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Techniques of Water-Resources Investigations
of the United States Geological Survey

Chapter A18

DETERMINATION OF STREAM REAERATION COEFFICIENTS BY USE OF TRACERS

By F.A. Kilpatrick, R.E. Rathbun, N. Yotsukura,
G.W. Parker, and L.L. DeLong

Book 3
APPLICATIONS OF HYDRAULICS

DEPARTMENT OF THE INTERIOR

MANUEL LUJAN, Jr., *Secretary*

U.S. GEOLOGICAL SURVEY

Dallas L. Peck, *Director*

UNITED STATES GOVERNMENT PRINTING OFFICE: 1989

For sale by the Books and Open-File Reports Section, U.S. Geological Survey,
Federal Center, Box 25425, Denver, CO 80225

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METRIC CONVERSION FACTORS

Multiply inch-pound unit	*By*	*To obtain metric unit*
inch (in)	25.40	millimeter (mm)
foot (ft)	0.3048	meter (m)
mile (mi)	1.609	kilometer (km)
pound (lb)	453.6	gram (g)
gallon (gal)	3.785	liter (L)
gallon (gal)	3,785	milliliter (mL)
foot per second (ft/s)	0.3048	meter per second (m/s)
square inch (in^2)	0.000645	square meter (m^2)
square foot per second (ft^2/s)	0.09294	square meter per second (m^2/s)
cubic foot (ft^3)	0.02832	cubic meter (m^3)
cubic foot per second (ft^3/s)	0.02832	cubic meter per second (m^3/s)
degree Fahrenheit (°F)	°C=5/9(°F-32)	degree Celsius (°C)
parts per billion ($lb/10^9lb$)	≈1.0	microgram per liter (μg/L)

SYMBOLS AND UNITS

Symbol	Explanation	Unit
A_c	Area of the observed dye concentration-time response curve	(µg/L)(min) or (µg/L)(h)
A_z	Area of the observed dye concentration-time response curve measured on a streamline, z	(µg/L)(min) or (µg/L)(h)
A_g	Area of the gas concentration-time response curve	(µg/L)(min) or (µg/L)(h)
B	Average width of stream	ft
$\bar{c}$	Weighted or average plateau dye concentration	µg/L
$\bar{c}_g$	Weighted or average plateau gas concentration	µg/L
C	Concentration of injected dye solution	µg/L
C_d	Observed dye concentration	µg/L
C_g	Observed gas concentration	µg/L
C_p	Observed peak dye concentration	µg/L
d	Mean depth of stream; also a subscript denoting downstream	ft
e	A constant, 2.72	---
E_z	Lateral or transverse mixing coefficient	ft^2/s
ΔH	Change in elevation in test reach	ft
i	Integration interval	---
J	Dye-loss correction factor	---
K_2	Oxygen absorption or reaeration rate coefficient	h^{-1}
K_{220}	Oxygen absorption or reaeration rate coefficient at 20 °C	h^{-1}
K_e	Ethylene gas desorption rate coefficient	h^{-1}
K_p	Propane gas desorption rate coefficient	h^{-1}
K_{PY}	Propane gas desorption rate coefficient at stream temperature Y	h^{-1}
K_{P20}	Propane gas desorption rate coefficient at 20 °C	h^{-1}
K_T	Tracer gas desorption rate coefficient	h^{-1}
K_{T20}	Tracer gas desorption rate coefficient at 20 °C	h^{-1}
K_{TY}	Tracer gas desorption rate coefficient at temperature Y	h^{-1}
K_{Tz}	Tracer gas desorption rate coefficient along a streamline, z	h^{-1}
L	Length of measurement reach	ft
L_o	Channel length required for optimum mixing; usually corresponds to about 95-percent mixing	ft
M	Mass of tracer injected	g
$M_{1,2,3...n}$	Series of slug injections of equal mass	g
P_g	Pressure at which the gas is metered	lb/in^2
q	Rate of constant dye tracer injection	mL/min
q_a	Air injection rate	ft^3/h
q_g	Rate of constant gas tracer injection	ft^3/h
q_p	Constant-rate gas injection at standard conditions of 70 °F and 14.7 psia	ft^3/h, L/min, or lb/h
Q	Total stream discharge	ft^3/s
ΔQ_z	Portion of total stream discharge applicable to a streamline	ft^3/s
Q_z	Cumulative stream discharge laterally in the cross section at streamline z	ft^3/s
Q_m	Maximum discharge in test reach	ft^3/s
R_e	Ratio of oxygen reaeration rate coefficient to ethylene gas desorption rate coefficient	---
R_p	Ratio of oxygen reaeration rate coefficient to propane gas desorption rate coefficient	---
RP	Reference point	---
R_r	Dye recovery ratio	---
s	Water-surface slope	ft/ft
S_G	Specific gravity	---
Δt	Numerical intergration interval	min or h
t	Elapsed time from start of injection	h
t_b	Interval of time for dye concentrations to build up from the leading edge to the peak	h
$t_{c,L,t,p}$	Traveltime of centroid, leading edge, trailing edge, and peak, respectively, of dye-response curve	h
t_i	Elapsed time to point i on response curve	h
$T_{c,L,t,p}$	Elapsed time to centroid, leading edge, trailing edge, and peak, respecively, of dye-response curve	h or min
T_d	Duration in time for dye cloud to pass any one point in a section	h or min
T_D	Duration in time for entire tracer cloud to pass section	h or min
u	Subscript denoting upstream	---
V_d	Volume of dye solution	mL
V_I	Volume of dye solution injected into stream	L or mL
V_s	Volume of stock dye solution	L or mL
V_w	Volume of diluent, usually distilled water	mL
v	Mean stream velocity	ft/s
v_p	Velocity of peak	ft/s
W_d	Weight of dye injected	g
W_r	Weight of dye recovered	g
z	Streamline index, expressed as a percentage of total river discharge: $z=100\ Q_z/Q$; also used as a subscript to denote a streamline	percent
ε	Absorption efficiency, expressed as a decimal	---
ϕ	Mass distribution	---
Y	Stream temperature	°C
Y_g	Temperature at which gas is metered	°F

DETERMINATION OF STREAM REAERATION COEFFICIENTS BY USE OF TRACERS

By F.A. Kilpatrick, R.E. Rathbun, N. Yotsukura, G.W. Parker, and L.L. DeLong

Abstract

Stream reaeration is the physical absorption of oxygen from the atmosphere by a flowing stream. This is the primary process by which a stream replenishes the oxygen consumed in the biodegradation of organic wastes.

Prior to 1965, reaeration rate coefficients could be estimated only by indirect methods. In 1965, a direct method of measuring stream reaeration coefficients was developed whereby a radioactive tracer gas was injected into a stream—the principle being that the tracer gas would be *desorbed* from the stream inversely to how oxygen would be *absorbed*. The technique has since been modified by substituting hydrocarbon gases for the radioactive tracer gas.

This manual describes the slug-injection and constant-rate-injection methods of measuring gas-tracer desorption. Emphasis is on the use of rhodamine WT dye as a relatively conservative tracer and propane as the nonconservative gas tracer, on planning field tests, on methods of injection, sampling, and analysis, and on techniques for computing desorption and reaeration coefficients.

Introduction

Stream reaeration is the physical absorption of oxygen from the atmosphere by a flowing stream. This is the primary process by which a stream replenishes the oxygen consumed in the biodegradation of organic wastes.

