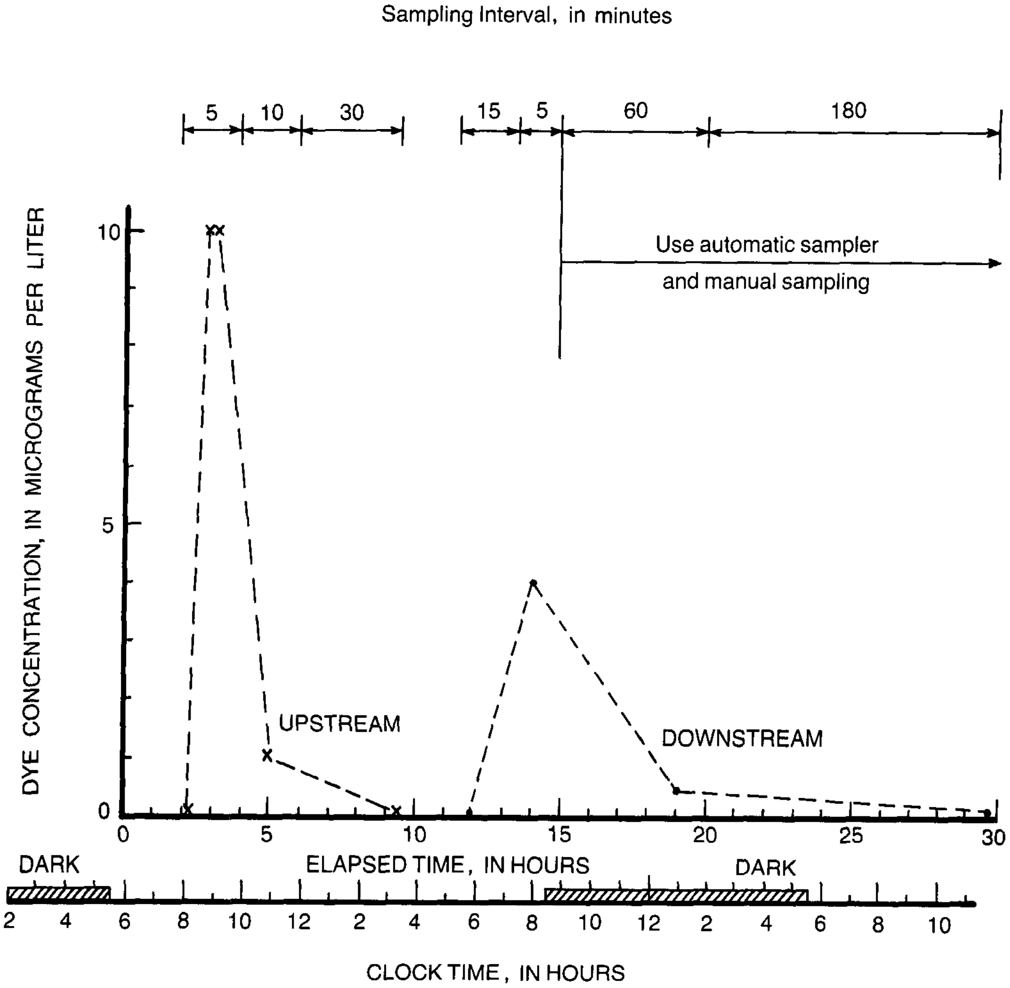


Figure 23.—Sketch of estimated response curves for guidance in scheduling slug-injection type of field reaeration test.

hoped, be sampled late in the same day. The remainder, or receding portion, of the downstream tracer response will have to be sampled through the night and morning hours of the second day. If only the SI-peak method is used, only dye samples will have to be acquired the second day. Samples from a floating automatic syringe sampler, supplemented occasionally with manually collected samples, would suffice for defining the downstream dye-response curve. Unfortunately, no automatic gas sampler is available for use under such circumstances. A sketch such as that in figure 23 also aids as shown in deciding on the sampling interval to ensure adequate definition.

It is very important to realize that the above estimates may be in considerable error and that *actual sampling must always be guided by real-time field results.*

Field test—Peak method

Preliminary preparation made the afternoon before the actual test included the following:

1. Reference points were established at the injection and upstream sections and a temporary staff gage was placed at the downstream section to measure gage heights and, hence, any change in discharge during the test.
2. A current-meter measurement was made at the upstream section, and the 50-percent discharge point was located distancewise from the left bank. Stage readings were made at all three sections at the same time the current-meter measurement was made.
3. The gas-diffuser and dye-injection lines were located and secured in the center of flow and made ready to hook up to the diffusers in the morning.

Table 6.—*Estimated properties of slug-response curves for scheduling sampling for slug-injection type of reaeration test*

[Note: Duration of dye injection, 1 hour. All times in hours. C_p, observed peak dye concentration]

Description	Upstream section	Downstream section
1. Elapsed time to peak, T_p	2.8	13.9
2. *Duration for truncation at 10 percent C_p: $T_{d_{10}}$	1.7	6.2
3. Buildup from leading edge to peak, t_b: $t_b = \frac{T_{d_{10}}}{3}$	0.6	2.1
4. Elapsed time to start sampling leading edge: $T_p - t_b$	2.2	11.8
5. Elapsed time to trailing edge at 10 percent C_p: (2) + (4) + 1 hour	4.9	19.0
6. *Duration, T_d, for truncation at 0 percent C_p	6.2	17.0
7. Elapsed time to trailing edge, T_t, at 0 percent C_p: (4) + (6) + 1 hour	9.4	29.8

*From figure 12

The actual test started early the next day and consisted of the following:

4. Gas and dye injection was begun at 6 a.m.; this time corresponded to $t=0$ for all data collection and analysis. The setup was as shown in figure 14.
5. While one hydrologist measured both the gas- and dye-injection rates[3], the other measured stage both before and after making the current-meter measurement at the upstream section of the test reach.
6. At $t=1$ h, background samples were collected at the upstream section and gas and dye injection was terminated at the injection site. The injection apparatus was removed immediately, and the dye pump and line and the gas line and diffuser were flushed with tap water.
7. Upon completion of the current-meter measurement, the tagline was left in place and the locations corresponding to the 16.7-, 50-, and 83.3-percent cumulative discharge points were flagged (see table 5).

(At this time, a minimum of two persons should be available in preparation for sampling both gas and dye.)

8. The fluorometer was set up in a vehicle and dye sampling was begun at the centerpoint at $t=2$ h. Samples were analyzed immediately and plotted on a graph similar to figure 24; stream water temperatures were measured and recorded.
9. With the first appearance of dye at the upstream section, dye sampling proceeded at approximately 5-min intervals at all three sampling points.
10. The immediate analysis and plotting of the dye concentrations continued until values approached about 7 µg/L (knowing that a brief plateau of about 10 µg/L was likely), at which time gas samples were collected at 5- to 10-min intervals through the dye peak. (Judgment should be exercised to sample for gas several times at each point just prior to the dye peak and as the peak occurs.)
11. Stage measurements were made periodically at the upstream section. (No additional discharge measurements are made, unless a 10 percent or greater change in discharge has occurred. A change in stage of a few hundredths of a foot can mean a significant change in discharge.)
12. Dye sampling continued at a less frequent rate, the fluorometer being used to immediately analyze samples and the data plotted concurrently.
13. The dye samples were retained and stored out of direct light.
14. The gas samples were checked, caps tightened, and those containing gas bubbles discarded; the remainder were stored in ice chests to maintain them at approximate stream temperature. Large

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[3] See figure 17 in background reference 3 (Kilpatrick and Cobb, 1985).

bottles of stream water were placed in the ice chests to help maintain them at the original stream temperature.

15. Except for checking injection rates, steps 5 through 14 were repeated at the downstream section, with the sampling schedule being altered as necessary on the basis of the results of the dye concentration-time plots at the upstream section.
16. Dye samples were taken manually at 1/2-h intervals until 9 p.m. (t=15 h), after which the automatic sampler anchored in the center was used. Manual dye samples were also collected at all three points at 9 and 10 p.m. and again at 6 and 10 a.m. of the second day. The center sample collected manually at the floating sampler was noted to coincide with the 12th sample taken by the floating sampler, which had been geared to sample every 45 min starting at 9 p.m.
17. All samples were returned to the office. Fluorometer dye standards were prepared from a sample of the 2×10^7-μg/L dye mixture that was injected. All dye samples, standards, background samples, and stream samples were analyzed at one temperature. These dye concentration-time data were plotted, and 12 gas samples, 4 from each point, were selected on the basis of their proximity to the dye peak. Twelve gas samples were also selected for downstream, and all 24 samples shipped to the laboratory. The laboratory was informed by phone to expect the 24 samples and as to the kind of analysis desired.

Data analysis and computations—Peak method

The dye-concentration data obtained from reanalysis of samples in the laboratory and the propane gas concentration data were all plotted as in figure 24 for both upstream and downstream sections.

