Techniques of Water-Resources Investigations
of the United States Geological Survey

Chapter A15

COMPUTATION OF WATER-SURFACE PROFILES IN OPEN CHANNELS

By Jacob Davidian

Book 3
APPLICATIONS OF HYDRAULICS

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PREFACE

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CONTENTS

FIGURES

UNIT CONVERSION

Multiply inch-pound unit	By	To obtain SI unit
ft (foot)	3.048×10^{-1}	m (meter)
mi (mile)	1.609	km (kilometer)
ft^3/s (cubic foot per second)	0.028	m^3/s (cubic meter per second)

SYMBOLS AND UNITS

Symbol	Definition	Unit
A	Area of cross section	ft^2
A	Drainage area of basin	ft^2, mi^2
a	Area of subsection	ft^2
B	Subscript denoting bypass channel	
C	Chezy discharge coefficient	$ft^{1/2}/s$
C	Subscript referring to a culvert	
C	Type of backwater profile for critical-flow conditions	
c	Subscript denoting composite section	
D_{max}	Maximum depth of flow in cross section	ft
d	Subscript denoting downstream cross section	
d_b	Depth of flow on flood plain	ft
d_m	Mean depth	ft
d_{50}	Median diameter of bed material	ft
F	Froude number	
FRDN	Index Froude number	
f	Darcy-Weisbach resistance coefficient	
g	Gravitation constant (acceleration)	ft/s^2
h	Static or piezometric head above an arbitrary datum	ft
h_e	Energy loss due to channel expansion or contraction	ft
h_f	Head loss due to friction	ft
h_v	Velocity head at a section	ft
i	Subscript referring to individual subsection	
K	Conveyance of a section	ft^3/s
K_q	Conveyance of the subsection containing the discharge that is not contracted to enter a single contracted opening	ft^3/s
k	Part of the total conveyance	ft^3/s
k	Coefficient for energy loss	
L	Length of reach	ft
L	Distance from left edge of water to a point in the cross section	ft
L	Subscript denoting left subsection or channel	
L	Subscript denoting the subsection having the largest conveyance	
M	Subscript denoting middle channel or main channel	
M	Type of backwater profile for subcritical-flow conditions	
n	Manning roughness coefficient	$ft^{1/6}$
P	Wetted perimeter of cross section of flow	ft
Q	Total discharge	ft^3/s
q	Part of the total discharge	ft^3/s
R	Hydraulic radius	ft
R	Distance from right edge of water to a point in the cross section	ft
R	Subscript denoting right subsection or channel	
R	Subscript referring to road embankment	

Symbol	Definition	Unit
S	Water-surface slope	
S	Subscript denoting a surcharge	
S	Type of backwater profile for supercritical-flow conditions	
S_0	Channel bed slope	
T	Width of a section at the water surface	ft
T	Subscript referring to total quantity for cross section	
t	Subscript referring to a tributary stream	
u	Subscript denoting upstream cross section	
V	Mean velocity of flow in a section	ft/s
v	Velocity in a subsection	ft/s
W	Length of an embankment between bridge openings	ft
y	Depth of flow	ft
y_c	Critical depth of flow	ft
y_n	Normal depth of flow	ft
1,2,3,4	Subscripts which denote relative order of cross sections or section properties; or which denote subsections of a cross section	
α	Velocity-head coefficient	
Δ	Difference in values, as Δh is the difference in head	
θ	Central angle at bed of triangular cross section	
Σ	Summation of values	
ϕ	Channel slope angle	
$=$	Equal to	
$\neq$	Not equal to	
$\cong$	Approximately equal to	
$>$	Greater than	
$<$	Less than	
$\int$	Integral	

COMPUTATION OF WATER-SURFACE PROFILES IN OPEN CHANNELS

By Jacob Davidian

Abstract

The standard step-backwater method of computing water-surface profiles is described in this chapter. The hydraulic principles and assumptions are reviewed, and the field data requirements are described. Certain special cases of backwater curves and certain special field conditions are discussed in detail. The technique is used to establish or extend stage-discharge ratings; to define areas which will be innundated by flood flows of a given frequency; and to compute profiles through various reaches, including multichannel flows, and past control structures such as bridges, culverts, and road embankments. A brief description of analysis of floodways and effects of encroachments is also presented.

Introduction

Water-surface profiles along stream channels can be computed quickly when electronic computers are applied to the commonly used step-backwater method. The method requires the evaluation of the energy losses between any two points on the water-surface profile.