The primary use of reaeration coefficients is to quantify the process of reaeration in dissolved-oxygen water-quality models. The reaeration coefficient is the rate constant for the absorption of oxygen from the atmosphere. These models, which simulate the exchange of dissolved oxygen, are used to calculate waste-load allocations for the stream so that dissolved-oxygen concentration standards are not violated. If the reaeration coefficient used in a model is smaller than the actual coefficient for the stream, the required degree of waste treatment indicated by the model results will be too large and the expenditure of funds will be greater than needed (St. John and others, 1984). Conversely, if the reaeration coefficient used in the model is larger than the actual coefficient for the stream, the degree of waste treatment indicated by the model results will be insufficient and dissolved-oxygen standards may be violated. Management of the dissolved-oxygen resources of the stream can be done either by increasing the degree of treatment of the wastes discharged into the stream or by reducing the quantity of waste discharged into the stream. In either case, the reaeration coefficient must be known accurately if the dissolved-oxygen resources of the stream are to be managed properly.

Prior to 1965, reaeration rate coefficients could be estimated only by indirect methods (Bennett and Rathbun, 1972), which were subject to significant errors. In 1965, E.C. Tsivoglou and others developed a direct method of measuring stream reaeration coefficients employing the radioactive tracer gas krypton-85. Tsivoglou's method is based on the principle that a gas tracer injected into a stream would be *desorbed* from the stream inversely to how oxygen would be *absorbed*. This provided the first direct means of measuring stream reaeration capacity.

Unfortunately, the radioactive-tracer technique has disadvantages. The primary disadvantage is the potential for radiation exposure by the personnel conducting the study and by the public along the stream. Tsivoglou (1967) reported that radiation exposure could be eliminated with a minimum of trouble for discharges of less than 200 ft^3/s (cubic feet per second). For larger flows, however, more stringent precautions are required because of the larger amounts of tracer necessary. Another disadvantage of using radioactive tracers is that the process for obtaining a license to use radioactive material in a public waterway may be quite involved.

In the early 1970's, R.E. Rathbun (Rathbun and others, 1975; Rathbun and others, 1978) modified the technique (hence, it is frequently called the "modified method"): a hydrocarbon gas such as ethylene or propane is substituted for radioactive krypton as the gas tracer. Theoretically, this method does not differ

substantively from the krypton method; the only differences are in performance techniques.

The purpose of this manual is to provide an understanding of the principles and techniques of performing reaeration tests on streams through use of gas tracers. The basic technique applied by Tsivoglou and Rathbun involved *slug* injection of the gas tracer into the stream and subsequent measurement of its desorption as it moved and dispersed downstream. To be consistent with other tracer manuals and with use in the U.S. Geological Survey (Kilpatrick and Cobb, 1985), the slug-injection method is referred to henceforth as the "SI method." This distinguishes it from the steady-state method of Yotsukura and others (1983) and Parker and Gay (1987), which features continuous injection of the gas tracer at a constant rate and is hereafter referred to as the "CRI method." The application of this method is also fully discussed in this manual.

Both the SI and CRI methods have been used primarily on small streams where complete vertical and lateral mixing in a cross section could be readily obtained. Preliminary tests have been performed and techniques developed (Yotsukura and others, 1984) to apply the CRI method to wide streams in which lateral mixing as well as longitudinal dispersion is not complete. Although preliminary, the field techniques and methods of analysis are presented in this manual to encourage their use and further development.

The authors wish to acknowledge the many contributions of others to the development of the techniques presented in this manual. In particular, we are indebted to E.C. Tsivoglou for providing the basic theory and techniques for indirectly measuring stream reaeration; his work has spawned many similar reaeration measuring techniques as well as broadened our understanding of gas transfer at water surfaces. More specifically, we are indebted to Merritt Blalock for developing the supporting computer programs and to Frederick Gay, David Stedfast, James Smoot, Kenneth Taylor, and numerous others for their data as well as helpful comments on equipment and procedures.

Background preparation

The reader is advised to become fully acquainted with the following Techniques of Water-Resources Investigations (TWRI) of the Geological Survey prior to using this manual and performing tracer reaeration tests:

1. Fluorometric Procedures for Dye Tracing
   By J.F. Wilson, Jr., E.D. Cobb, and F.A. Kilpatrick
   TWRI, Book 3, Chapter A12, 1986
2. Measurement of Time of Travel and Dispersion in Streams by Dye Tracing
   By E.F. Hubbard, F.A. Kilpatrick, L.A. Martens, and J.F. Wilson, Jr.
   TWRI, Book 3, Chapter A9, 1982
3. Measurement of Discharge Using Tracers
   By F.A. Kilpatrick and E.D. Cobb
   TWRI, Book 3, Chapter A16, 1985

Particular attention should be given to the last background reference, because an understanding of the superposition principle and of the various factors involved in performing slug and constant-rate dye injections is similar for reaeration tests and is vital to their successful performance. For continuity and convenience, some of the information available in these reports is presented again. For consistency, symbols and definitions are the same as presented in these other reports when feasible.

Accuracy

Good engineering practice dictates that measures employed to ensure accuracy of data not exceed measures necessary for an adequate analysis to meet the intended use. For example, background reference 2 suggests low and high levels of study commensurate with the objectives of the study for time-of-travel tests. Conversely, background reference 3 emphasizes the need for greater accuracy and care when performing dye-dilution discharge measurements. This requirement is to be further emphasized in regard to performing tracer reaeration tests in virtually all aspects of planning, data collection, laboratory analysis, and computations.

Theory

Behavior of tracers in streams

Dispersion and mixing

Tracers used successfully in hydrologic studies are water soluble and have essentially the same physical characteristics as water (Feuerstein and Selleck, 1963; Smart and Laidlaw, 1977). Thus, when introduced into a flowing stream, they experience the same dispersion and dilution as does the element of water tagged. The use of tracers to simulate flow and transport in a water body as well as to measure the quantity of flow is widespread (Pritchard and Carpenter, 1960; Wright and Collings, 1964; Taylor, 1970; Boning, 1973; Yotsukura and Kilpatrick, 1973; Kilpatrick and Cobb, 1985; Kilpatrick and Taylor, 1986).

An understanding of how tracers behave in a stream is essential to an understanding of their application in