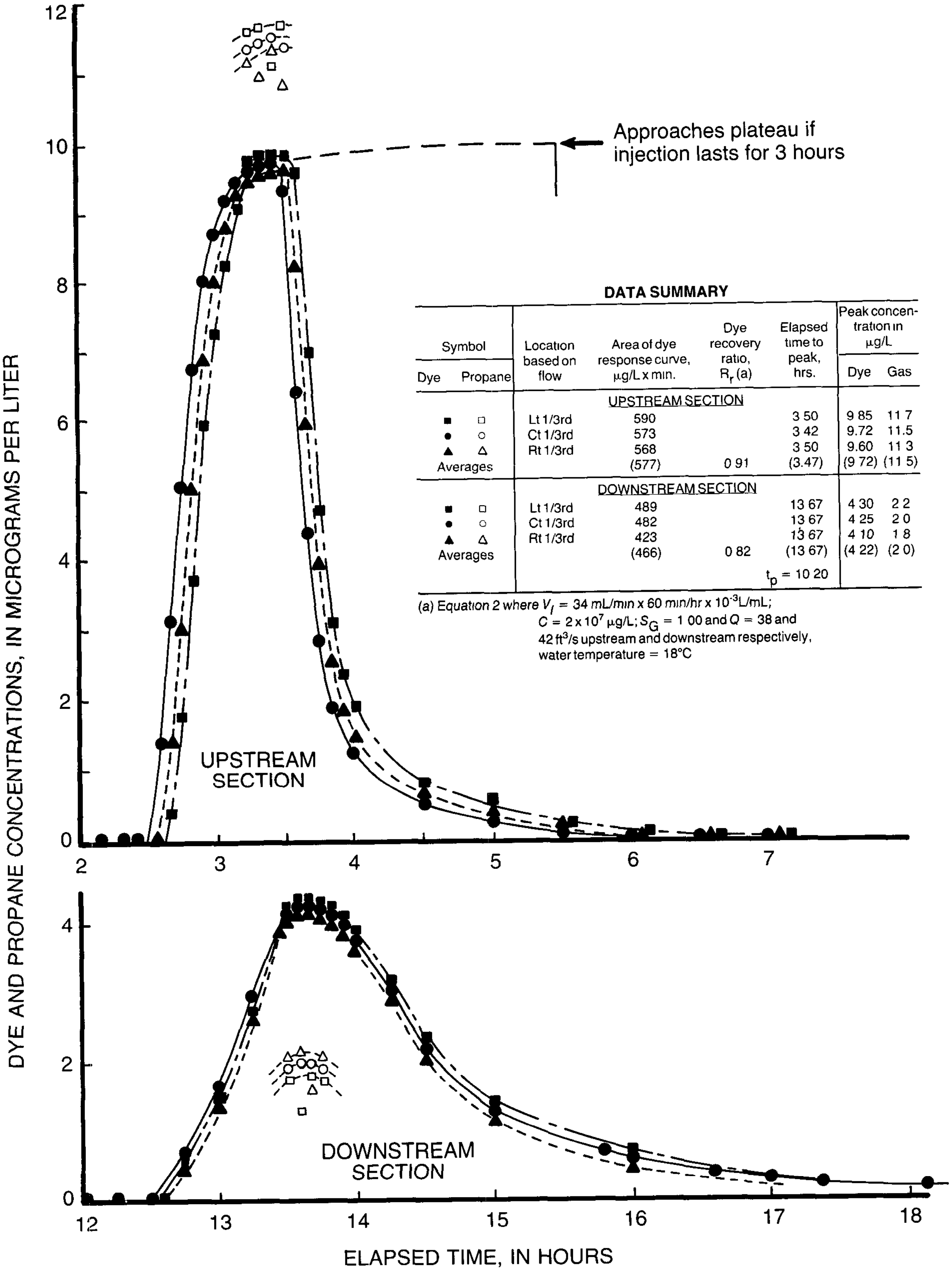
Dye data

As can be seen in figure 24, the dye concentration-time data plot smoothly and the response curves at the three sampling points are essentially the same, being slightly faster in the center than along the sides. Normally, it is advisable to plot each point response curve separately and draw a smooth-fitting curve through each set of data. It will be noted that at the upstream section the dye first arrived in the center at t=2.5 h elapsed time and was essentially gone at t=6.5 h; thus, its duration was 4.0 h. Had the dye injection been an instantaneous slug injection instead of one lasting 1 h, the duration would have been 3 h. Thus, at this specific section and point, dye and (or) gas tracer would have to be injected for a minimum of 3 h for a steady-state plateau to develop. As the data show, the dye curve is climbing toward the plateau level of 10 μg/L as intended, which was the basis for picking a dye-injection rate of 34 mL/min from figure 15. The plateau is never fully reached, as dye injection was for 1 h instead of 3 h. The point is that no attempt should be made to fit a horizontal line to the peak data (unless the injection is longer than the duration of the response); instead, it should be realized that this last 2 h, during which the dye would be climbing toward a plateau, is a mirror image in shape of the recession. This knowledge can be used to fit the curve to the peak data, as has been shown in figure 24 for the upstream section. Conversely, the data for the downstream section indicate a response duration of about 8 h (if extended to essentially background) and hence there is no indication that a plateau is forming, as continuous injection would have had to take place for at least 7 more hours.

As seen, the areas of the three dye-response curves at each of the two sections are very nearly the same, indicating good mixing. The areas of the individual time-concentration curves should be determined by planimetering or mathematically using a numerical integration scheme such as that employed in computing a current-meter discharge measurement. A program available from the Geological Survey Office of Surface Water will numerically integrate such data to obtain areas. Accurate results will be obtained if a sufficiently small time interval is used to define adequately the shape, and hence the area, of the response curve. Normally, it is best to draw a smooth curve through the data and pick the values from the plot rather than attempt to use actual data. Frequently, the placement of a smooth curve through the bulk of the data will require ignoring some data points; hence, only data conforming to the best fit curve should be used.

Furthermore, it should be kept in mind that the response curves and area computations must include all of the tracer mass; thus, computations should include dye data extended to essentially background levels.

In figure 24, the average areas of the upstream and downstream response curves are 577 and 466 μg/L×minutes, respectively. The reduction in the areas of the downstream dye-response curves is due to dilution by a 4-ft^3/s gain in stream discharge and by actual dye loss. Equation 2 allows for the increase in discharge, since the actual discharge at each section during cloud passage is that used in the computations. Recovery ratios of 0.91 and 0.82 were computed for the upstream and downstream sections, respectively.



DATA SUMMARY

Symbol		Location based on flow	Area of dye response curve, µg/L x min.	Dye recovery ratio, R_r (a)	Elapsed time to peak, hrs.	Peak concentration in µg/L	
Dye	Propane					Dye	Gas
		UPSTREAM SECTION					
■	□	Lt 1/3rd	590		3.50	9.85	11.7
●	○	Ct 1/3rd	573		3.42	9.72	11.5
▲	△	Rt 1/3rd	568		3.50	9.60	11.3
Averages			(577)	0.91	(3.47)	(9.72)	(11.5)
		DOWNSTREAM SECTION					
■	□	Lt 1/3rd	489		13.67	4.30	2.2
●	○	Ct 1/3rd	482		13.67	4.25	2.0
▲	△	Rt 1/3rd	423		13.67	4.10	1.8
Averages			(466)	0.82	(13.67)	(4.22)	(2.0)
					t_p = 10.20		

(a) Equation 2 where V_l = 34 mL/min x 60 min/hr x 10^{-3}L/mL; $C = 2 \times 10^7$ µg/L; S_G = 1.00 and Q = 38 and 42 ft³/s upstream and downstream respectively, water temperature = 18°C

Figure 24.—Tracer data acquired for slug injection, peak method of reaeration measurement.

The traveltime between the peaks can be calculated by picking the average elapsed time to the upstream and downstream sections, respectively, and subtracting to get the difference. These data and results are shown in the "Data Summary" in figure 24.

Gas data

The propane gas concentrations corresponding to times around the peak are also plotted in figure 24 for both upstream and downstream cross sections. The data scatter more than the dye data, but they more or less conform to the shape of the dye curves. The exceptionally low gas concentrations are ignored, as these are attributed to propane losses prior to laboratory analysis. The peak gas concentrations chosen from the plots are picked to coincide in time with the peak dye concentrations chosen from the response curve for each point. Note that the gas concentrations are greater than the dye concentrations at the upstream section but, because of desorption, are less than the dye concentrations at the downstream section.

Computations

The data shown in the "Data Summary" in figure 24 can be used with equation 12 to compute the desorption coefficient for propane as

$$K_p=\frac{1}{10.20\text{ h}}\ln\frac{0.91\,\frac{11.5}{9.72}}{0.82\,\frac{2.0}{4.22}}$$

$$K_p=\frac{1}{10.20\text{ h}}\ln(2.77)$$

$$K_p=0.10\text{ h}^{-1}$$

This is very close to the estimated value of 0.098/h used initially. Using equation 8, K_2 at 20 °C can be computed, adjusting K_p as measured at 18 °C:

$$K_{2_{20}}=1.39K_{p_Y}(1.0241)^{(20-Y)}$$

$$K_{2_{20}}=(1.39)(0.10/\text{h})(1.0241)^2=0.15\text{ h}^{-1}$$

Field test—Area method

The reaeration coefficient can also be measured using the area method, with essentially the same procedures as just described for the peak method except that gas and dye samples must be collected to define the entire tracer-response curves for both. Since gas-tracer concentrations cannot be readily measured in the field, the dye-response curve as measured onsite must be used to guide gas sampling. Extra gas samples should be collected and only selected ones forwarded to the laboratory for analysis. Because the gas analyses are relatively expensive, maximum use should be made of the dye-response curves to guide gas sampling and to limit the number submitted for laboratory analysis.

For example, in the previous example, mixing of the dye tracer is good at the upstream section and excellent at the downstream section, as can be seen from the dye-concentration data. Thus, even though gas samples would be collected throughout the duration of the response curves at all three points in each section, a subsequent plot of the dye-concentration data would indicate that only definition of the gas-response curve just in the center of the flow at the downstream section would be necessary.

Figure 25 shows a plot of concentrations of selected gas samples collected in the center of the flow at the upstream section along with the companion dye-response curve. The dye-response curve as measured in the center at this section and presented previously in figure 24 is shown again in figure 25 on an expanded time scale. It can be seen again that some of the gas-concentration data plot low and should be ignored. Note, too, that the gas peak seems to drop quicker than the dye peak. This reflects the desorption of the propane gas, which tends to erode the trailing edge of the gas-response curve. Similar plots of both gas- and dye-response curves for the left and right third of the section were made but for the sake of clarity are not shown.

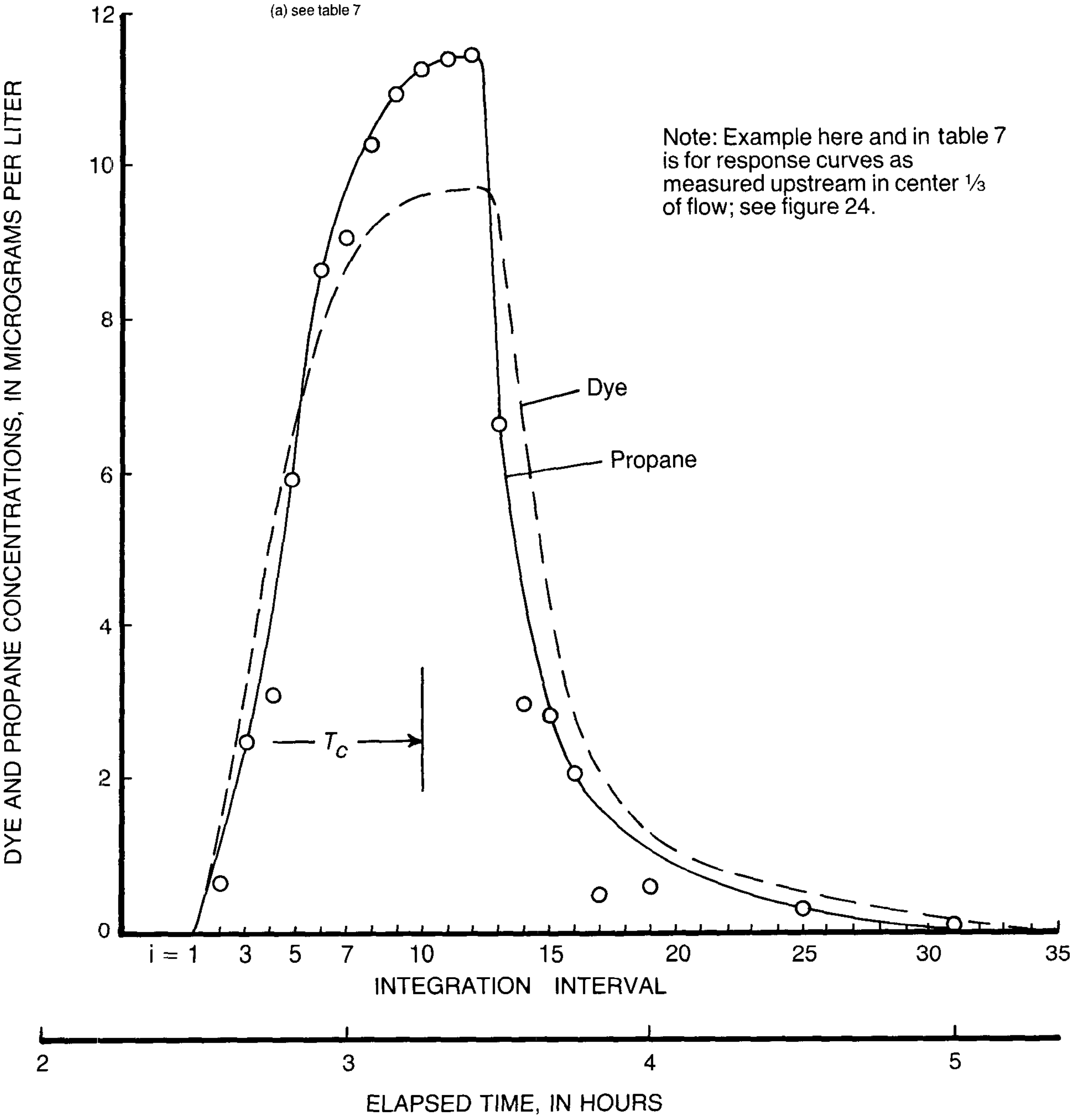
At the downstream section, only the gas samples collected in the center were analyzed and plotted.