Water-surface profile computations by the step-backwater method are a major part of most studies leading to the delineation of flood plains in urban and suburban areas. Flood plains must be delineated before they can be properly zoned to reduce flood damages.

The method is also applied to floodway analysis. Given a stream profile for a flood of a certain magnitude, a surcharge, or increase in stage, is chosen for the entire profile. Then the encroachments on the flood plains are determined by the step-backwater method such that the total stream conveyance remains unchanged. This application is useful in planning and in flood-insurance studies.

The method is also used in establishing or extending stage-discharge relations at gaging stations or at other sites along a stream. This information is valuable in the design of structures.

A survey of the geometry of the stream channel along the reach for which the profiles are needed produces the data required—principally, cross sections of the channel drawn to a common datum at intervals along the reach and the value of the roughness coefficient. From these data, the water-surface profile for any known or assumed discharge may be computed.

This manual describes the standard step-backwater method of computing water-surface profiles and the channel geometry data needed. Certain sources of error and some special field conditions are discussed in detail. Frequent reference is made to computer program E431 of the U.S. Geological Survey. Because computer programs are frequently changed, details of that program are not discussed here. Program E431 is described by Shearman (1976) in detail, with respect to data handling and preparation, computed results, error messages, and assumptions.

Hydraulic Principles

Steady, uniform flow

Almost all open-channel flows are both unsteady (depth at a point varies with time) and nonuniform (depth changes from point to point along a channel). Because these flows are difficult to analyze, a steady, uniform flow is frequently assumed. The assumption of steadiness is justified by the fact that at peak flows the discharge hydrograph flattens out and flow approximates steady conditions. High-water marks are left along the channel by these relatively steady, peak flows. Uniformity of flow is achieved by dividing channels into shorter lengths that are considered reasonably uniform between cross sections, both on the basis of high-water marks and on channel geometry.

When a steady discharge flows in a long channel of uniform hydraulic characteristics, it flows at a constant depth, called the normal depth. But adjacent subreaches are not identical in channel dimensions, roughness, or bed slope, and so the normal depth in each is different. A natural water-surface profile, therefore, is a series of curves relating the normal depth in one subreach to the normal depth in the next. The normal depth line in a long stretch of river is, thus, rarely a long, smooth curve.

Backwater curves

The water-surface profiles resulting from nonuniform-flow conditions are called backwater curves. Various types of such profiles for gradually varied flows are described by Chow (1959) and Woodward and Posey (1941). Of the many backwater curves possible, those for subcritical flows on mild slopes (fig. 1), and those for supercritical flows on steep slopes (fig. 2), are generally of most concern. In figures 1 and 2, the dashed lines represent the critical depth, y_c, and the straight, solid lines represent the normal depth, y_n. The heavy, solid, curves lines represent the water-surface profiles from a control point to the normal-depth profile. (The expression "control point" or "control section" is defined as any section at which the depth of flow is known or can be controlled to a required stage.)

Curve M1 in figure 1*A* could be the result of a dam or a constriction downstream, or it could represent the backwater in a tributary which is flowing into a flooding main stream. Curve M2 could result from a dropoff (falls or riffles) farther downstream where the water surface drops below the critical-depth line. The flow then passes into the supercritical regime. For both the M1 and M2 curves, the control is