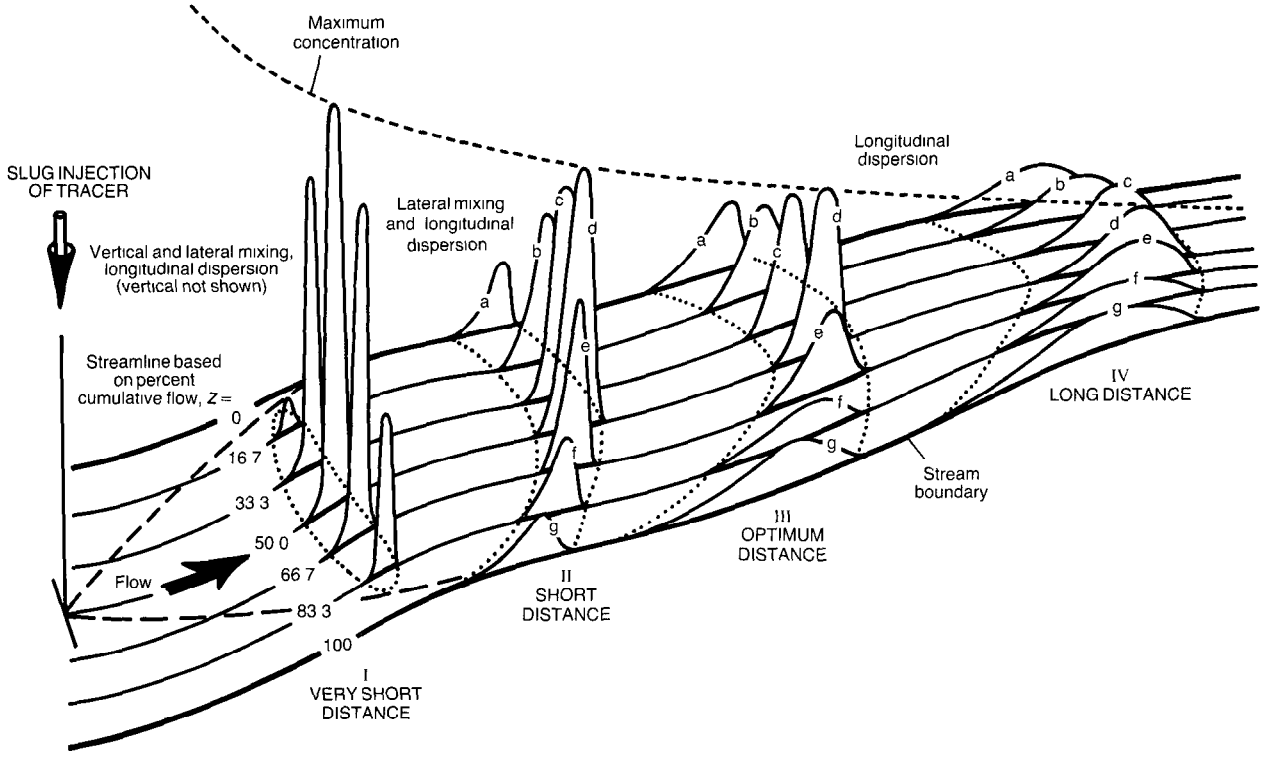


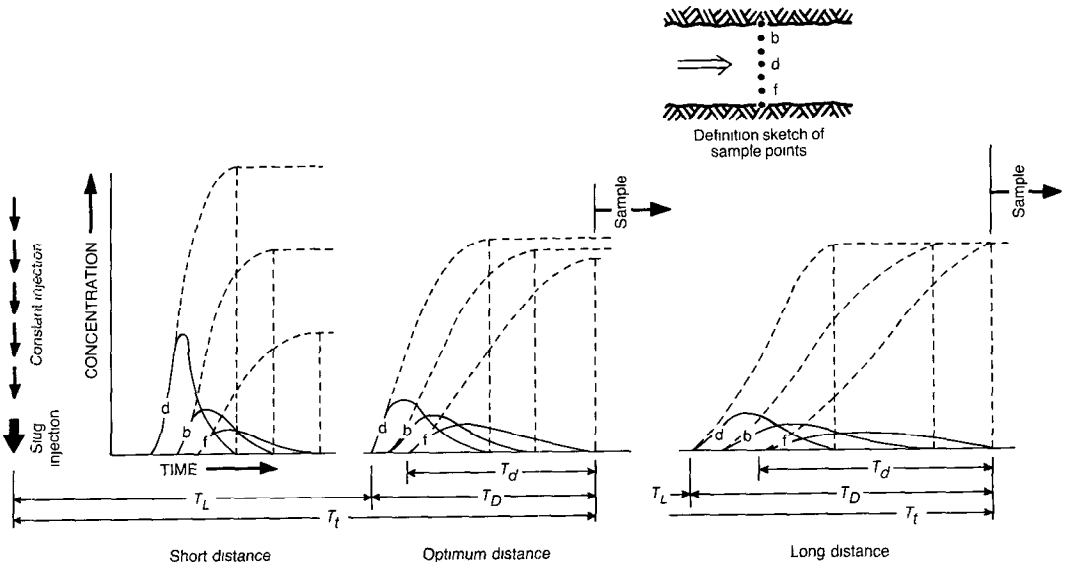
Figure 1.—Lateral mixing and longitudinal dispersion patterns and changes in distribution of concentration downstream from a single, center slug injection of tracer.

reaeration tests. Figure 1 depicts the downstream behavior of a tracer injected as a slug in the center of flow. Note that in figure 1 the response curves are shown as concentration versus longitudinal distance and not versus time. In figure 1, the stream has been arbitrarily divided laterally into six stream tubes of equal discharge; these are defined by seven streamlines, a through g. In practice this is done by plotting the cumulative discharge, such as measured by a current meter, versus lateral distance in the measurement cross section. The concept of stream tubes and streamlines was advanced by Chow (1964) and Yotsukura and Cobb (1972). It is useful in describing the behavior of a tracer and is an essential part of the technique of measuring reaeration, particularly in the application of the CRI method to wide streams.

The dispersion and mixing of a tracer in a receiving stream takes place in all three dimensions of the channel (fig. 1). In this manual, vertical and lateral dispersion and diffusion are referred to in a general way as "mixing." The elongation of a tracer response cloud longitudinally is referred to as "longitudinal dispersion." Vertical mixing is normally completed first, and lateral mixing later, depending on the width of the stream and on velocity variations. Longitudinal dispersion, having no boundaries, continues indefinitely. Thus at section I (fig. 1), vertical mixing may be complete (meaning that at any one streamline and time, the tracer concentration is the same near the bed of the stream as at the water surface), while lateral mixing is still in progress. At section II, lateral mixing is still taking place, as the tracer mass in transport along the different streamlines is not equal since the response curves do not have the same areas. Therefore, mixing and dispersion in two dimensions exists between sections I and II.

It should be kept in mind that measurements over relatively short distances do not necessarily yield representative results, because the entire flow is not yet tagged and simulated. The SI method of measuring reaeration using radioactive tracers is usually used for such short distances. Similarly, the CRI two-dimensional method as employed for wide rivers is used for a reach that is short relative to the distance required for complete lateral mixing.

An optimum mixing distance, section III, is reached when the tracer-response curves as observed laterally, a, b, c,...g, have about the same area, even though the individual response curves may vary considerably in shape and dimensions; dispersion is approaching the one-dimensional state. Nevertheless, peak concentrations in the center of the channel may be considerably greater than peak concentrations along the banks, while curves along the bank are



NOTE Only the duration of passage, T_d, of the longest response curve is labeled

Figure 2.—Time-concentration curves for slug and simulated constant injection observed at three points laterally across channel, three different distances below the injection point. Abbreviations are defined in text and in list of "Symbols and Units."

longer both physically and in time of passage. Also, the tracer cloud may be skewed, advancing faster in the center of the flow than at the channel banks. As shown in figure 1, the tracer cloud may not be uniformly skewed. Figure 1 illustrates why sampling of the tracer cloud at several points laterally is advisable.

If the tracer-response curves are next examined at a long distance, section IV, the curve areas will be found to be nearly identical and peak concentrations more nearly the same laterally. Thus, a nearly one-dimensional dispersion state exists between sections III and IV, with longitudinal dispersion dominating and continuing indefinitely downstream. With time and distance, peak concentrations become attenuated and the cloud lengths get longer and longer (Kilpatrick and Taylor, 1986).

When using hydrocarbon gas and dye tracers to measure reaeration by the SI method, a stream reach is sought wherein both vertical and lateral mixing can be essentially complete. A test reach normally consists of an upstream and a downstream measuring section. An effort is made to select an injection site a sufficient distance upstream that, as a minimum, optimum mixing is obtained in the first measuring section (the upstream section) of the reach. It is essential that an accurate measurement and adjustment for dye losses be made, and this can be more accurately done if lateral mixing is essentially complete in the test reach. Complete lateral mixing is not a factor when using the SI method with radioactive tracers because the radioactive dispersion tracer is essentially conservative over the stream reaches and times employed and because loss of this tracer is not a factor in analysis and computations. The same is true for the CRI method using hydrocarbon gas and dye tracers, although results more representative of the entire stream will be obtained if a stream reach is used in which optimum mixing has been obtained.

Characteristics of response curve

The conventional manner of illustrating the *response* of a stream to a slug injection of tracer is to plot concentration variation with elapsed time as observed at one or more points laterally at a stream cross section. To illustrate this, the tracer-response curves for a single centerpoint slug injection (see fig. 1) for streamlines b, d, and f are shown as solid lines in figure 2 for the short, optimum, and long distances. If the complete tracer cloud is to be observed, samples must be taken for the entire duration of the time of passage, T_D, if sampled at just b, d, and f. If samples are taken very close to the banks, a and g in figure 1, T_D will be much longer. Similarly, as shown, T_D will increase with distance downstream.