Unlike the peak method, the area method uses in the computation of the desorption coefficient the traveltime of the centroid of the dye cloud. This requires that the centroid of each *dye*-response curve be computed relative to $t=0$ (see eq. 15). The numerical integration to accomplish this is shown in table 7 using data *picked* from the dye-response curve shown in figure 25 for just the center curve. The computation should be performed independently on each dye-response curve and averaged for upstream and downstream sections. Note that Δt_i, the numerical integration interval, has been chosen as 5 min, which results in 35 increments and defines the response curve with minimum error. The computer program available from the Geological Survey Office of Surface Water will accomplish this same calculation as well as the entire computation of the desorption coefficient. The elapsed centroid times for the upstream and downstream dye-response curves are shown in the "Data Summary and

DATA SUMMARY AND COMPUTATIONS

Location based on flow	Area of Gas Response Curve, µg/L x min		Elapsed time to centroid of dye response curve, T_c, hours		Travel time t_c, hours	Propane desorption coefficient, K_p at 18°C in h^{-1}
	Upstream	Downstream	Upstream	Downstream		
Lt 1/3rd	588	---	3 49	14 41	10 92	0 097
Ct 1/3rd	561(a)	185	3 23(a)	14 35	11 12	0 091
Rt 1/3rd	562	---	3 40	14 24	10 84	0 093
Averages	(571)	(185)	(3 37)	(14 33)	(10 97)	(0 094)

(a) see table 7



Note: Example here and in table 7 is for response curves as measured upstream in center ⅓ of flow; see figure 24.

Figure 25.—Preparation of gas-response curve using dye-response curve as a guide and selection of numerical integration interval for computation of curve area and centroid.

Table 7.—*Numerical integration of dye-response curve to obtain elapsed time to centroid*

[Data are for upstream section at center of channel. µg/L, micrograms per liter; h, hours; min, minutes]

i	Elapsed time, t, from t = 0 in hours (1)	Dye concentration C_d, in µg/L (2)	Area increment, $\Delta t \times C_d$ (a) (µg/L x h) (3)	Incremental moment $t \times \Delta t \times C_d$ (4)
1	2.50	0	0	0
2	2.58	1.40	0.117	0.302
3	2.67	3.00	.250	.668
4	2.75	5.10	.425	1.169
5	2.83	6.40	.533	1.508
6	2.92	8.00	.667	1.948
7	3.00	8.70	.725	2.175
8	3.08	9.23	.769	2.368
9	3.17	9.52	.793	2.514
10	3.25	9.62	.802	2.606
11	3.33	9.65	.804	2.677
12	3.42	9.72	.810	2.770
13	3.50	9.20	.767	2.059
14	3.58	6.40	.533	1.908
15	3.67	5.20	.433	1.589
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
28	4.75	.32	.027	.128
29	4.83	.28	.023	.111
30	4.92	.23	.019	.093
31	5.00	.20	.017	.085
32	5.08	.15	.012	.061
33	5.17	.10	.008	.041
34	5.25	.05	.004	.021
35	5.33	0	0	0

(a) Δt = 5 min = 0.0833 h

Σ = 9.55 µg/L x h

Σ = 30.889

A_c = 573 µg/L x min

$$T_c = \frac{30.889}{9.55} = 3.23 \text{ h}$$

Computations" table in figure 25 along with other pertinent data for computing the desorption coefficient.

The desorption coefficient can be computed, using equation 14, as

$$K_p = \frac{1}{10.97} \ln \frac{(571 \times 38)}{(185 \times 42)}$$

$$K_p = \frac{1}{10.97} \ln (2.71) = 0.094 \text{ h}^{-1}$$

and K_2 at 20 °C can be computed, using equation 8, as

$$K_{2_{20}} = (1.39)(0.091/\text{h})(1.0241)^2 = 0.14 \text{ h}^{-1}$$

Note that in the "Data Summary and Computations" table (fig. 25) the desorption coefficient can also be computed for each third of the flow with very similar results. This assumes that the area of the gas-response curve is the same at all three points in the downstream section. Each third of the flow is treated as a separate stream tube.

Performance of Constant-Rate-Injection Reaeration Measurement

General

The planning for a CRI type of reaeration measurement is essentially the same as discussed previously for the SI method. Differences relate to the fact that the dye injection is an instantaneous slug injection and the gas injection is constant rate and continuous for a duration somewhat longer than the duration of passage of the most downstream dye-response curve.

Table 8.—*Estimated properties of slug-response curves for scheduling sampling for constant-rate-injection type of reaeration test*

[Note: Dye injection is instantaneous and gas injection is constant for 20 hours. All times in hours. C_p, observed peak dye concentration]

Description	Upstream section	Downstream section
DYE RESPONSE CURVE		
1. Elapsed time to peak, T_p	2.8	13.9
2. *Duration for truncation at 10 percent C_p: $T_{d_{10}}$	1.7	6.2
3. Buildup from leading edge to peak, t_b: $t_b = \frac{T_{d_{10}}}{3}$	0.6	2.1
4. Elapsed time to start sampling leading edge: $T_p - t_b$	2.2	11.8
5. Elapsed time to trailing edge at 10 percent C_p: (2) + (4)	3.9	18.0
6. *Duration, T_d, for truncation at 0 percent C_p	6.2	17.0
7. Elapsed time to trailing edge, T_t, at 0 percent C_p: (4) + (6)	8.4	28.8
GAS PLATEAU		
8. Elapsed time to termination of steady-state plateau (4) + 20 hours continuous gas injection	22.2	31.8
9. Elapsed time between peaks, t_p from line 1 above		11.1
10. Recommended elapsed time to sample downstream		30
11. Recommended elapsed time to sample upstream (hourly immediately following stable plateau at 8.4 hours)	9.0	
12. Samples likely to be used in concert with those collected downstream at 30 hours (30 hours - 11.8 hours, rounded to nearest hour; collect samples every one-half hour to bracket 18 hours)	18.0	

*From figure 12

This may influence overall scheduling of the test, in particular the timing of gas sampling.

Narrow streams—One-dimensional dispersion

Rath Creek, previously used in the example of an SI measurement, is also used to illustrate the CRI method applied to a narrow stream for the case when one-dimensional dispersion exists. The same channel and flow conditions were assumed and the same test reach was used (see fig. 22)

Dye injection

To use figure 16 in estimating the quantity of 20-percent rhodamine WT dye to slug-inject, QL/v for the downstream section was computed as

$$\frac{QL}{v}=\frac{(42\ \text{ft}^3/\text{s})(20{,}000\ \text{ft})}{(0.4\ \text{ft/s})}=2.1\times10^6$$

Entering figure 16 indicated that 80 mL of 20-percent rhodamine WT dye should produce a peak of about 1 μg/L. A peak of about 5 μg/L would be acceptable and well below the 10 μg/L maximum allowed. Thus, $V_s=5\times80$, or 400, mL of dye should be injected.

Gas injection and tank size

Using figure 11, $Q_m e^{T_p K_p}$ was computed for the most downstream section as before, and it was determined that a gas injection rate of 3.23 ft^3/h was needed for a plateau of 1 μg/L (assuming an absorption efficiency of 10 percent). Because a plateau concentration of 5 μg/L was desired at the most downstream section, a rate of 5×3.23 ft^3/h, or 16.15 ft^3/h, was necessary. (A rate of about 16 ft^3/h is equivalent to about 1.8 lb of propane per hour (scale on right side of fig. 11).)

The traveltime to the downstream dye peak was originally estimated to be 13.9 h. From figure 12 it was determined that a constant-rate gas injection of approximately 17 h would be necessary to produce a steady-state gas plateau; a 20-h continuous injection was assumed, to allow time for sampling. Thus, a tank containing in excess of 36 lb of propane (1.8 lb/h×20 h) was required. A 50-lb tank would be sufficient and not involve using the last 20 percent of the tank.

Preliminary test schedule

In planning the test, the arrival and duration of passage of the dye response was estimated as was done for the SI test. Table 8 is similar to table 6, but for a slug injection of dye and a 20-h continuous injection of propane gas. Figure 26 was prepared from these data to aid in selecting injection and sampling schedules to best use daylight hours and provide for comprehensive data collection. It is important to note that a gas plateau will exist at the upstream section well in advance of its appearance downstream. The elapsed time to the trailing edge of the dye-response curve corresponds to the time at which a steady-state plateau is reached at upstream and downstream sections, respectively, assuming the gas and dye injection start at the same time and location.

The dye-response curves at the upstream and downstream sections were scheduled for sampling exactly as was done for the SI test. Since dye sampling is far more extensive than gas sampling, scheduling should favor this effort. A 6 a.m. slug injection of dye still seemed the best, as it allowed most of the sampling of the downstream dye-response curve to take place during daylight hours; automatic sampling was thought to be sufficient from about 9 p.m. on.

Gas injection was scheduled to start at 6 a.m. ($t=0$) and to terminate 20 h later, at 2 a.m. of the second day. Note that this is 10 h *before* sampling the plateau downstream! In practice, the gas could be turned off any time the following morning; this is one reason for having a tank with extra capacity. In the example, if streamflow is steady, gas samples could then be collected upstream at any time from $t=8.4$ h to 22.2 h. It is recommended that gas samples be taken upstream as soon as a stable plateau is known to exist; if the number proves to be excessive, the extra samples can be discarded. It is desirable to sample the same fluid mass if possible, especially if the flow is changing slightly. The estimated time of travel of the peaks was 11.8 h. Therefore, it appeared likely that the upstream gas samples that would be used would correspond to about $t=18$ h, 12 h before gas sampling downstream at $t=30$ h. A logical schedule for the upstream site seemed to be to sample for gas every hour starting at $t=9$ h and every ½ h at about $t=16$ h.

The time at which a steady-state plateau is reached downstream would be apparent from observation when the recession of the dye-response curve reached background levels. This could be well before the estimated 28.8 h. Sampling upstream at $t=9$ h and later would allow *picking* the samples to *match the time of travel* of the dye-response curves, data not available until completion of the test. *Preliminary planning is highly advisable, but the dye-response curves should be measured in the field and should dictate actual gas sampling.*

Field test—Constant-rate-injection method—One-dimensional dispersion

Preliminary preparation made the afternoon before the actual test included

1. Reference points were established at the injection and upstream sections and a temporary staff gage was placed at the downstream section to measure gage heights and, hence, detect any change in discharge during the test.
2. A current-meter measurement was made at the upstream section, and the 50-percent discharge point was located distancewise from the left bank. Stage readings were made at all three sections at the same time the current-meter measurement was made.
3. The gas diffuser line was located and secured in the center of flow and made ready to hook up to the diffuser plates the next morning.

The actual test started early the morning of the next day and consisted of the following:

4. Continuous propane gas injection was started at 6 a.m. concurrent with an *instantaneous* slug injection of 400 mL of 20-percent rhodamine WT dye; 6 a.m. corresponds to $t=0$ for all data collection and analysis. Based on equation 34, an air injection rate of 7 ft^3/h was set on the rotameter. A platform scale was placed under the gas tank, and weight change with time noted periodically (see fig. 27).[4] The 400 mL of 20-percent rhodamine WT was premixed with 9,600 mL of river water prior to slug injection; a 100-mL sample was retained for future preparation of standards and fluorometer calibration.[5]
5. One hydrologist stayed at the injection site until the injection rate was observed to be constant. As can be seen by the data defined by a dashed

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[4] See figure 17 in background reference 3 (Kilpatrick and Cobb, 1985).