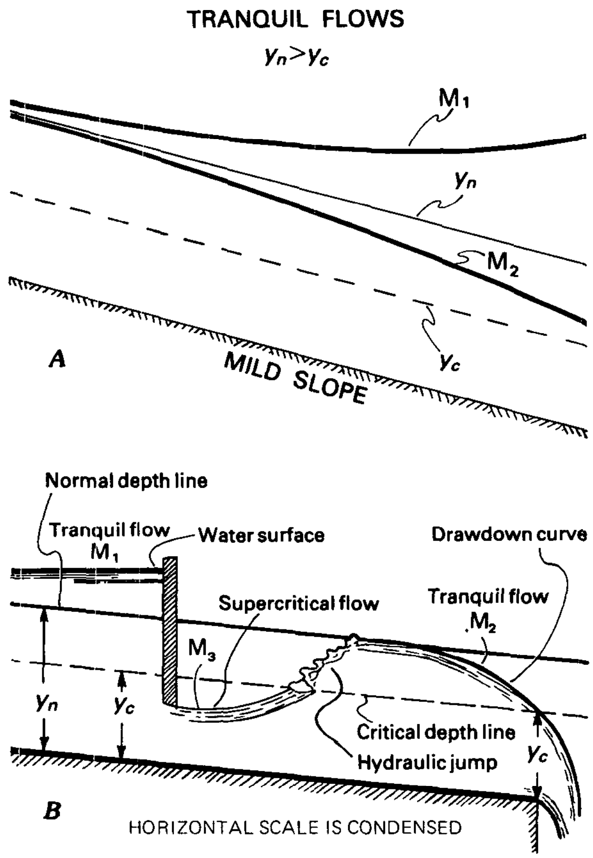

Figure 1.—Water-surface profiles on mild slopes. *A*, Tranquil flows, showing relations among M1, y_n, and y_c curves; *B*, Sketch showing typical instances of M curves.

downstream; if the channel were long enough, both curves would asymptotically approach the normal-depth line upstream. Typical examples M curves are shown in figure 1*B*.

In figure 2*A*, curve S3 could represent the profile of flow downstream from a sluice gate. Curve S2 could be the profile of flow which has just passed into the critical regime from a milder slope upstream, as might occur at a riffle. The control point for both these curves is upstream; if the channel were long enough, both curves would asymptotically converge toward the normal-depth line downstream. Typical examples of the S curves are shown in figure 2*B*.

Examination of figures 1*A* and 2*A* will show that the lowest possible M1 curve and the highest possible M2 curve will each coincide with the normal-depth curve, which represents

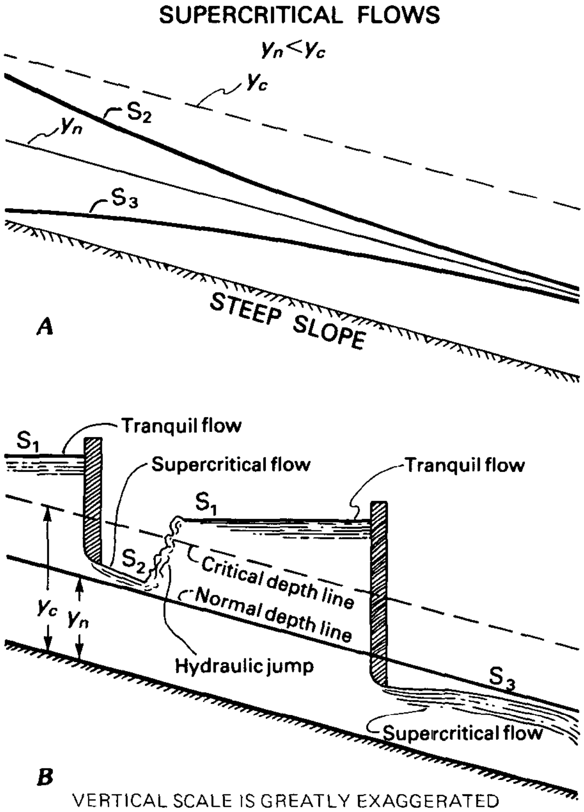

Figure 2.—Water-surface profiles on steep slopes. *A*, Supercritical flows, showing relations among S2, S3, y_n and y_c curves; *B*, Sketch showing typical instances of S curves.

the no-backwater condition on the mild slope; and that the lowest S2 and highest S3 curves will each similarly coincide with the no-backwater, normal-depth line on the steep slope.

Because most natural streamflow is subcritical, with only occasional stretches of supercritical-flow conditions, this manual emphasizes M1 and M2 curves, refers to the S2 and S3 curves where appropriate, and briefly mentions the rare M3 and S1 curves.

Water-surface elevations along M1 and M2 curves should be computed only in an upstream direction, away from a control, to ensure that successive points on the computed curve will asymptotically approach, or converge toward, the normal-depth line. Similarly, water-surface elevations along S2 and S3 curves should be computed only in a downstream direction, away from a control, to ensure that the computed curves will converge toward the normal-depth line. Computations in the wrong direction will define profiles that diverge from the normal-depth line, and they are, therefore, erroneous. It is possible, however, to make a few computations in the wrong direction without introducing large errors if the velocity head is small. The small errors locally introduced and reflected in the computed profiles in such instances are quickly dissipated if further computations of the water-surface profiles are in the correct direction for the remainder of the reach being investigated.

It should be noted that the description of a reach as mild or steep, or as subcritical, or critical, or supercritical refers not to the numerical value of the slope of the reach, but, rather, to the regime of the discharge flowing through the reach. Whether the numerical value of the slope is small or large, a reach could be considered "mild" because of the subcritical flow it carries, but it could later be considered "steep" because a different discharge is supercritical during the passage of a flood wave through the reach.