As will be seen, it is the single longest duration response curve *at the most downstream cross section* that determines the *minimum* duration of gas injection in the CRI method of reaeration measurement. An understanding of this unique characteristic of the

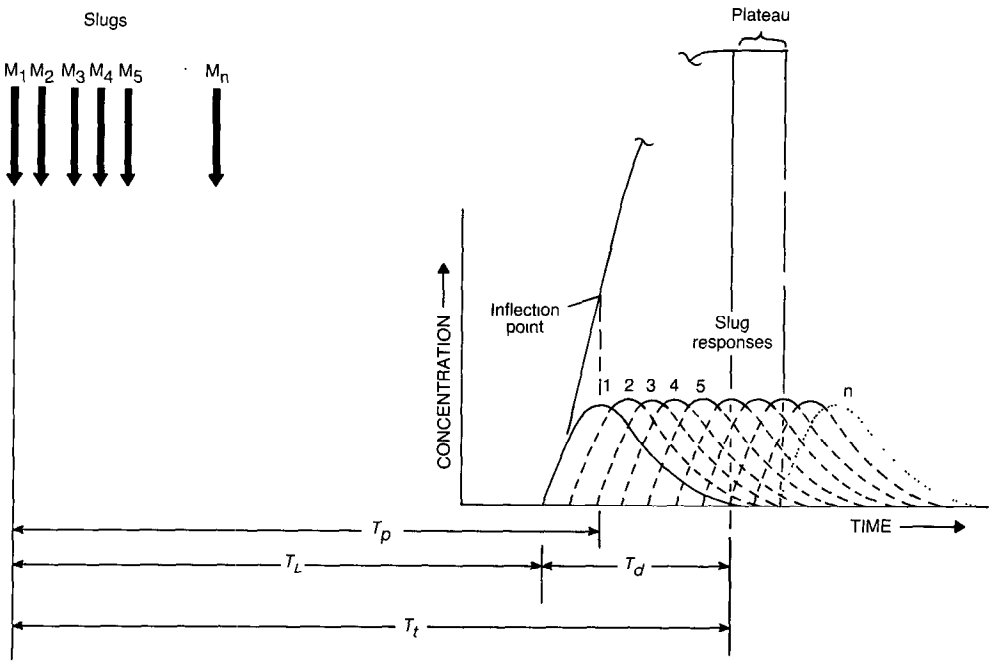


Figure 3.—Superposition of slug-response curves to simulate constant-injection buildup to a plateau at one location in stream section. Abbreviations are defined in text and in list of "Symbols and Units."

time-concentration response curve is essential to performance of the CRI method and can be best understood by using the superposition principle.

Superposition principle

A continuous, constant-rate injection of tracer can be simulated from the response curve of a slug injection using the superposition principle (Yotsukura and Kilpatrick, 1973). This can be understood best by using *any one* of the slug-response curves in figure 2 to simulate the response of a continuous injection *at the same location.* In figure 3, the solid response curve is due to the slug injection, M_1, and T_L and T_t are the elapsed times to the arrival of the leading edge and trailing edge, respectively, of this response curve. Assuming that streamflow is steady, continuing to inject a series of tracer slugs of equal amounts (M_2, M_3,...M_n) at short, uniform time intervals (a constant-rate injection) would yield a series of identical response curves offset by the interval between injections. Of course, if the same soluble tracer is continuously injected, the individual response curves cannot be distinguished and there is an ever-increasing buildup of concentration with time until T_t is reached. In effect, the superimposed slug-response curves are being added as they overlap. The initial buildup is ever increasing until an inflection point is reached at a time corresponding to the time of the peak, T_p, of the first slug-response curve. Buildup continues at a decreasing rate past this inflection point until a plateau is reached at T_t, the time when the trailing edge of the first slug response passes the point of observation. This plateau of constant concentration is *first* reached at time T_t *for that point in the channel.* At this time, *for a constant injection,* an equilibrium condition is reached. Continued injection after T_t would result in a plateau of constant concentration *at that point* as long as stream discharge and rate of tracer injection did not vary.

The same results illustrated in figure 3 are shown in figure 2 for the same three distances and the three lateral locations b, d, and f. The slug- response curves are used to simulate (dashed lines) the responses to a continuous injection of tracer and are obtained by superimposing the uniformly repeated slug-response curves.

It can be seen in figure 2 that for the short distance, plateaus of different concentrations are obtained laterally. This shows that mixing is poor and is identical for continuous and slug injections.

At the optimum distance[1], the plateaus laterally tend to converge to about the same concentration, indicating that mixing is adequate. Note that the tracer would need to be injected continuously for at

[1]The optimum distance is usually where mixing is about 95 percent complete. The concept of degree of mixing and how it is computed is discussed in appendix A of background reference 3 (Kilpatrick and Cobb, 1985).

least a time equal to the longest duration of passage at any location *across the channel between points b and f* at that distance (see fig. 2) to just produce a stabilized plateau. Of course, it would be necessary to inject tracer for a slightly greater duration to allow for sampling the fully developed plateau. It is important to remember that to develop a plateau close to the immediate streambanks, points a and g, injection would have to be longer and would be determined by the duration of the slowest moving tracer along one bank or the other. It is therefore advisable not to sample too close to one bank or the other and to be suspicious of any near-bank samples that yield low concentrations of tracer.

At a long distance, the resulting plateau concentrations are virtually identical if the constant injection is made over sufficient time and *if sufficient time is allowed for the buildup* to be accomplished across the entire channel. What was said for the optimum-distance superposition also applies to the long-distance superposition. Most important, it is the longest duration of passage *for the most downstream site* that determines the *minimum* gas injection period for the CRI method. This is because the duration of the plateau at that site will be shorter than at any site farther upstream.

In summary, the time-concentration curves resulting from a slug injection of tracer can be used to determine the minimum length of continuous gas injection necessary when the CRI method of reaeration measurement is to be used. Furthermore, the time of departure of the dye tracer at the sampling section, which is the response to a slug injection, corresponds to the time at which a gas plateau will have just been reached as a result of a continuous, constant-rate injection of gas upstream. A CRI reaeration test should be designed to avoid sampling along streambanks so that extremely long gas-injection periods are not needed.

Dye-tracer losses and recovery ratios

All of the methods of directly measuring stream reaeration involve injection of a tracer gas into the streamflow and subsequent measurement of its desorption as it is transported downstream (Rathbun and others, 1975; Rathbun and Grant, 1978). The amount of desorption is determined by measuring the loss of the gas tracer between two or more sections downstream from the injection. This desorption is measured against a totally conservative tracer that is injected at exactly the same location and undergoes the same dispersion and dilution as the gas tracer.

The radioactive tracer and hydrocarbon gas-dye tracer SI methods of reaeration measurement are the same in principle but different in application, primarily because the dye tracer used in the latter is not totally conservative, in contrast to the tritiated water used in the former. It is, therefore, essential that an accurate measurement of dye recoveries be made, as this enters directly into the computation of the reaeration coefficient when using the peak method; in effect, the dye-concentration data are made conservative by use of recovery information.