[5] See section "Alternative Method of Analysis and Computation" in background reference 3 (Kilpatrick and Cobb, 1985).

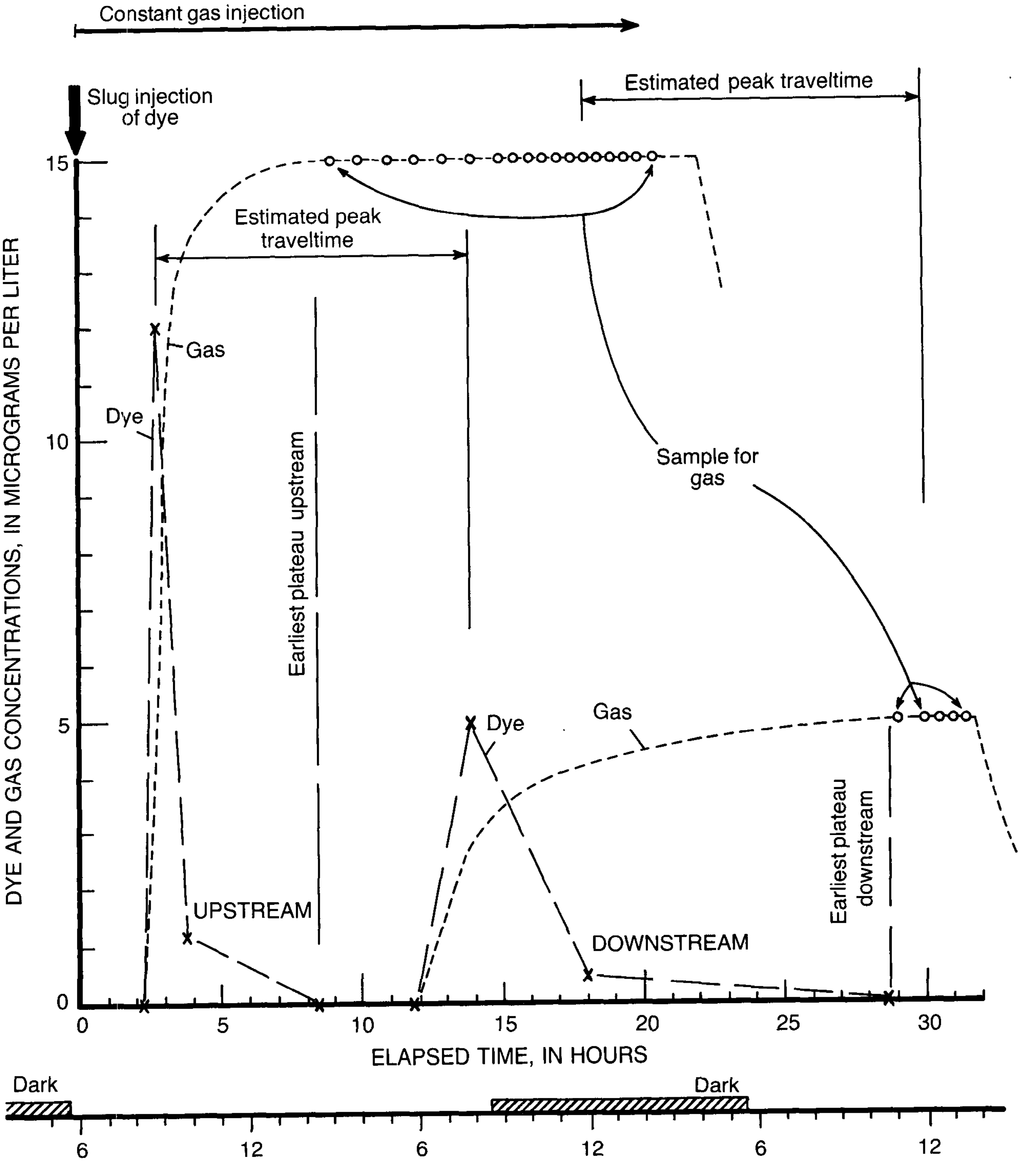

Figure 26.—Sketch of estimated response curves for guidance in scheduling constant-rate-injection type of field reaeration test.

line in figure 27, a constant rate was not attained until 7 a.m. The hydrologist also collected several background samples for use in fluorometric analysis.

6. Another hydrologist proceeded to the upstream section constituting the test reach and made a current-meter discharge measurement and observed stage both before and after.

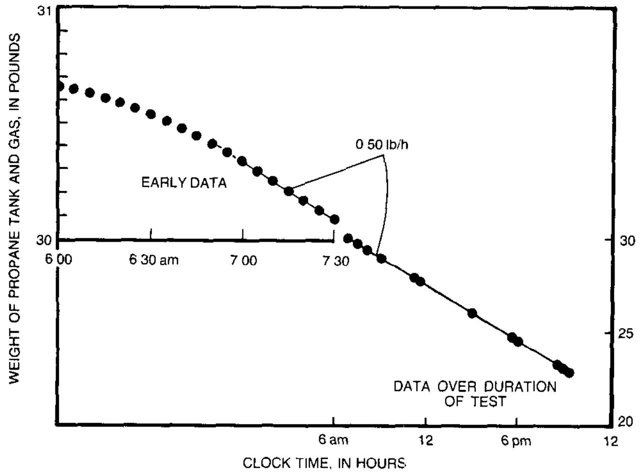

Figure 27.—Determination of rate and constancy of propane gas injection by measurement of weight change with time.

7. Upon completion of the current-meter measurement, the tagline was left in place and the locations corresponding to the 16.7-, 50-, and 83.3-percent cumulative discharge points flagged (see table 5).
8. The fluorometer was set up in a vehicle and dye sampling at 5-min intervals was begun at the centerpoint at $t=2$ h. Samples were analyzed immediately and plotted on a graph such as that shown in figure 28. (Note that fig. 28 is a final data plot based on careful laboratory analyses of dye and gas samples; for brevity it is discussed here as if it were the *field analysis* of the dye data.)
9. Stream water temperature was observed to be 18 °C.
10. With the first appearance of dye at the upstream section, dye sampling proceeded at approximately 5-min intervals *at all three sampling points*; for clarity, only the center dye-response curve is presented in figure 28.
11. As the dye peak passed, sampling was less frequent.
12. The dye samples were retained and stored out of direct light.
13. The data plot of dye concentration (fig. 28 or dial readings) versus time was examined and used to predict new times of travel to the downstream section.[6] For example, while the measured leading edge and estimated time of arrival were identical, the trailing edge was at about $t=5$ h instead of 8.4 h as had been estimated; sampling schedules at the downstream section were modified as necessary.
14. Examination of figure 28 indicated the trailing edge of the upstream dye-response curve to be at $t=5$ h. Since the gas injection was an hour late in stabilizing (see fig. 27), the gas plateau at the upstream section would be fully stabilized at $t=6$ h. Starting at $t=6$ h (12 noon), sets of three gas samples were taken at ½-h intervals upstream.
15. Stage measurements were made periodically at the upstream section, and no additional discharge measurements were made unless a 10-percent or greater change in discharge had occurred.
16. All equipment was moved to the downstream section and steps 6 through 12 were repeated as newly scheduled.
17. As the downstream dye cloud started to recede, the centroid traveltimes upstream and downstream were estimated as 3 and 14 h, respectively; thus, gas samples taken upstream and downstream 11 h apart represented the same fluid element undergoing gas desorption.
18. As the downstream dye-response curve continued to recede (fig. 28), it was estimated that its duration would be about 6 h (18.5−12.5 h). Furthermore, if the gas injection had been stable initially, a stable gas plateau would be reached at $t=18.5$ h. Since it didn't stabilize until an hour after the start (see fig. 27), a stable gas plateau would be reached at $t=19.5$ h. Since this corresponds to 1:30 a.m. on the second day, a decision had to be made as to whether to complete the test that night or the next morning.

If the test was to be completed that night, gas sampling would be at $t=19.5$ h downstream to match samples taken upstream earlier at $t=8.5$ h (chosen so that $t_c=11$ h).

If the test was to be completed the next morning, the gas samples could be taken at $t=26$ h (8 a.m.) downstream, and the gas samples collected upstream at $t=15$ h (9 p.m.) could be used as representative matching samples ($t_c=11$ h).

Note that continuous gas injection for 7 h (19.5–12.5 h) would be sufficient to *just* establish a gas plateau at $t=19.5$ h. Thus, if the test was to be completed that night, the gas injection could be terminated at 1 p.m. or if the next morning, at 7:30 p.m. on the first day (26.0 h−19.5 h+7 h=13.5 h, or 7:30 p.m.). In either case, gas injection could be safely terminated well in advance of sampling.

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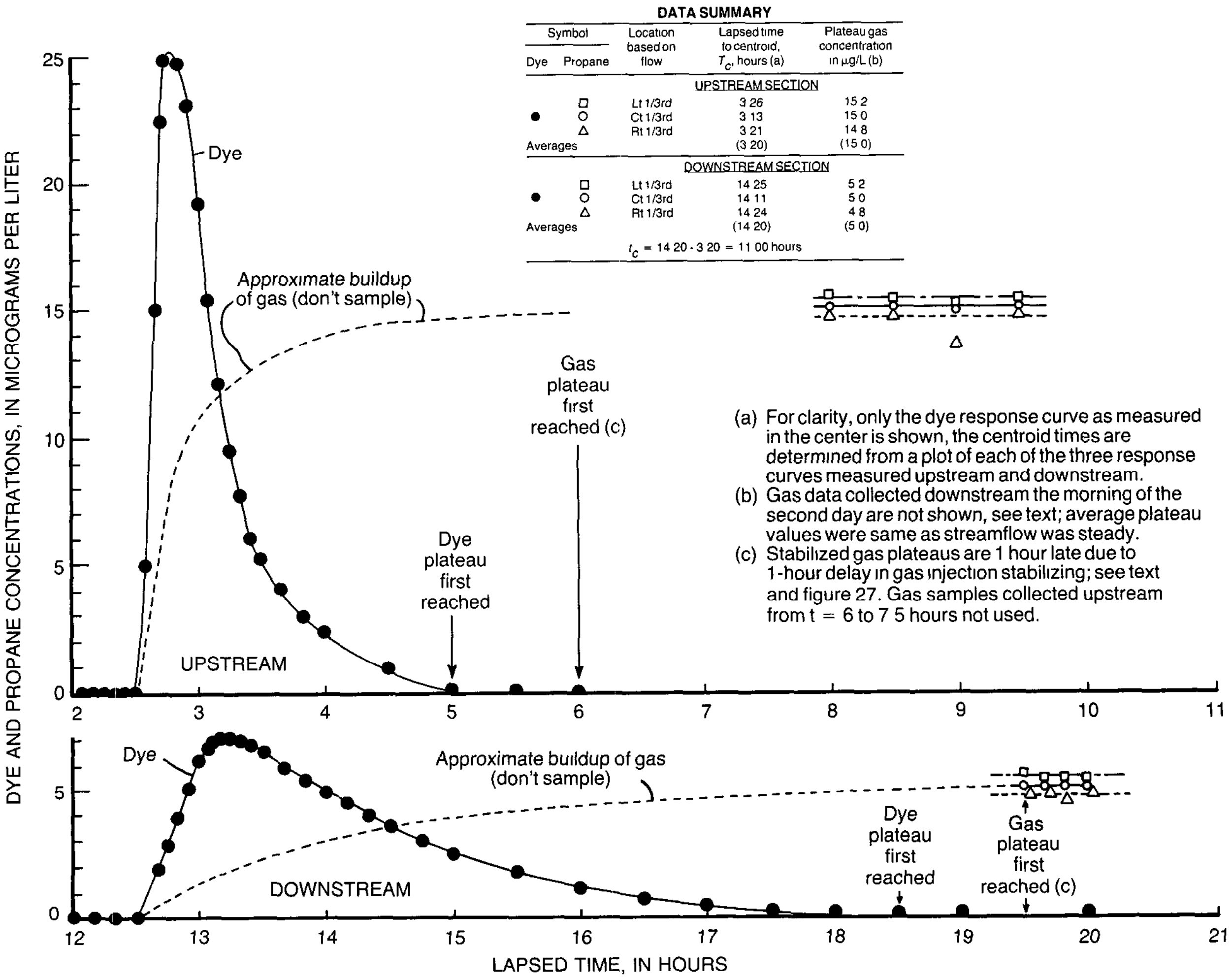
[6] See figure 15 in background reference 2 (Hubbard and others, 1982).