Transition curves

Figures 3–8 show some backwater transition curves for flow passing from one reach into another. In figure 3, the normal-depth lines are at different depths; the M1 curve of the mild slope, therefore, smoothly joins the normal-depth line of the milder slope.

In figure 4, a change in roughness (n) on a mild slope will result in different normal depths. The transition curve is, therefore, very similar to that of figure 3.

In figures 5 and 6, the normal depths on mild slopes are high in the upstream reach for a very mild slope or a very high roughness, and they are low in the downstream reach for a steeper slope or a lower roughness. The water-surface profile in the transition is, therefore, an M2 curve which smoothly joins the downstream normal-depth line.

Figure 7 shows a break in channel bed slope from mild to steep. The tranquil flow upstream from the control point has a normal depth higher than the critical depth; the supercritical flow downstream from the control point has a

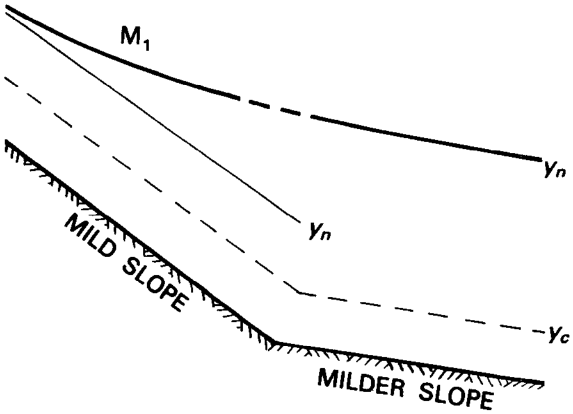

Figure 3.—Backwater-curve transition for mild slope to milder slope.

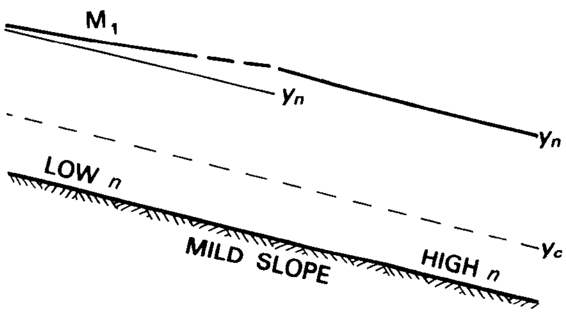

Figure 4.—Backwater-curve transition from small n to large n on mild slope.

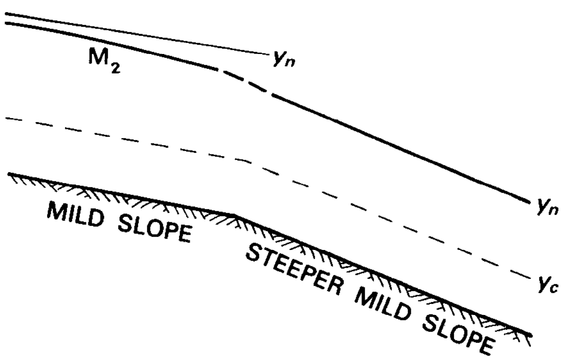

Figure 5.—Backwater-curve transition for mild slope to steeper mild slope.

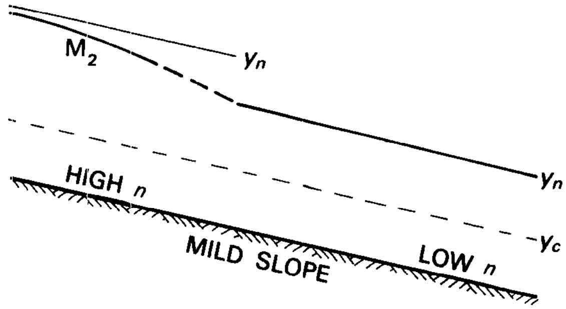

Figure 6.—Backwater-curve transition from large n to small n on mild slope.

normal depth lower than the critical depth. The water-surface profile must, in the transition between the two normal-depth lines, pass through critical depth by means of M2 and S2 curves. Such a transition is known as a draw-down curve or a hydraulic drop.

The transition from a steep slope ($y_n < y_c$) to a mild slope ($y_n > y_c$) in figure 8 requires a hydraulic jump in order for the water surface to pass through critical depth. The location of the jump depends on the relative elevations between the upstream and downstream normal depths. Associated with the hydraulic jump could be a short segment of an S1 or an M3 curve, which is not ordinarily of concern because of its proximity to the jump.