Thus, as discussed previously, the SI method using hydrocarbon gas is best performed in a stream reach in which mixing is essentially complete. It is suggested that at least three sampling points be located in each measuring section in equal or known portions of the stream discharge, such as at b, d, and f of figures 1 and 2. The dye time-concentration curves should be well defined and extended to background at each sampling point. This may require sampling until concentrations are about 2 percent of the peak or 0.1 μg/L (microgram per liter), whichever is larger. The remainder of the curve should be extrapolated to zero. Under no circumstances should the time-concentration curves be truncated at 10 percent of the peak concentration, as is suggested for time-of-travel studies. Information about the total mass of dye passing the measuring section is desired, and hence the best measurement of the *total* areas of the time-concentration curves is necessary. The dye recovery ratio, R_r, at any measuring section is expressed by the equation

$$R_r = \frac{W_r}{W_d} \tag{1}$$

where W_r is the weight recovered and W_d is the weight of dye injected. Since $W_r = QA_c$ and $W_d = V_I C S_G$

$$R_r = 1{,}699\frac{QA_c}{V_I C S_G} \tag{2}$$

where

Q = stream discharge, in cubic feet per second;

A_c = representative *total* area under the observed dye time-concentration curve, in micrograms per liter times *minutes*. If three or more time-concentration curves are observed at a section, A_c is the average, weighted on the basis of discharge. If the sample points are chosen to represent equal increments of discharge, weighting has already been accomplished. Observed dye concentrations must be based on a fluorometric calibration that also considers the specific gravity, S_G, of the dye solution (see background reference 1, Wilson and others, 1986);

V_I = volume of dye injected, in liters having a concentration C;

C = concentration of the injected solution, in micrograms per liter (20 percent=20×10^7 µg/L); and

S_G = specific gravity of the injected dye solution—1.19 for rhodamine WT, 20 percent solution as received from the manufacturer. It should be noted that if S_G is actually other than 1.19, no error will occur if the same value is used in the preparation of standards for calibrating the fluorometer and hence is reflected in A_c.

As indicated above, $W_r=QA_c$; where good mixing exists and multiple sampling points have been selected to represent equal increments of stream discharge, A_c may be a simple average of the area of the individual dye-response curves. If mixing is not good, that is, if a two-dimensional dispersion state exists and (or) if sampling points were not selected to represent equal increments of discharge, QA_c in equation 2 must be the summation of the dye mass in transport along each streamline z. Thus equation 2 for the two-dimensional case may be written

$$R_r=1{,}699\frac{\Sigma(\Delta Q_z A_z)}{V_I C S_G} \tag{3}$$

where

ΔQ_z = portion of total stream discharge applicable to streamline, in cubic feet per second; and

A_z = area of the observed dye concentration-time response curve measured on streamline z.

A dye-loss correction factor, J, is frequently used in computation of the gas desorption coefficient when using the SI method (Rathbun and others, 1975). The factor J may also be expressed as

$$J=\frac{(R_r)\text{ upstream}}{(R_r)\text{ downstream}} \tag{4}$$

using the recovery ratio for the upstream and downstream measuring sections, which together make up the reaeration measurement test reach. Rather than using J outright, it is suggested that recovery ratios be computed for each measuring section so that their validity can be examined. In practice, the recovered amount of a dye tracer must always be less than 100 percent; R_r will usually be 0.80 to 0.95 in short stream reaches typically used for reaeration tests. If computations produce other results, usually Q or A_c is in error; commonly, A_c has not been adequately or accurately defined. In short stream reaches, A_c measured just in the center of the stream will typically yield recoveries in excess of 100 percent. Measuring the dye time-concentration curve at fewer than three points in a section is discouraged.

It is important that the fluorometer used be calibrated using a sample of the injected dye. Poor fluorometry techniques may yield erroneous recovery values. Errors can often be detected by computing recoveries and comparing them over the stream reach. Comprehensive field-data collection and accurate laboratory analyses will avoid most problems.

Gas desorption and oxygen absorption

The principle involved in measuring stream reaeration coefficients using gas tracers is that the *desorption* of the gas tracer is related to the *absorption* of oxygen under similar flow conditions—specifically, the ratio of the reaeration rate coefficient to the desorption rate coefficient of the tracer gas is constant, independent of temperature or turbulence intensity.

Tank tests

Tank stirring tests that have involved careful concurrent measurement of oxygen absorption and gas desorption for the hydrocarbon gases ethylene and propane have been performed by Rathbun (Rathbun and others, 1978) and Rainwater and Holley (1983, 1984). Tsivoglou (1967) performed essentially the same tests for krypton-85 gas, which is used in the radioactive method. Rathbun and others (1978) determined that the ratios for ethylene (R_e) and propane (R_p) are

$$R_e=\frac{K_2}{K_e}=1.15 \tag{5}$$

and

$$R_p=\frac{K_2}{K_p}=1.39 \tag{6}$$

where K_e and K_p are the desorption rate coefficients for ethylene and propane, respectively, and K_2 is the reaeration rate coefficient for oxygen. The tests by Rainwater and Holley confirmed these ratios.

The use of any gas tracer to simulate oxygen is predicated on the assumption that removal of the gas is by desorption only and that no other physical or chemical processes interfere. Experience (Holley and Yotsukura, 1984) indicates that ethylene gas may be subject to biodegradation. Ethylene, which is an unsaturated hydrocarbon, is more chemically reactive than saturated hydrocarbons such as propane. Also, hydrocarbons are known to be chemically reactive with halogens such as fluorine and chlorine, which may be found downstream from industrial or domestic sewage-treatment plants (Stedfast and Draper, 1986). Therefore, propane has become the more commonly used hydrocarbon gas in reaeration tests.

Temperature corrections

The reaeration coefficient is usually expressed as being at 20 °C (Elmore and West, 1961). Stream water is seldom at 20 °C, and field-determined desorption coefficients can be adjusted to 20 °C using the equation

$$K_{T_{20}}=K_{T_Y}(1.0241)^{(20-Y)} \tag{7}$$

where K_{T_Y} is the desorption coefficient as measured by a tracer gas at stream temperature Y. Similarly, the reaeration coefficient at 20 °C when propane is the tracer gas can be expressed as

$$K_{2_{20}}=1.39K_{P_Y}(1.0241)^{(20-Y)} \tag{8}$$

Gas desorption

All methods of measuring stream reaeration involve injecting a tracer gas into the streamflow and subsequently measuring its desorption as it is transported downstream. In effect, a nonconservative gas tracer is being purposely used to assess the ability of the stream to *desorb* the gas; this ability is then related to the capacity of the same stream reach to *absorb* oxygen. It may be assumed that the desorption of the tracer gas through the stream surface is a first-order process of the form

$$\frac{dC_g}{dt}=-K_TC_g \tag{9}$$

where C_g is the gas concentration, t is time, K_T is the tracer gas desorption rate coefficient, and dC_g/dt is the rate of change of the gas concentration as a result of desorption. The amount of desorption is usually calculated by measuring the loss of the gas tracer between two or more sections downstream from the injection. Equation 9 can be integrated and applied to a stream reach delineated by upstream and downstream measuring sections to give

$$\ln\frac{(C_g)_u}{(C_g)_d}=K_T(t_d-t_u) \text{ or } K_T=\frac{1}{(t_d-t_u)}\ln\frac{(C_g)_u}{(C_g)_d} \tag{10}$$

where the subscripts u and d refer to upstream and downstream.

Slug-injection method

The SI method actually involves a short-term continuous injection of the tracer gas into the flow. This "quasi-slug" injection of gas is in contrast to an *instantaneous* slug injection and is necessary to accomplish sufficient absorption of gas into the water for measurement purposes.

As seen in figures 1 and 2, downstream concentrations resulting from a slug injection of a gas tracer will be reduced by longitudinal dispersion, independent of the gas lost by desorption; they will be further reduced if there is flow accrual serving to further dilute the tracer cloud. To measure actual gas desorption, it is necessary to inject a conservative tracer and the gas tracer *concurrently so that they undergo identical dispersion and dilution.* Therefore, the dye tracer must be slug-injected in an identical manner as the gas, that is, in a short-term, continuous, "quasi-slug" injection. Thus the loss of the gas tracer relative to the conservative tracer is independent of dispersion and dilution effects. If the dye tracer is used as the conservative dispersion-dilution tracer, equation 10, with rearranging, can be expressed as

$$K_T=\frac{1}{t_d-t_u}\ln\frac{\left[\frac{C_g}{C_d}\right]_u}{\left[\frac{C_g}{C_d}\right]_d} \tag{11}$$

in which both the gas-tracer concentration and the dye-tracer concentration, C_d, are measured at a characteristic time on the upstream and downstream response curves. As shown in figure 4, the most commonly used characteristic time is that for peak concentration; hence, this procedure is often called the "peak method."