DATA SUMMARY

Symbol		Location based on flow	Lapsed time to centroid, T_c, hours (a)	Plateau gas concentration in µg/L (b)
Dye	Propane			
		UPSTREAM SECTION		
	□	Lt 1/3rd	3.26	15.2
●	○	Ct 1/3rd	3.13	15.0
	△	Rt 1/3rd	3.21	14.8
Averages			(3.20)	(15.0)
		DOWNSTREAM SECTION		
	□	Lt 1/3rd	14.25	5.2
●	○	Ct 1/3rd	14.11	5.0
	△	Rt 1/3rd	14.24	4.8
Averages			(14.20)	(5.0)

$t_c = 14.20 - 3.20 = 11.00$ hours

(a) For clarity, only the dye response curve as measured in the center is shown, the centroid times are determined from a plot of each of the three response curves measured upstream and downstream.
(b) Gas data collected downstream the morning of the second day are not shown, see text; average plateau values were same as streamflow was steady.
(c) Stabilized gas plateaus are 1 hour late due to 1-hour delay in gas injection stabilizing; see text and figure 27. Gas samples collected upstream from t = 6 to 7.5 hours not used.

Figure 28.—Tracer data acquired for constant-rate-injection method of reaeration measurement.

This example illustrates the importance of a timely field plot of the dye data for both upstream and downstream sections and knowledge of the important properties of the dye-slug response in scheduling gas injection and sampling.

19. Since it was necessary to sample the downstream dye-response curve through the night with the automatic sampler and recover the samples the next morning, it was decided to collect gas samples downstream every 10 min for ½ h starting at 1:30 a.m. and 8 p.m., to be matched by gas samples upstream at about 2:30 p.m. and 9 p.m. Thus, duplicate sets of data would be available for analysis.
20. Gas injection was terminated at 10 p.m. on the first day, and several rate checks were made at this time (see fig. 27). This allowed time for sampling; furthermore, it was this hour ($t = 16$ h) before sufficient dye data were available at the downstream section to permit the above decisions to be made.
21. All dye samples were stored out of direct light, gas samples were checked, caps tightened when necessary, and any containing gas bubbles discarded; the remainder were stored in ice chests with little or no chilling as required to maintain them at approximately stream temperature.
22. All samples were returned to the office. Fluorometer dye standards were prepared from a sample of the dye mixture that was injected. Note that the solution injected has a concentration of 0.952×10^7 µg/L, in contrast to 20×10^7 µg/L for straight rhodamine WT.[7] This shortens the time for preparation of fluorometric standards as well as improves accuracy because the first serial dilution has in effect been made in the field and *is the same solution that was injected.*
23. All dye samples, standards, background samples, and stream samples were analyzed at one

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[7] Using equation 7 in background reference 3 (Kilpatrick and Cobb, 1985).

temperature. These dye concentration-time data were plotted as in figure 28. Since two sets of gas data were collected, 48 gas samples were chosen for shipment to the laboratory. Each data set consisted of 24 samples, 12 upstream and 12 downstream (4 from each of three sampling points). The upstream and downstream samples were taken 11 h apart so as to measure the gas desorption in essentially the same element of flow.

Data analysis and computations—Constant-rate-injection method—One-dimensional dispersion

The dye-concentration data obtained from reanalysis of samples in the laboratory and the propane gas-concentration data were all plotted in figure 28 for both upstream and downstream sections. Just the dye data collected in the center are shown for clarity.

As can be seen in figure 28, the dye concentration-time data plot smoothly, with the peak falling from 25.0 µg/L upstream to 7 µg/L downstream. A numerical integration of the six dye-response curves such as is illustrated in columns 2 through 5 in table 9 for just the center response curves yields the centroid travel-times shown in the "Data Summary" in figure 28. The average time of travel from upstream to downstream sections, t_c, was 11.00 h.

Plateau gas concentrations were found to have dropped on the average from 15.0 µg/L upstream to 5.0 µg/L downstream. These data, along with stream discharges, can be used with equation 17 to compute an estimated propane gas desorption coefficient:

$$K_p = \frac{1}{t_c}\ln\frac{(\bar{c}_gQ)_u}{(\bar{c}_gQ)_d} = \frac{1}{11.00\ \text{h}}\ln\frac{(15.0\mu\text{g/L}\times 38\ \text{ft}^3/\text{s})}{(5.00\mu\text{g/L}\times 42\ \text{ft}^3/\text{s})}$$
$$=0.091\ \text{h}^{-1}$$

This value can be used as a basis for selecting trial values of K_p in equation 18. Equation 18 for propane can be expressed in the form

$$\left[\frac{(\bar{c}_gQ)_u}{(\bar{c}_gQ)_d}\right]_z = \left[\frac{\left[\sum_{i=1}^{N}\frac{C_d\Delta t}{Ae^{K_pt_i}}\right]_u}{\left[\sum_{i=1}^{N}\frac{C_d\Delta t}{Ae^{K_pt_i}}\right]_d}\right]_z \qquad (39)$$

where the individual dye-response curves observed on a given streamline, z, upstream and downstream are *numerically integrated.* The computational process is shown in abbreviated form in columns 6 through 12 in table 9. Note that the normalizing of the dye-concentration data involves dividing each piece of data by the total area of the specific response curve. The data and computations in table 9 are just for the center streamline ($z=50$ percent) using dye data picked from the response curves shown in figure 28; a similar procedure should be used for data on other streamlines such as at $z=16.7$ and 83.3 percent if three points are chosen for observation. As seen in table 9, column 1, $N=32$ and 34 were chosen to adequately define each response curve and resulted in Δt, the integration interval being 5 and 10 min upstream and downstream, respectively. The ratio $\Delta t/A_z$ becomes a constant for a given response curve; C_d values are the dye concentrations at i points, t_i being the elapsed time to each point. Since the approximate value of K_p was 0.091 h^{-1}, values of 0.090 and 0.100 h^{-1} were chosen for the first two trial computations (columns 7 through 10, table 9).

As shown at the bottom of table 9, these two trial calculations produced ratios of 2.67 and 2.98. Interpolating for a *measured* value of 2.71 indicated $K_p=$ 0.091 h^{-1}, or the same as estimated by equation 17. A third trial computation using $K_p=0.091\ \text{h}^{-1}$ verified this calculation.

Similar computations were performed with the dye and gas data collected on streamlines $z=16.7$ and $z=83.3$; the results are summarized in table 10.

Information on a computer program to perform all the computation shown in table 9 is available from the Geological Survey Office of Surface Water in Reston, Va.

Wide streams—Two-dimensional dispersion

Example

A reach on the Pat River is used in an example of a CRI type of reaeration test on a wide river where two-dimensional dispersion will likely exist; the reach is shown in figure 29. A reach approximately 4 mi long and 200 ft wide, averaging 1.5 ft deep and having an average slope of 0.003 ft/ft, was selected for the test. The reach was chosen for its uniformity, lack of inflow, lack of islands, and accessibility. A preliminary estimate of stream discharge was 150 ft³/s; thus, the average velocity is about 0.5 ft/s. The test reach was composed of four sections, an upstream section at $L=0$, where the dye and gas injections were made, and three sampling sections downstream.

Preliminary tests, planning, and preparations

It was planned to perform the field test over a 3-day period. The first day would consist of site preparation and discharge and stage measurements. The second

Table 9.—*Computation of areas, centroids, and desorption coefficient along center streamline* ($z=50$ *percent*), *constant-rate-injection method, narrow stream*

[See text and list of "Symbols and Units" for explanation of variables. µg/L, micrograms per liter; hrs, hours; h, hours; ft³/s, cubic feet per second]

						Trial computation of K_p					
						Trial 1: K_p=0.090/h		Trial 2: K_p=0.100/h		Trial 3: K_p=0.091/h	
i	Elapsed time, t_i hours	Dye concentration C_d, (a) µg/L	Area increment $C_d\ \Delta t$ (b) µg/L x hrs	Incremental moment $t_i \times \Delta t \times C_d$ µg/L x (hrs)^2	$\frac{\Delta t}{A_z}$	$e^{K_p t_i}$	$\frac{\Delta t\ C_d}{A_z e^{K_p t_i}}$	$e^{K_p t_i}$	$\frac{\Delta t\ C_d}{A_z e^{K_p t_i}}$	$e^{K_p t_i}$	$\frac{\Delta t\ C_d}{A_z e^{K_p t_i}}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	UPSTREAM, CENTERLINE DYE RESPONSE CURVE										
1	2.58	5.00	0.42	1.08	0.005	1.261	0.020	1.294	0.019	1.265	0.020
2	2.67	15.00	1.25	3.34	.005	1.272	.058	1.306	.057	1.275	.058
3	2.75	25.00	2.08	5.72	.005	1.281	.096	1.317	.094	1.284	.096
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
30	5.00	0.25	0.02	0.10	.005	1.568	.001	1.649	.001	1.576	.001
31	5.08	.15	.01	.05	.005	1.580	0	1.662	0	1.588	0
32	5.17	.05	0	0	.005	1.592	0	1.677	0	1.601	0
		$A_z = \Sigma = 16.81$		$\Sigma = 52.68$		$\Sigma = 0.755$		$\Sigma = 0.732$		$\Sigma = 0.753$	
		$T_{c_z} = 52.68/16.81 = 3.13$ hrs									
	DOWNSTREAM, CENTERLINE DYE RESPONSE CURVE										
1	12.68	1.85	0.31	3.93	0.012	3.13	0.007	3.55	0.006	3.17	0.007
2	12.83	3.85	.64	8.21	.012	3.17	.015	3.61	.013	3.21	.014
3	13.00	6.30	1.05	13.65	.012	3.22	.023	3.67	.021	3.26	.023
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
32	17.83	0.15	0.03	0.53	.012	4.98	0	5.95	0	5.07	0
33	18.00	.10	.02	.36	.012	5.05	0	6.05	0	5.14	0
34	18.17	.05	.01	.18	.012	5.13	0	6.15	0	5.23	0
		$A_z = \Sigma = 13.92$		$\Sigma = 196.40$		$\Sigma = 0.283$		$\Sigma = 0.246$		$\Sigma = 0.278$	
		$T_{c_z} = 196.40/13.92 = 14.11$ hrs									
		Calculated $\left[\left(\sum_{i=1}^{N} \frac{\Delta t\ C_d}{A_z e^{K_p t_i}}\right)_u \Big/ \left(\sum_{i=1}^{N} \frac{\Delta t\ C_d}{A_z e^{K_p t_i}}\right)_d\right]_{z=50}$			=	$\frac{0.755}{0.283} = 2.67$		$\frac{0.732}{0.246} = 2.98$		$\frac{0.753}{0.278} = 2.71$	

Measured $\left[[\bar{c}_g Q]_u / [\bar{c}_g Q]_d\right]_{z=50}$ = (38 ft³/s x 15.00 µg/L)/(42 ft³/s x 5.00 µg/L) = 2.71

(a) Dye concentrations are picked from response curves shown in figure 28.
(b) Δt = 5 minutes (0.083 hours) for upstream section and 10 minutes (0.167 hours) for downstream section.

day would consist of a preliminary dye slug test, with sampling at two or more sections downstream to determine the best test reach. These data would also be used in the integrations on the right side of equation 20 if stream discharge did not vary significantly during the 3 days. A constant-rate gas injection would be performed the third day with or without a concurrent dye slug injection, depending on whether or not there was a significant change in discharge by the third day and whether the initial sections chosen proved satisfactory based on analysis and interpretation of the preliminary dye slug test data. These data would provide true time-of-travel information and might require revision of the sections to obtain sufficient residence time or better mixing.