Determination of normal depth

Average profile in a long reach

The simple transitions described in figures 3-8 are only a few of many possible combinations that could result from changes in channel

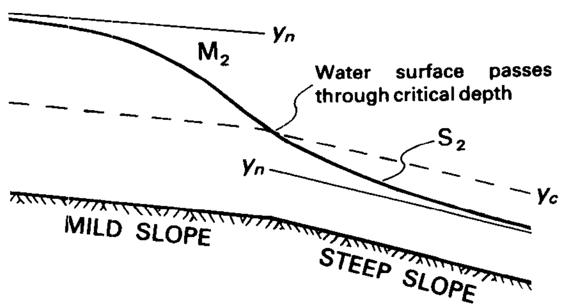

Figure 7.—Backwater-curve transition for mild slope to steep slope.

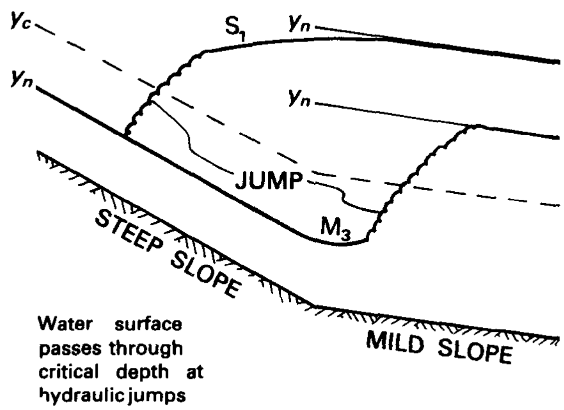

Figure 8.—Backwater-curve transitions for steep slope to mild slope.

characteristics such as slopes, roughnesses, widths, depths, or any combination of these between adjacent subreaches in a long reach. Obviously, in a long stretch of river that has been divided into shorter subreaches of "uniform" characteristics, the water-surface profile is a series of transition curves from the normal-depth line in one subreach to the normal-depth line in the adjacent subreach. Unless there are radical changes in characteristics, as for example at control points and hydraulic jumps, one could speak of the uniform-flow profile in a long reach as an average of the numerous transition curves that are reflecting local nonuniformities in the channel.

The normal depth line is determined using the backwater curves for channels of mild or steep slopes.

Use of M1 and M2 curves

A characteristic of the tranquil-flow M1 and M2 backwater curves is that one can start at any point on either of them and, by solving the energy equation, determine the elevation of the water surface at another point farther upstream. The intervening geometry and roughness, as well as the discharge, have to be known. As may be seen in figure 1*A*, if the procedure were carried far enough upstream in increments, this step-by-step computed profile would asymptotically approach the normal-depth line. This characteristic of the backwater curves is used in determining normal depth in a channel of mild slope.

Computed profiles that start with an elevation higher than normal depth would be M1 curves; those starting with elevations lower than normal depth would be M2 curves. When computed values of water-surface elevations along two profiles converge mathematically, it is generally assumed that the normal-depth profile has been reached. Computations along either profile continued farther upstream would be identical, regardless of whether the profiles had been two M1's, two M2's, or one of each, at the start of computations at the downstream end of the reach. Upstream from the point of convergence, the computed profile would define a locus of normal depth at each cross section. It would be the expected profile because the nonuniformities in the channel geometry and roughness would be translated to a series of minor, transitional backwater curves.

Determination of normal depth for subcritical flow in a channel by the procedure described above thus has two distinct phases. First, two starting backwater curves are assumed to exist in the channel, caused by different, assumed, control conditions downstream. Normal depth at a point (gaging station, bridge, or other place of interest) is determined when the two profiles have converged. The two computed profiles up to this point are imaginary and become useless after serving as a ploy for determining the normal-depth profile. The second phase begins with the point of convergence. All subsequent computations of the profile farther upstream represent the expected, or normal, water-surface elevation in this channel, to be used for inundation studies, flood-insurance studies, bridge-backwater studies, and so on.

Use of S2 and S3 curves

To determine normal depth in a steep channel, the supercritical-flow S2 and S3 backwater curves are used. The procedures correspond to those described for M1 and M2 curves, but in the downstream direction. Starting at any point on the S2 or S3 curve, the elevation of the water surface at any point farther downstream can be determined by solving the energy equation. The intervening geometry and roughness, as well as the discharge, have to be known. As may be seen in figure 2*A*, if the procedure is carried out far enough downstream, in increments, this step-by-step computed profile asymptotically approaches the normal-depth line.