As has been discussed, rhodamine WT dye, which is about the most conservative dye tracer available, still experiences small losses. This can be resolved by adjusting the measured dye concentrations using the dye recovery ratios measured at each section such that for the "peak method"

$$K_T=\frac{1}{t_p}\ln\frac{\left[\frac{C_g}{C_p/R_r}\right]_u}{\left[\frac{C_g}{C_p/R_r}\right]_d} \tag{12}$$

Note that at both the upstream and downstream sections the measured dye concentration is made conservative by dividing by the respective recovery ratios.

If the dye-loss correction factor, J, is used

$$K_T=\frac{1}{t_p}\ln\frac{\left[\frac{C_g}{C_p}\right]_u J}{\left[\frac{C_g}{C_p}\right]_d} \tag{13}$$

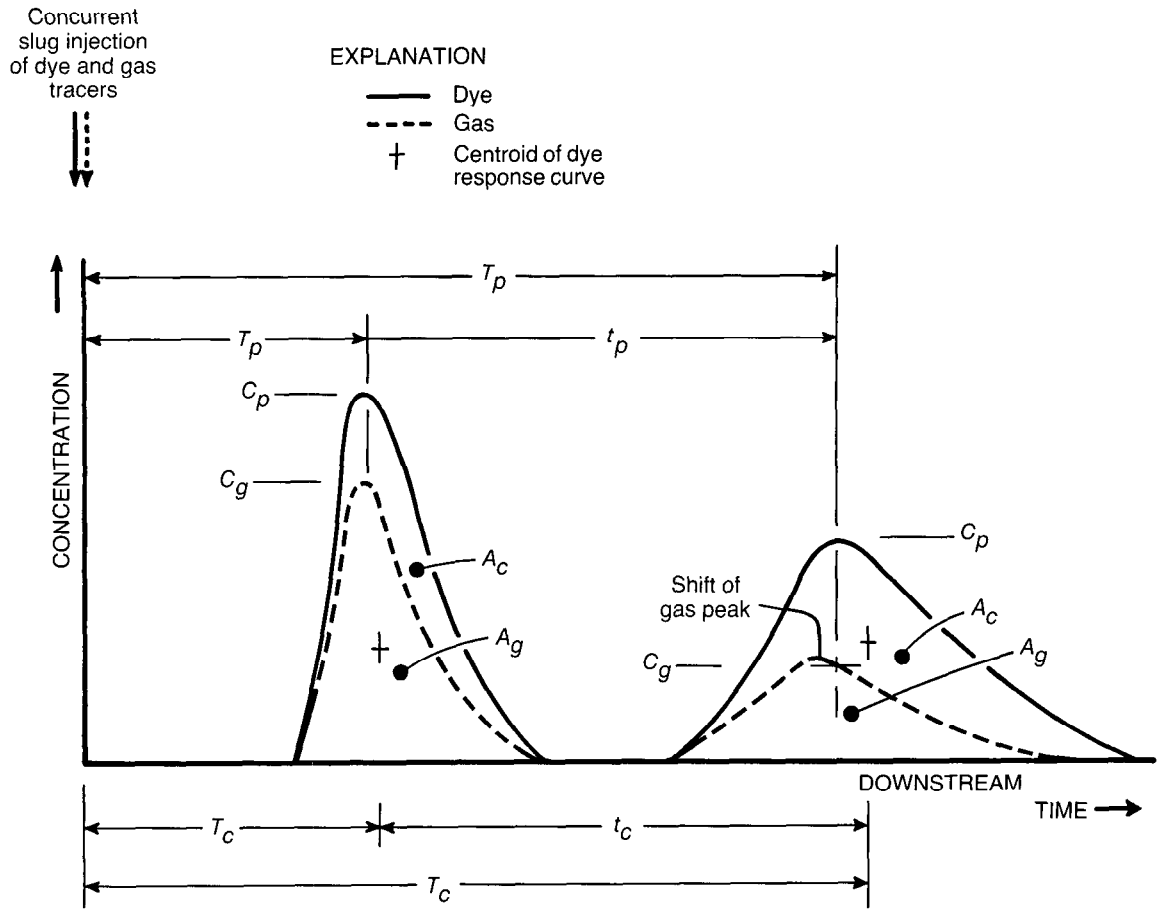


Figure 4.—Definition sketch of slug-injection type of reaeration measurement. Abbreviations are defined in text and in list of "Symbols and Units."

where t_p is the traveltime from the upstream to the downstream peaks of the dye-response curves, C_p is the observed peak dye concentration, and C_g is the gas concentration observed at the same time as the dye peak. It will be noted in figure 4 that the peak concentration of the downstream gas-response curve has shifted slightly with respect to the dye-response curve. This may occur where t_p is long and (or) K_T is large, the gas tracer being desorbed with increasing time so as to "erode" the gas peak. Hence, the gas concentrations to be used are those existing *at the time* corresponding to the dye-tracer peak and may not be the maximum gas concentrations. In most instances, though, the gas peaks will occur at the same time as the dye peaks.

An alternative to using peak dye concentrations and the corresponding gas concentrations is to determine the reduction in mass of the gas tracer over a selected test reach, frequently referred to as the "area method." Referring to figure 4, the gas desorption coefficient can be determined as

$$K_T=\frac{1}{t_c}\ln\frac{(A_g\,Q)_u}{(A_g\,Q)_d} \tag{14}$$

where t_c is the time of travel of the centroid of the dye-response curve between the upstream and downstream measuring sections and A_g and Q are the area of the gas-response curve and stream discharge at the upstream and downstream sections, respectively. Dye-tracer data are available since dye is injected to aid in sampling the gas cloud. Dye recovery computations are not needed since the entire gas mass is being measured at the two sections, rather than just selected concentrations as in the peak method. Although the area method avoids the requirement to evaluate dye-tracer recoveries accurately, it requires much more extensive gas sampling and hence is less often used. Note that even though the total mass of the gas tracer is being used, the time-of-travel measurement is from centroid to centroid *of the dye-tracer response curves*. The elapsed time to the centroid (T_c) of each dye-response curve is needed and is best calculated by taking moments about the dye-response curves with respect to $t=0$. The first moment to obtain T_c for any one curve can be expressed as

$$T_c = \frac{\int_{t=0}^{t=\infty} C_d t dt}{\int_{t=0}^{t=\infty} C_d dt} \quad (15)$$

where C_d is the observed dye concentration at any time, t. The dye-response curve is defined completely through its recession to the trailing edge, t_t. The computation can be performed by numerical integration. For a given test reach, t_c is equal to the difference between T_c for the upstream and downstream sections.

The areas of the gas-response curves, A_g, can be determined either by planimetering or by numerical integration, as was performed with the dye-concentration data. Information on a computer program that will perform area and centroid computations for either dye- or gas-concentration response curves is available from the U.S. Geological Survey Office of Surface Water in Reston, Va.

For clarity, the response curves in figure 4 are as observed along a common streamline at upstream and downstream sampling sections. Preferably, the response curves for the dye are measured at three or more points (streamlines) laterally at each section and gas samples are taken as necessary. The equations just presented deal with the entire flow; nevertheless, they could be just as easily applied along streamlines.

Constant-rate-injection method—One-dimensional dispersion state

As seen in figures 2 and 3, if a tracer is injected at a constant rate for a sufficient length of time, a plateau concentration will be established laterally (assuming the tracer has moved far enough downstream for adequate mixing). If stream discharge, Q, is constant, the resulting plateau concentration when a conservative tracer is used is expressed by the dilution discharge equation (Kilpatrick and Cobb, 1985) as

$$Q = q \frac{C}{\bar{c}} \quad (16)$$

where

q = rate of tracer injection, assumed to be very small relative to Q[2];

C = concentration of the tracer being injected; and

$\bar{c}$ = resulting plateau concentration after dilution by Q.