Selection of test reach

Equation 23 indicated that the optimum mixing length for an injection in the center would be

$$L_o = 0.1\frac{vB^2}{E_z} = 0.1\frac{(0.5)(200)^2}{0.11} \approx 18{,}200 \text{ ft}$$

Table 10.—*Summary of desorption and reaeration coefficient computations for constant-rate-injection method—One-dimensional dispersion*

[See text and list of "Symbols and Units" for explanation of variables. h^{-1}, per hour]

Streamline z, percent of flow	Measured $\frac{[\bar{c}_g Q]_u}{[\bar{c}_g Q]_d}$ (a)	Final K_p as calculated by trial h^{-1}	Reach average $K_{2_{20}}$ h^{-1} (c)
16.7	$\frac{38 \times 15.20}{42 \times 5.20} = 2.64$	0.089	
50.0	$\frac{38 \times 15.00}{42 \times 5.00} = 2.71$	0.091 (b)	0.133
83.3	$\frac{38 \times 14.80}{42 \times 4.80} = 2.79$	0.094	

(a) See figure 28

(b) See table 9

(c) By equation 7

Thus, mixing might be expected to be good at the very end of the 4-mi reach; that is, a nearly one dimensional (longitudinal) dispersion state would be reached. Two-dimensional dispersion (lateral mixing and longitudinal dispersion) were expected to exist at sections closer to the injection point.

Equation 28 can be used to estimate a propane desorption coefficient for a 15,000-ft reach:

$$K_p = (0.039)\frac{\Delta H}{t_c}$$

$$= (0.039\ \text{ft}^{-1})\ \frac{(0.003\ \text{ft/ft})(15{,}000\ \text{ft})}{(15{,}000\ \text{ft}/0.5\ \text{ft/s})/(3{,}600\ \text{s/h})}$$

$$= (0.039)\ \frac{(45)}{(8.33\ \text{h})} = 0.21\ \text{h}^{-1}$$

Therefore, if the test were performed in a 15,000-ft reach, $K_p t_c \approx (8.33\ \text{h})(0.21\ \text{h}^{-1}) \approx 1.75$, which is greater than 1.0 and would suggest that a shorter reach could be used. Since these are very approximate estimates, the initial decision was to locate measuring sections at 5,000, 10,000, and 20,000 ft downstream from the injection point located on the most upstream section.

Estimated elapsed times and durations

The elapsed times to the peaks and the leading and trailing edges, as well as dye-response-curve durations at the three sections, were estimated using an average velocity of 0.5 ft/s, figure 12, and equation 38 and are summarized in table 11.

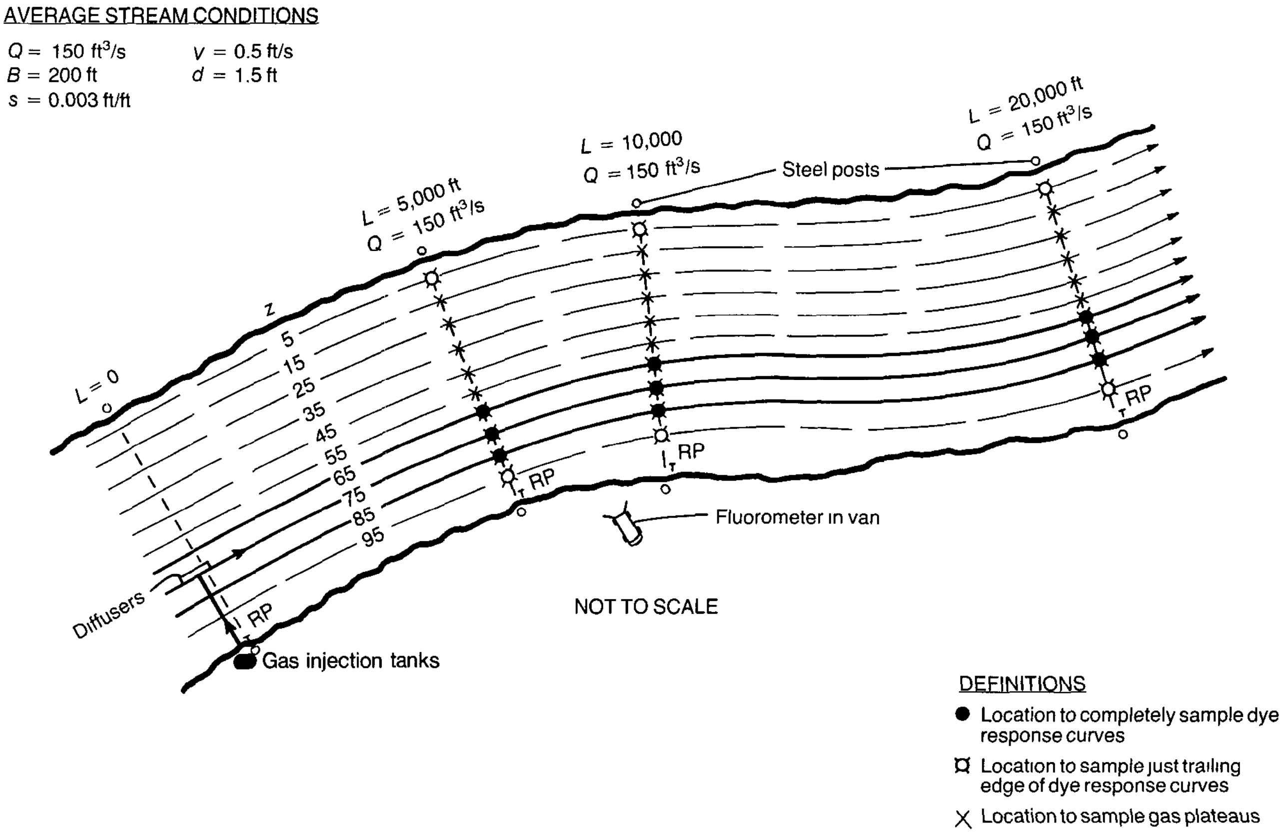

Figure 29.—Sketch of test reach on the Pat River showing layout for a constant-rate-injection type of reaeration test on a wide river with two-dimensional dispersion.

Table 11.—*Summary of average dye-response curve properties at three sections in proposed test reach on the Pat River*

[ft, feet]

Property	Section at 5,000 ft	10,000 ft	20,000 ft
Elapsed time to peak, T_p, hours	2.78	5.56	11.11
Duration of dye response curve, T_d, hours	6.15	9.79	15.56
(from figure 12 or equation $T_d = 3.1\ T_p^{0.67}$)			
Elapsed time to leading edge, T_e, hours	2.18	4.47	9.13
($T_e = T_p - t_b$ where $t_b = (T_p^{0.86}/4)$)			
Elapsed time to trailing edge, T_t, hours	8.33	14.26	24.69
($T_e + T_d$)			

These are estimated average properties, and the dye cloud might be expected to arrive sooner along the center streamlines and later along the banks. Caution must therefore be used to avoid missing the early data. The decision was to begin sampling on streamlines $z = 65$, 75, and 85 percent at elapsed times of 1.5, 4, and 8 h at the three sections.

Dye injection

Using the equation in figure 16, the quantity of 20-percent rhodamine WT dye to produce a peak of 5 μg/L at 20,000 ft would be 4,500 mL. An average peak of about 5 μg/L should be obtained at the 20,000-ft section, with lower concentrations near the left streambanks, especially at the 5,000-ft section. For this reason, if in doubt, an amount of dye to produce an average peak of 10 μg/L or more could be used if no water-supply intakes existed in the test reach.

Gas injection

Water depths at the point of injection were known to be more than 5 ft. This, and the availability of new diffusers, suggested that the efficiency of the gas-injection system might be as high as 20 percent. An average plateau concentration of 5 μg/L at the downstream section was sought, and this value used in computing the gas-injection rate. Using figure 11, with a peak traveltime of 11.11 h (for 20,000 ft), an estimated K_p of 0.21 h^{-1}, an average gas plateau of 5 μg/L, and ε equal to 0.2 yielded a propane injection rate of 75 ft^3/h. This exceeds the upper limit of the gas-rate controller. The scale on the right of figure 11 indicates that this rate is equivalent to about 8.5 lb/h. Table 11 indicates a maximum cloud duration of 15.56 h, which is the length of gas injection needed if preliminary estimates are correct. This amounts to a total of about 132 lb of propane; thus, two 100-lb tanks would be required, allowing the injection rate for each tank to be about 38 ft^3/h of propane. Using equation 34, the air rate to be set on each rotameter was about 17 ft^3/h.

Performance of test

Preparations

The first day, the four sections making up the test reach were monumented with steel posts on both banks. Stage RP's (reference points) were established on the right bank of all four sections. Taglines were stretched across at the four sections, and conventional boat-type current-meter discharge measurements made; stage was measured concurrently with each discharge measurement. The measurements were computed immediately, with discharge recorded accumulatively. Ten points were then located on each taglined section at the 5,000-, 10,000-, and 20,000-ft distances. As shown in table 5, the 10 points were located at the 5-, 15-, 25-,...75-, 85-, and 95-percent cumulative discharge points at each section. To avoid collisions with passing boats, the taglines were removed after the sampling points had been marked by buoys.

The decision was made to locate the point of injection, or source streamline, on the 75-percent streamline; this allowed the propane tanks to be located on the right bank and the diffusers to be fed via 100-ft plastic lines. Furthermore, at this point water depths were found to be about 6 ft, and thus the efficiency of the gas injection might be expected to be high.