Local effects on profiles

The importance of the effects of local channel nonuniformities on the computed profiles is a relative matter. In the first phase of computations along an M1 or an M2 curve, the profile usually has more slope and the effect of a local disturbance will show in the profile at that point, but it will probably diminish within a short distance upstream. For example, a bridge in the channel through which an M1 or an M2 curve is being computed will cause a local jump in computed elevations as in figure 9. An M1 curve will continue along another, higher M1 curve. An M2 curve could jump up to an M1 curve, or it could continue along another, higher M2 curve. The effect of the bridge is quickly dissipated.

If, however, the channel bed has a very small slope or if the bridge is located in the reach where backwater curves have already converged and for which the expected water surfaces are being computed, the local effects will be reflected farther upstream. In such instances, the effects of several bridges in a series would be additive. The resultant M1 curve could require a long distance before converging with the average normal-depth line for the channel.

All M1 curves where the computed profile or the channel bed have very little slope should be examined and interpreted with care.

Convergence of backwater curves

There are several factors to consider in the technique of using converging backwater curves to determine normal depth. The work "converge" is not the proper word to use in describing the relation of a backwater curve to the normal-depth profile. Backwater curves approach the normal-depth profile asymptotically, and the relation between two M1 curves or two M2 curves is an asymptotic convergence. During computation for two adjacent profiles, results may be identical or within an allowable difference or tolerance. Then the profiles are considered to have converged for all practical purposes.

If an M1 and an M2 curve are used as a pair and if the two profiles converge, normal depth is assured. It is not always feasible, however, to work with this ideal pair. An M1 curve lies above the normal-depth line (see figure 1*A*). Cross-sectional properties, therefore, must be available for elevations higher than normal depth at all of the cross sections in the reach. To calculate such properties is frequently impossible, particularly for large discharges where normal depths may be near bankfull stage; there simply may not be any ground points available above bankfull stages to which the cross sections could be vertically extended. This problem is more acute, of course, for the most downstream cross sections, where the M1 curve would be at its highest elevations with reference to the normal-depth line.

Another characteristic of the M1 curve is that it requires a longer reach to converge with the normal-depth line than does an M2 curve, both having started an equal vertical distance from the normal depth at the starting cross section downstream. A longer reach of channel must, therefore, be surveyed whenever an M1 curve is used and an even longer one if a pair of them are used.

The use of two M2 curves to determine normal depth is just as effective, but there are a few precautions to consider. The starting elevations preferably should be a foot or more apart. They should not be near the channel bed because the length of reach required before the backwater curve converges with the normal-depth line will be longer. The starting elevation of an M2 curve to be used for convergence purposes should never be taken below the critical-depth elevation. The starting elevation for computing an M2 profile upstream from a control would, on the other hand, have to start at critical depth. The difference in this instance is that the computations are defining the actual water surface in the reach, rather than a profile to be used to converge with the normal-depth profile.

Convergence of any two or more M1 curves or any two or more M2 curves is no guarantee that normal depth has been reached. Although they may seem to converge mathematically,