If there is an increase in stream discharge, the plateau concentration is reduced to reflect the added dilution. Unlike the SI method, if tracer injection is continuous and at a constant rate, and if mixing is complete, longitudinal dispersion will not affect the magnitude of the plateau concentration; in theory, a single sample for measurement of the plateau concentration is sufficient.

If a nonconservative gas tracer is injected continuously and at a constant rate, then desorption of the gas will be reflected in an ever-decreasing plateau concentration, $\bar{c}_g$, with distance downstream, as depicted in figure 5. If gas injection lasts long enough, longitudinal dispersion ceases to be a factor in determining concentration levels; only dilution by flow accrual, if any, will reduce plateau levels below the level caused by the desorption of the gas. Thus, an *approximate* solution for the gas desorption coefficient is

$$K_T = \frac{1}{t_c} \ln \frac{(\bar{c}_g Q)_u}{(\bar{c}_g Q)_d} \quad (17)$$

This equation is approximate as it is based (Yotsukura and others, 1983) on the assumption that the stream channel is straight and prismatic. The product, $\bar{c}_g Q$, may be thought of as the gas mass in transport at a section. In this instance, $\bar{c}_{g_u}$ and $\bar{c}_{g_d}$ are the weighted average plateau concentration of the gas at the upstream and downstream measuring sections, respectively. As was the case in the SI technique using the area method, t_c is the *time of travel of the centroid* between upstream and downstream measuring sections of a *dye*-tracer response cloud. These data are normally acquired by the instantaneous *slug* injection of the dye tracer into the streamflow at the same location and time as the continuous gas injection.

The centroid time for any one curve, T_c, can be calculated by taking moments of the *dye*-response curve with respect to $t = 0$ as defined by equation 15.

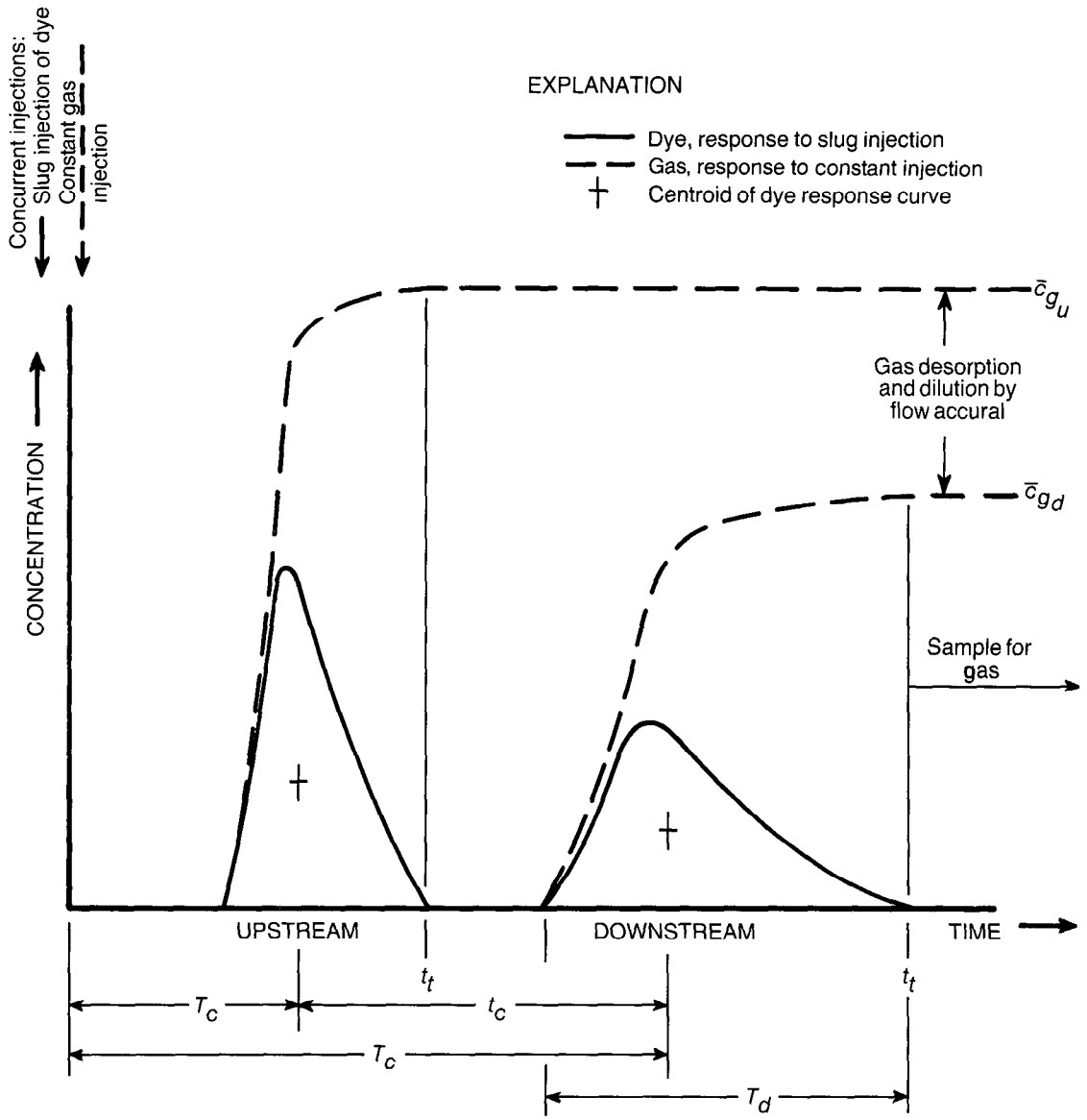
A more exact solution for the desorption coefficient is obtained by the trial solution of the equation

$$\frac{(\bar{c}_g Q)_u}{(\bar{c}_g Q)_d} = \frac{\left[\int_{t=0}^{t=\infty} f(C_d,t) \exp(-K_T t) dt\right]_u}{\left[\int_{t=0}^{t=\infty} f(C_d,t) \exp(-K_T t) dt\right]_d} \quad (18)$$

in which the left term is the ratio of the upstream gas mass in transport to the downstream gas mass in transport. The bracketed expressions on the right are the integration of the normalized dye-response curve, $f(C_d, t)$, and the first-order decay function with time for the upstream and downstream measuring sections. Normalizing a given set of dye time-concentration data is done by dividing by the area of that response curve such that

$$f(C_d, t) = C_d(t) / \int C_d dt \quad (19)$$

[2]For clarity, units are ignored here.



NOTE: T_d is minimum duration of constant gas injection to just produce a plateau at the downstream sampling section at the point of observation of the dye response curve.

t_t is earliest time at which gas sampling should begin.

Figure 5.—Definition sketch of constant-rate-injection type of reaeration measurement. Abbreviations are defined in text and in list of "Symbols and Units."

The solution of equation 18 involves numerical integration calculations employing a number of trial K_T values estimated from the value determined by equation 17. When the ratio on the right side of equation 18 agrees with the ratio of observed data on the left, the trial K_T is considered the correct desorption coefficient.

It is important to note that in the CRI method, dye losses are not a factor because normalized concentrations are used. Only a limited number of gas tracer samples need be collected on the plateaus at the measuring sections. Furthermore, the dye-tracer injection, while being made at the same location as the gas injection, need be only a simple instantaneous slug injection, in contrast to the SI method, an apparent contradiction. In fact, an instantaneous slug injection is the most desirable. It simplifies the measurement of traveltimes and, as previously described in the section on the "Superposition Principle," the duration of passage of the dye-response curve provides exact information on how long a gas injection must be, and its

time of final departure, t_t, provides the exact time to sample the gas-concentration plateau.