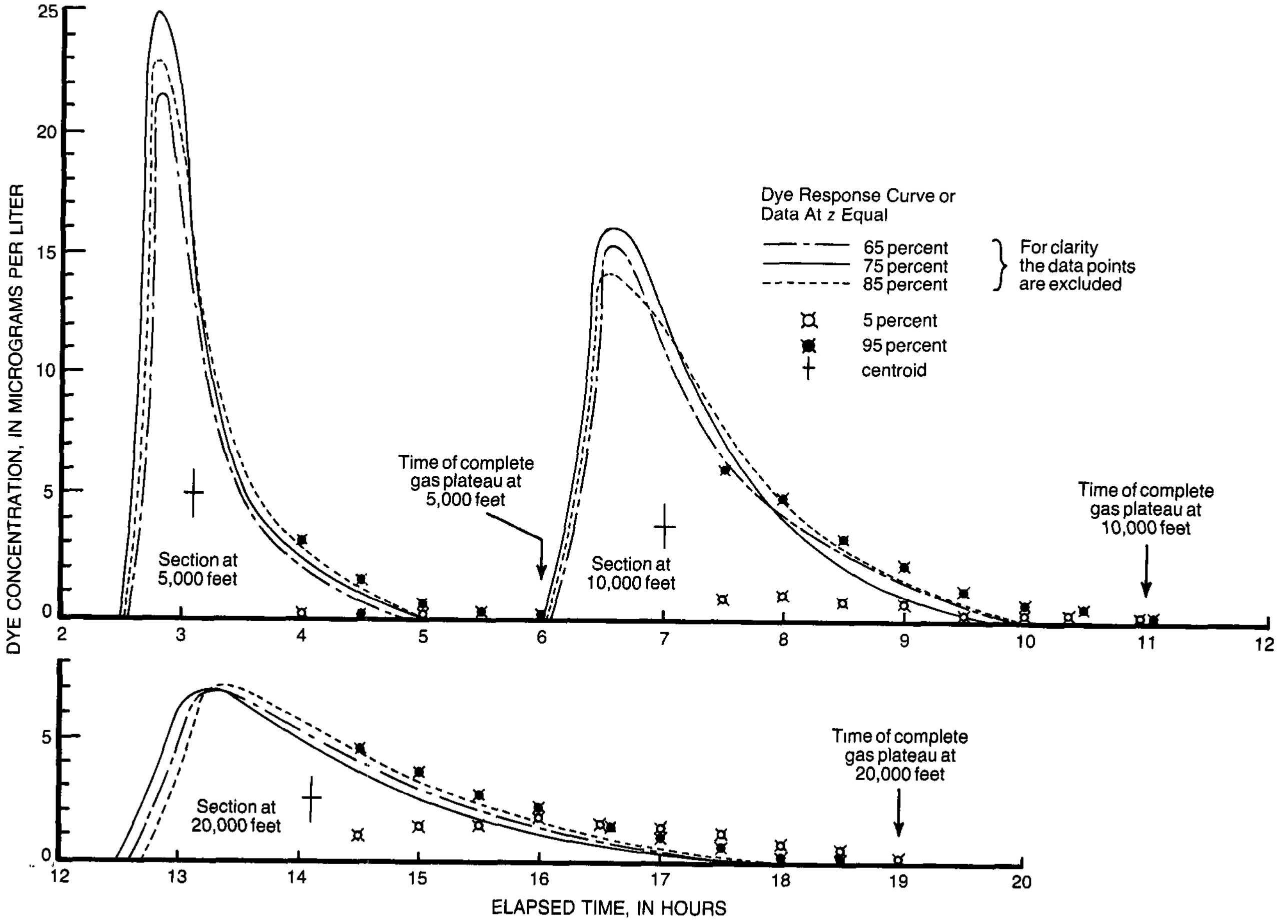

Figure 30.—Graph of dye-tracer response curves and data for constant-rate-injection method of reaeration measurement with two-dimensional dispersion.

Dye test

At 6 a.m. ($t=0$) on the second day, a slug of 4,500 mL of 20-percent rhodamine WT premixed with 15,500 mL of river water was injected at $z=75$ at section 1. A 100-mL sample of the mixture was retained for later fluorometric analysis.

Dye sampling was initiated at $t=1.5$, 4, and 8 h downstream at the 5,000-, 10,000-, and 20,000-ft sections *only* on streamlines 65, 75, and 85. Dye samples were rapidly analyzed using a fluorometer located on the right bank near the three sections. As seen in figure 30, along the $z=75$ streamline, the leading edges arrived at $t=2.58$, 6.00, and 12.68 h, which was considerably slower than estimated in table 11.[8]

At $z=65$, 75, and 85 at the three sections, dye sampling was continued until background levels were reached or could be estimated. As soon as dye concentrations approached background levels at $z=65$, 75, and 85, dye samples were collected at $z=5$ and 95 until the dye cloud was found to have essentially passed in its entirety. The longer durations measured along the banks would dictate the length of gas injection the next day and also determine when gas concentrations had plateaued across each section and could be sampled to determine ϕ. Figure 30 is a plot of these data for the three sections.

Based on these field plots of the dye-response curves, the decision was made to use all three sections in the gas test; thus, two sets of data were available: 5,000 to 20,000 ft and 10,000 to 20,000 ft.

All dye samples were forwarded to the laboratory for final analysis.

Frequent stage measurements indicated stream discharge to be constant from $t=0$ through the passage of the dye cloud. Nevertheless, a current-meter measurement was made at the 10,000-ft section at $t=10$ h.

Propane test

The dye-response curves and data obtained as a result of an actual slug injection and sampling as

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[8] The reader will note that for simplicity and comparison purposes, the dye-response curves used in the previous example are used here as applying to $z=75$ at the 5,000- and 20,000-ft sections, respectively. Furthermore, the data plotted in figure 30 are those obtained later in the laboratory following careful fluorometric analysis but are discussed here as if they were the field-data plots.

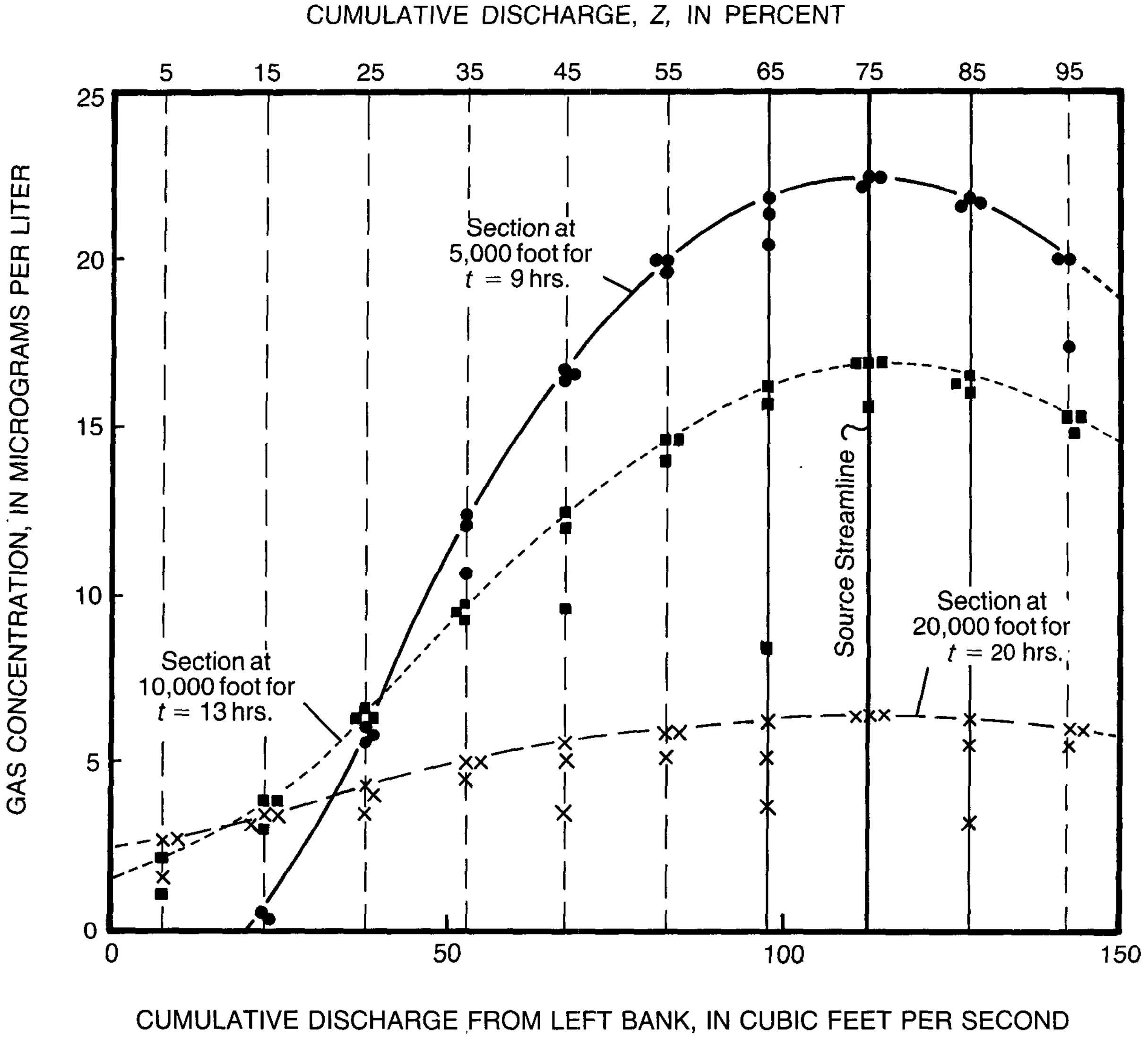

Figure 31.—Graph showing lateral distribution of plateau gas concentrations at the three measuring sections on the Pat River.

shown in figure 30 are extremely valuable in determining *exact* information on the required duration of gas injection and the time at which stable gas plateaus will exist downstream for sampling. The complete dye-response curves shown in figure 30 are for the 65-, 75- (source streamline), and 85-percent streamlines; their uniformity suggests good mixing for the part of the channel sampled. However, had dye-response curves been measured at all streamlines, particularly those left of the source streamline, ever smaller curves would have been observed, indicating incomplete mixing over the channel width. As the dye samples taken at z=5 and 95 percent indicate, very low concentrations would exist along the banks, particularly the left bank at the 5,000-ft section. Similarly, lateral mixing would not be too good even at the 20,000-ft section, as injection at the 75-percent streamline amounts to a partial side injection.

The fully defined dye-response-curve data were used subsequently in the computation of K_p. The selected dye samples taken at z=5 and 95 percent were obtained to show when stable gas plateaus would exist at each section, particularly at the 20,000-ft section. In the example, if gas injection were sufficiently long, stable plateaus would exist at 6.0, 11.0, and 19.0 h elapsed times at the 5,000-, 10,000-, and 20,000-ft sections, respectively.

Inspection of figure 30 indicates centroid traveltimes of approximately 11 h between sections 5,000 and 20,000 ft and 7 h between 10,000 and 20,000 ft. Thus, if gas sampling were performed at t=19 h at the 20,000-ft section, it should be performed at t=8 and 12 h at the 5,000- and 10,000-ft sections, respectively, to represent the same water mass in transport in which gas is being desorbed. Inspection of figure 30 indicates that stable plateaus will have existed for 2 h (8–6 h) at the 5,000-ft section and 1 h (12–11 h) at the 10,000-ft section.

The dye data at the 20,000-ft section also indicate that a *minimum* gas injection of 6.5 h (19.00–12.50 h) would be required. Obviously it would be desirable to inject longer to allow time for sampling the gas at 20,000 ft.

Two 100-lb propane tanks were positioned on a platform scale and shielded with a protective enclosure. Eight diffuser plates were placed in a longitudinal line at $z=75$, four diffusers being supplied by each tank. A gas injection of approximately 38 ft^3/h was started from each tank at 10 p.m. ($t=0$ for the gas data) on the second day; it lasted for 10 h and was discontinued at 8 a.m. on the third day. Gas sampling was performed at $t=9$ h (7 a.m.), 13 h (11 a.m.), and 20 h (6 p.m.) at 5,000, 10,000, and 20,000 ft, respectively, on the third day. At these times at each of the 10 streamlines, four gas samples were taken in quick succession.