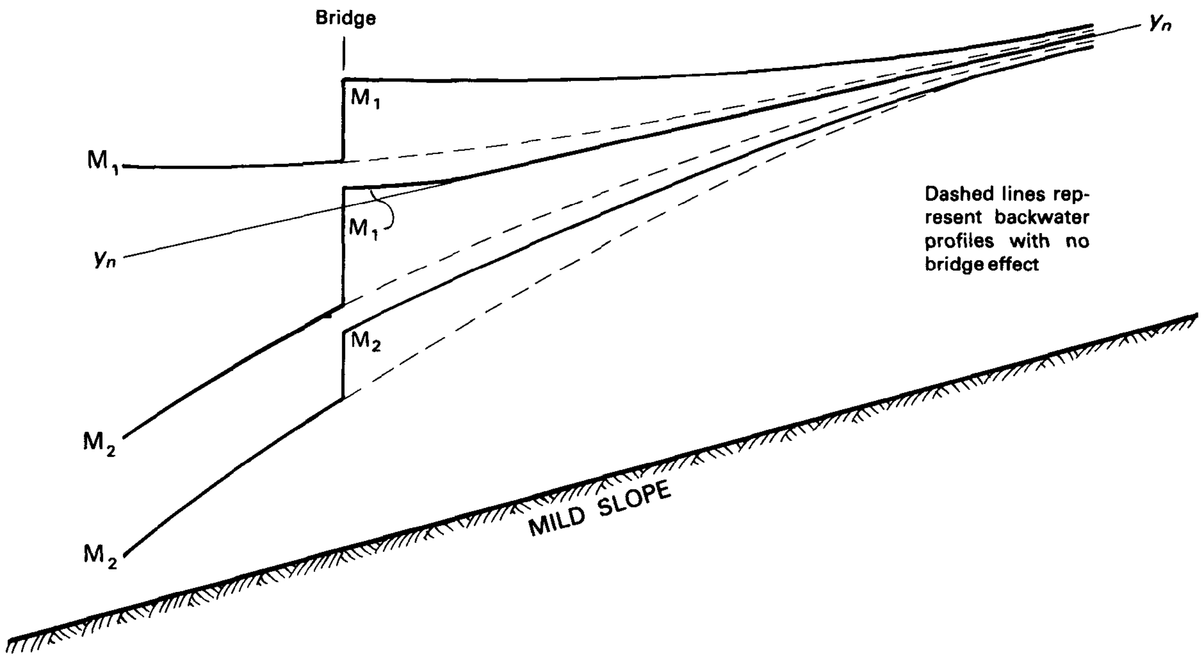

Figure 9.—Local effect of a bridge on an M1 or an M2 backwater curve.

the true normal-depth profile could still be some distance away in the vertical. For example, in figure 10, several M1 curves have apparently converged, and several M2 curves have apparently converged. None of them, however, have reached the normal-depth profile. Regardless of how many M1 curves have converged, therefore, the one starting with the lowest elevation is the most nearly correct. Similarly, the M2 curve starting with the highest elevation is the most nearly correct of the M2 curves. Figure 10 illustrates graphically that when an M1 and an M2 curve converge, the normal-depth profile has been reached. The starting elevations of all M1 and M2 curves should preferably be within the range 0.75–1.25 of the estimated normal depth.

Special Cases of Backwater Curves

Flows on very small slopes

Sometimes a stream will have so small a slope that the rate of convergence between two M2 curves is negligible. Two curves starting a foot apart might, after 10,000–20,000 feet, still be 0.75 foot apart at the upper end of the reach, and there may be no more stream channel available for extending the reach downstream; for example, the stream might flow into a lake or large river.

In such a case, a technique that can produce satisfactory results is the artificial extension of the reach. First, the average streambed slope is determined from a plot of the thalweg and extended downstream on the plot. An average cross-sectional shape can then be determined. This may be done by superposing all of the cross sections on a plot and averaging them, not only as to geometric shape, but as to roughness values as well.

Extension of the reach length and placement of an average cross section at a few intervals along it, should provide enough length for two M2 backwater curves to converge downstream from the reach under study. If a suitable computer program is used, an extended reach could be as long as 2,000 miles (Shearman, 1976) to try for convergence.

The procedure described might not prove satisfactory in streams having extremely small slopes. It should be kept in mind that the nor-