It should be noted that thus far the approach has been to evaluate dye and gas concentrations, response curve areas, and properties at each measuring section in total. This may be a satisfactory approach if the tracers are sampled far enough downstream that mixing is good and average or discharge-weighted values can be used with confidence. Referring to figure 1, in some instances, especially on wide streams, it may be necessary to calculate values along given streamlines. Thus, if sampling points b, d, and f are established to represent one-third of the total discharge in each section, say sections II and III in figure 1, traveltimes and moments might best be computed for a given streamline and then averaged for any final computation. For example, even between sections III and IV in figure 1, the time of travel along streamline f may be significantly different from time of travel along streamline d because of the skewed nature of the tracer-response curves.

Constant-rate-injection method—Two-dimensional dispersion state

Most gas-tracer reaeration tests are performed in relatively short test reaches and small streams, the former because the nonconservative gas tracer must be desorbed to accomplish the measurement. From a practical standpoint, only a small amount of the gas (such as propane or ethylene) initially injected into the flow is actually absorbed by it. This, coupled with the desired desorption, limits the length of reach that can be measured and the levels of gas concentration that can still be measured accurately.

Furthermore, most field tests have been on small streams, as it was desirable to perform the measurement in a reach where vertical and lateral mixing of the tracers was essentially complete and only longitudinal dispersion continued, such as between sections III and IV in figure 1. With increased stream size, particularly width, adequate lateral mixing cannot be obtained without increasing reach length, and this may result in gas concentrations too small to measure. In effect, for wide rivers, the distance required to achieve transverse uniformity may be longer than the study reach possible for gas-desorption measurements.

Nevertheless, there is a need for reaeration information on larger, wider streams. Yotsukura and others (1984) have expanded the application of the gas-tracer reaeration measurement technique by developing the theory and techniques of measuring gas desorption in the nonuniform plume region such as between sections I and II in figure 1. Although this technique is new and its performance in the field is demanding, it is presented in this manual to encourage its use and further development.

The CRI method applied to wide rivers uses an *instantaneous slug injection* of conservative dye tracer at the beginning of a *continuous injection* of gas tracer at some fixed source location in a cross section. This injection section is located upstream from an upstream and a downstream measuring section which constitute the test reach. At several transverse locations in the measurement cross sections, measurements are made for only dye concentration while the dye cloud is passing and then measurements are made for gas concentration, as was described previously when applying the CRI method to narrow streams. The primary difference is that now measurements *must* be made between cross sections *along streamlines* (see fig. 1). For example, injection of the tracers might be at the 50-percent streamline, d (the source streamline), and sampling at the 33.3-, 50-, and 66.7-percent streamlines, at sections I and II (or, preferably, at sections II and III, as depicted in figure 1).

The logistics of performing a continuous injection in the center of a wide channel may dictate that the tracer injection take place closer to one streambank or the other, say streamline c or e in figure 1. Tracer sampling would then be performed downstream on the injection streamline and, if desired, at one or more parallel streamlines. Obviously, using the CRI method in a wide stream means that the desorption characteristics of the stream are being measured in only a portion of the flow.

In figure 5, the reduction in plateau gas concentrations from an upstream to a downstream measuring section could be attributed entirely to gas desorption and flow accrual if the measurement reach were far enough downstream from the injection for adequate mixing, for example, section III to IV in figure 1. At short distances, it can be readily visualized from figure 1 that the reduction of *both* gas concentrations and dye-tracer concentrations would be further reduced by *lateral* mixing and tracer transport.

An equation similar to equations 17 and 18 that applies the CRI method to the two-dimensional nonuniform plume situation and to measurement along a given streamline, z, is

$$\left[\frac{(\bar{c}_g Q)_u \phi_d}{(\bar{c}_g Q)_d \phi_u}\right]_z = \left[\frac{\int_{t=0}^{t=\infty} f(C_d,t)\exp(-K_T t)dt_u}{\int_{t=0}^{t=\infty} f(C_d,t)\exp(-K_T t)dt_d}\right]_z \quad (20)$$

All terms apply along the streamline, z, chosen for the measurement. Hence, it becomes vital that preliminary discharge measurements be made at both upstream and downstream measuring sections as well as at the section upstream chosen for the injections. From plots of cumulative discharge versus lateral distance, common streamlines are chosen for the measurements. Plateau gas concentrations, $\bar{c}_{g_u}$ and $\bar{c}_{g_d}$, are those applicable to the upstream and downstream sections *along the same streamline.*

In equation 20, ϕ_z is a mass distribution factor to account for the change in the nonuniform, cross-sectional distribution of tracer masses passing each measuring section. For a given location on a streamline, it is defined as

$$\phi_z = \frac{A_{c_z}}{\Sigma A_{c_z} \frac{\Delta Q_z}{Q}} \tag{21}$$

The computation is made for a given streamline for each dye-tracer response curve for both upstream and downstream sections. The area of each response curve, A_{c_z}, should be for the slug-response curve carried to background concentration; ΔQ_z is the portion of the total discharge applicable to the chosen streamline. The more uniform in area the tracer response curves are laterally, the more nearly ϕ approaches 1.00.

The data requirements to compute ϕ_z by equation 21 for a wide stream may be extensive. On a wide stream, the complete dye-response curves at 7 to 10 points laterally at both upstream and downstream sections would be required. An alternative to collecting such extensive dye data is to determine ϕ based on the steady-state gas plateau concentrations, $\bar{c}_{g_z}$, measured laterally at each streamline; replacing A_{c_z} in equation 21 by the plateau gas concentrations gives

$$\phi_z = \frac{\bar{c}_{g_z}}{\Sigma \bar{c}_{g_z} \frac{\Delta Q_z}{Q}} \tag{22}$$

If possible, the gas injection should be sufficient to yield measurable concentrations near the streambanks, where they might be expected to be lowest. Thus, maximum gas-injection rates should be employed and measuring sections chosen carefully to minimize zero gas concentrations along the streambanks.

A second alternative for determining ϕ is to perform a continuous dye injection long enough to establish steady-state plateau concentrations and to use these data in place of $\bar{c}_{g_z}$ data in equation 22. This is the least preferred alternative, as it is difficult to sustain an accurate constant rate of dye injection because as much as a day or more may be required. This approach would, however, be the preferred method on very wide streams where zero or very low gas-tracer concentrations might be expected along one or both banks.

Regardless, complete dye slug response curves must still be measured along the streamline(s) *chosen for the measurement.* Of the choices and in most cases, the following is recommended: (1) a slug injection of dye tracer at a point on a source streamline at the injection section, with complete sampling of the response curves on the same streamline and on the two adjacent streamlines at the two downstream sections; (2) concurrent with or within a short time following, continuous gas injection at the same point on the same source streamline followed by gas sampling *on all streamlines once steady-state gas concentrations have been reached.*

As with equation 18, equation 20 is solved by trial after estimating K_T from equation 17.

General case

It should be kept in mind that equation 20 is the general equation applicable to the two-dimensional situation, whether on a narrow or a wide stream. Thus, if circumstances prohibit selection of a test reach on a narrow stream in which mixing is complete throughout the test reach, equation 20 can be used as long as the necessary data are acquired to evaluate ϕ. This may in some instances permit a lengthening of the test reach (by moving the upstream section closer to the injection point) such that $K_T t_c$ is equal to or greater than 1.00. The reason for this is discussed later. If in fact there is any doubt about the adequacy of mixing, data should be collected to compute ϕ such that equation 20 may be used.

Planning the Reaeration Study

The most important consideration in planning a reaeration test is to select a test period when flow conditions are close to those ultimately to be modeled. This will probably be a period of light wind and steady discharge. If regionalization is an objective, a certain flow duration, or maybe two or more flow durations, will probably be desirable.