Observations of stage throughout the third day indicated only minor changes. Discharge measurements were made at the 5,000-ft section at 9 a.m. and at the 20,000-ft section at 5 p.m. No significant changes were noted. Had there been significant change, say ±10 percent, it would have been necessary to repeat the dye test along $z=65$, 75, and 85 percent.

Analysis and computations

Dye data

All dye samples along with background samples and a sample of the dye solution injected were sent to the laboratory for careful analysis. Figure 30 shows the dye-response curves as measured at $z=65$, 75, and 85 at all three sections. The response curves on $z=75$ are solid, as it is these data for the 5,000-ft and 20,000-ft sections that are used subsequently in the computations.

Gas data

Of the 120 gas samples collected, selected ones were submitted for laboratory analysis. The results are shown in figure 31 plotted from left to right bank with cumulative discharge on streamlines. As can be seen, lateral mixing between the injection point and the 5,000-ft section was insufficient to yield any gas concentrations along the left bank and yielded only very low concentrations at the 10,000-ft section. Similarly, lateral mixing was still not complete at the 20,000-ft section, owing primarily to the injection being at $z=75$ percent instead of 50 percent. This amounts to a partial side injection. Theoretically (Kilpatrick and Cobb, 1985), a side injection requires four times the distance to mix as a center injection; these data would tend to confirm this.

Average gas-plateau concentrations were selected for computing the distribution factor ϕ using equation 22. These computations are shown in table 12 for the 5,000-, 10,000-, and 20,000-ft sections. Note that zero concentrations were present at the 5,000-ft section but posed no problems in computing ϕ.

Computation of desorption coefficient

Equation 20 can now be numerically integrated in a manner similar to the numerical integration of equation 18 to yield an equation that includes the ratio of the distribution factors applicable to each streamline:

$$\left[\frac{(\bar{c}_g Q)_u \phi_d}{(\bar{c}_g Q)_d \phi_u}\right]_z = \left[\frac{\left[\sum\limits_{i=1}^{N} \frac{C_d \Delta t}{A e^{K_p t_i}}\right]_u}{\left[\sum\limits_{i=1}^{N} \frac{C_d \Delta t}{A e^{K_p t_i}}\right]_d}\right]_z \qquad (40)$$

Computations must be along given streamlines—in this example, streamlines $z=65$, 75, and 85. The numerical integration computations are as previously demonstrated in table 9 for the one-dimensional case and are not repeated. As before, an initial value of K_p can be estimated from equation 17 (along streamline $z=75$):

$$K_p=\frac{1}{t_c}\ln\frac{(\bar{c}_g Q)_u \phi_d}{(\bar{c}_g Q)_d \phi_u}$$

$$=\frac{1}{10.98}\ln\left[22.50\times\frac{150}{10}\Big/6.40\times\frac{150}{10}\right]\left[\frac{1.245}{1.589}\right]$$

$$K_p=\frac{1}{10.98}\ln\,[3.52]\,[0.784]=0.092\ \mathrm{h}^{-1}$$

Note that the appropriate distribution factors have been applied to equation 17. This estimate is considerably less than the value of 0.21 h^{-1} originally used in planning, but since it is based on actual data and is corrected for distribution it should be close to the correct value. Trial computations were made along streamlines 65, 75, and 85 for both reaches until the ratio on the right side of equation 37 was in agreement with the ratio on the left. Table 13 shows a summary

Table 12.—*Computation of distribution factor* ϕ *at the 5,000-, 10,000-, and 20,000-foot sections using gas-concentration plateaus for the Pat River*

[See text and list of "Symbols and Units" for explanation of variables. µg/L, micrograms per liter]

z in percent	$\bar{c}_g$ µg/L	$\frac{\Delta Q_z}{Q}$	$\bar{c}_g \frac{\Delta Q}{Q}$	$\phi_z = \frac{\bar{c}_g}{\Sigma \bar{c}_g \frac{\Delta Q}{Q}}$
5,000-FOOT SECTION				
5	0	0.1	0	
15	0.50	.1	.05	0.035
25	6.00	.1	.60	.424
35	12.20	.1	1.22	.862
45	16.70	.1	1.67	1.179
55	20.0	.1	2.00	1.412
65	21.8*	.1	2.18	1.540*
75	22.5**	.1	2.25	1.589**
85	21.9*	.1	2.19	1.547
95	20.0	.1	2.00	1.412
		1.0	Σ = 14.16	
10,000-FOOT SECTION				
5	2.1	0.1	0.21	0.184
15	3.7	.1	0.37	.324
25	6.6	.1	0.66	.578
35	9.7	.1	0.97	.850
45	12.3	.1	1.23	1.078
55	14.6	.1	1.46	1.280
65	16.2	.1	1.62	1.420*
75	16.9	.1	1.69	1.402**
85	16.6	.1	1.66	1.455*
95	15.4	.1	1.54	1.350
		1.0	Σ = 11.41	
20,000-FOOT SECTION				
5	2.6	0.1	0.26	0.506
15	3.4	.1	0.34	.661
25	4.2	.1	0.42	.817
35	4.9	.1	0.49	.953
45	5.5	.1	0.55	1.070
55	5.9	.1	0.59	1.148
65	6.2*	.1	0.62	1.206*
75	6.4**	.1	0.64	1.245**
85	6.3*	.1	0.63	1.226*
95	6.0	.1	0.60	1.167
		1.0	Σ = 5.14	

*Used to compute K_p along streamlines adjacent to source streamline.

**Used to compute K_p along source streamline; see table 13.

of K_p and K_2 as determined for z=65, 75, and 85 for both reaches. Note that in column 2 of table 13 the gas transport ratios are approximately 2.75, as the residence time was sufficient from 5,000 to 20,000 ft to produce a large desorption of gas. In contrast, from 10,000 to 20,000 ft the desorption was less and the gas transport ratios in column 4 vary from 2.22 to 2.34; thus, more weight was given the data for the longer reach.

A point to be made is that because initial estimates of K_p may be considerably in error, the product $K_p t_c$ used in initial planning should probably be significantly larger than 1.00 (or $(\bar{c}_g Q)_u/(\bar{c}_g Q)_d$ significantly larger than 2.72). Furthermore, if data collection is comprehensive, sufficient residence time can be obtained by placing the upstream measuring section closer to the injection point in a region where two-dimensional dispersion exists but adjusting the data by computing distribution factors.

Regionalization

As in most hydrologic studies, it is impractical to attempt to measure all streams for their reaeration coefficient; hence, numerous empirical equations have been developed to apply to a given region (Parker and Gay, 1987). Many of these depend on the evaluation of other parameters of pertinence to the reaeration capabilities of streams. Thus, when performing gas- and dye-tracer reaeration measurements, other parameters should be measured or estimated to further regionalization efforts or the development of better empirical equations and models.

Other data needs

In all instances, measurements of channel and hydraulic properties should be made. These normally include width, mean depth, hydraulic roughness, and longitudinal slope. The type of stream should be noted; for example, is it pool and riffle, or does channel control exist in most of the test reach. All of the above change with river stage; hence, reaeration measurements at more than one stage on the same stream may be advisable.

On wide and (or) exposed streams, the measurement of wind speed and direction relative to the orientation of the stream is desirable. Yotsukura and others (1984) suggest that one well-placed wind-speed and wind-direction meter would be sufficient in most cases.

Since relative humidity and water and air temperatures influence desorption, their measurement is advisable.

The gas-transfer process can also be affected by changes in water-quality parameters such as methylene blue active substances as an indicator of detergent concentrations, color as an indicator of the concentration of organic acids, specific conductance as an indicator of dissolved-solids concentrations, and suspended solids as an indicator of suspended inorganic concentrations in the water column (Bennett and Rathbun, 1972).

When sufficient data become available, it may be possible to estimate reaeration coefficients from photographs of the stream reach. Thus, representative photographs of the stream reach *at the flow being tested* are advisable.

Table 13.—*Summary of desorption and reaeration coefficient computations for constant-rate-injection method—Two-dimensional dispersion*

[See text and list of "Symbols and Units" for explanation of variables. h^{-1}, per hour]

Streamline z, percent of flow	Reach 5,000 to 20,000 feet		Reach 10,000 to 20,000 feet		Average $K_{2_{20}}$ for 5,000- to 20,000-foot reach h^{-1} (c)
	Measured $\left[\frac{[\bar{c}_gQ]_u\phi_d}{[\bar{c}_gQ]_d\phi_u}\right]_z$ (a)	Final K_D as calculated by trial h^{-1} (b)	Measured $\left[\frac{[\bar{c}_gQ]_u\phi_d}{[\bar{c}_gQ]_d\phi_u}\right]_z$ (a)	Final K_D as calculated by trial h^{-1} (b)	
(1)	(2)	(3)	(4)	(5)	(6)
65	$\frac{21.8 \times 15 \times 1.206}{6.2 \times 15 \times 1.540} = 2.75$	0.092	$\frac{16.2 \times 15 \times 1.206}{6.2 \times 15 \times 1.420} = 2.22$	0.110	0.13
75	$\frac{22.5 \times 15 \times 1.245}{6.4 \times 15 \times 1.589} = 2.76$	0.092	$\frac{16.9 \times 15 \times 1.245}{6.4 \times 15 \times 1.402} = 2.34$	0.115	
85	$\frac{21.9 \times 15 \times 1.226}{6.3 \times 15 \times 1.547} = 2.75$	0.092	$\frac{16.6 \times 15 \times 1.226}{6.3 \times 15 \times 1.455} = 2.22$	0.110	

(a) See figure 31 and table 12
(b) By equation 40
(c) For stream temperature of 18 °C

Summary and Conclusions

This manual describes in detail how to perform reaeration measurements on streams using dye and gas tracers injected concurrently or in tandem to simulate oxygen absorption. Two basic approaches, differing in the mode of injection and, hence, the mode of sampling are presented. Both have advantages and disadvantages, and both require greater effort and care in their performance than do most other types of tracer studies if accurate results are to be obtained. However, both are fully within the capabilities of the average hydrologist. Hydrologists should become familiar with routine dye tracer and fluorometry techniques before attempting tracer reaeration measurements; these are well described in the other manuals in this series (Hubbard and others, 1982; Kilpatrick and Cobb, 1985; Wilson and others, 1986).

The equipment required for both types of tests is essentially the same. It is suggested that the constant-rate-injection method has certain advantages over the slug-injection method because

1. A simple slug injection of dye rather than a constant-rate injection is required.
2. The CRI method may require less gas sampling, although sampling of the dye-response curves is essentially the same for both methods; an exception is for wide rivers, where more extensive sampling is required.
3. The CRI method can be used in narrow as well as wide rivers where two-dimensional dispersion exists. Because the test can be performed closer to the point of injection and, hence, in a region where two-dimensional dispersion may exist, longer test reaches can be used, and therefore the greater accuracy that results from the use of greater gas-residence times can be obtained.

While not emphasized during the description of the SI method, the occurrence of two-dimensional dispersion with such a test (intentionally or otherwise) can be adjusted for by use of the same lateral distribution correction factors that are used with the CRI method. To do so would require extensive definition of the dye-response curves laterally at the measuring sections. The determination of ϕ by measurement of plateau gas concentrations laterally, as is done in the CRI method, is a much more practical approach; hence, the CRI method is particularly recommended for wide streams.

This report has provided sufficient detail and examples to allow manual computation of desorption coefficients by a variety of methods. Some of the computations are very laborious and more easily performed by computer. The Geological Survey Office of Surface Water has prepared a program to perform most of the computations for both the SI and CRI methods. The program is tailored to conform to the data-collection and analysis procedures discussed in this report; its use is encouraged.

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