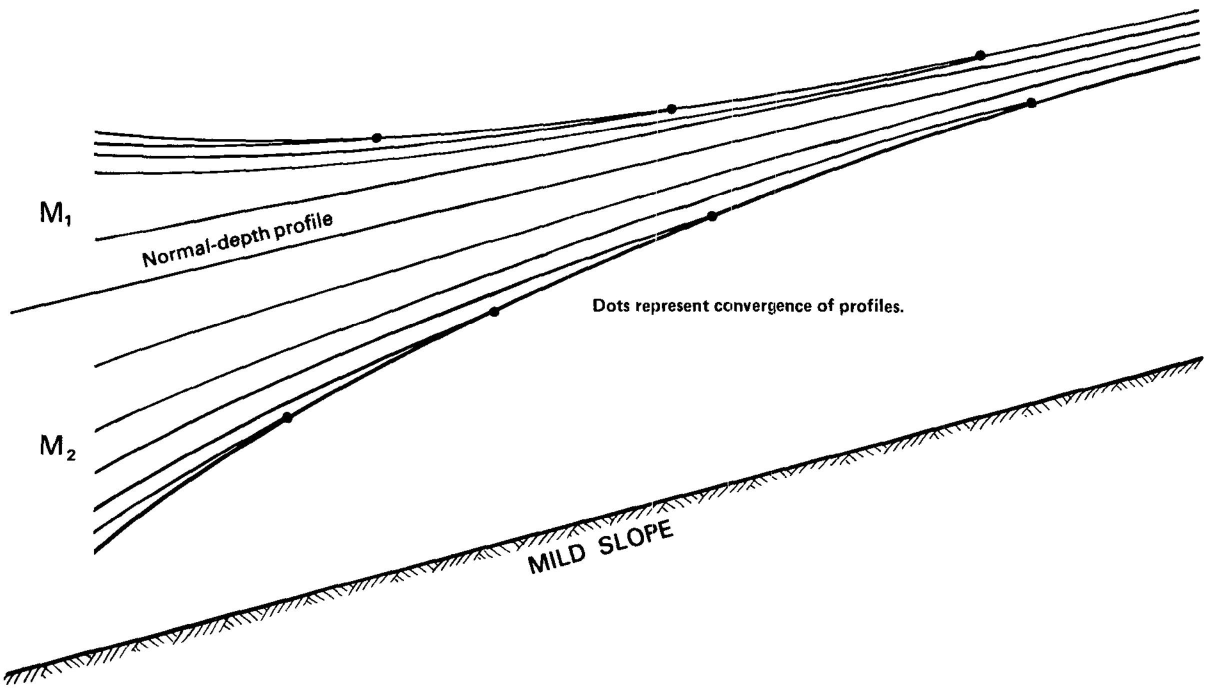

Figure 10.—Sketch of a family of M1 and M2 backwater curves. Dots indicate points of convergence.

mal depth for a slope of zero is infinity. If the slope is extremely small, even a 2,000-mile artificial extension of channel might prove inadequate in length, or the channel cross sections might not extend high enough to contain a flow that requires a very large normal depth. Flow could spill out over banks into adjacent drainages such that large discharges would have no distinguishable channels in which normal depths could be computed.

The use of an artificially extended reach could force the use of negative numbers, not only for section reference distances, but for elevations as well. Most computer programs will handle negative reference distances. But, to avoid the use of negative numbers and to lessen the likelihood of errors resulting from them, the datum can be adjusted by several hundred, or a thousand feet, throughout the entire reach.

A factor to consider in interpreting computed profiles on very small flat slopes is that relatively minor local departures from the general uniformity of the reach, both with respect to cross-sectional geometry and roughness and to bed slope, will affect the profile markedly. The local effects will extend far upstream (farther the flatter the slope), and they will more likely be additive than compensatory. In contrast, local disturbances in a reach that has a steeper bed slope but is otherwise the same will not affect the profile as far upstream.

Flows on Steep Slopes

Water-surface profiles on slopes, where supercritical flows are possible, should be carefully examined. Not only is it possible to get a variety of incorrect profiles, but, if the water surface breaks, the usual computations become inapplicable and no solution is possible. Sections of a stream that are likely to cause problems are at falls and riffles and at places where chute flows and hydraulic jumps are likely. A convex break in bed slope, where the slope is mild or flat upstream and steeper downstream, is particularly suspect, as are extreme contractions of area, owing either to smaller widths, to smaller depths, or to a combination of smaller widths and depths. Figure 11 shows several possible water-surface profile transitions involving critical or supercritical flow. These are the most common types. Other examples of flow profiles and transition curves at control sections are shown by Chow (1959, p. 229-236).

Locus of critical-depth stages

When critical-flow conditions are suspected for a given discharge in a reach, it is advisable to compute the critical depth at each cross section in the general vicinity and to connect these points on a plot to show the locus of critical-flow stages through that part of the reach. The procedure to follow is straightforward.

At each cross section investigated, let the Froude number, **F**, equal 1.0:

$$\mathbf{F}=1=V\sqrt{\alpha}\,/\,\sqrt{g d_m \cos\phi}\,.$$

The value of $\cos\phi$, where ϕ represents the channel slope angle, is generally close enough to unity to be ignored. Let velocity be equal to discharge, Φ, divided by total area, A, and let mean depth, d_m, be equal to total area divided by T, the top width. The velocity-head coefficient is represented by α and g is the gravitation constant in feet per second per second. Then,

$$\mathbf{F}=1=\frac{\sqrt{\alpha}\;Q/A}{\sqrt{g}\;\sqrt{(A/T)}}\,.$$

Now solve for Q:

$$Q=\sqrt{g}\;A^{3/2}/(\alpha T)^{1/2}.$$

Find the elevation at each cross section for which this holds true.

The computations are greatly simplified by the fact that cross-section properties, including area, velocity-head coefficient, and top width, are tabulated by computer at many elevations for each cross section. The Geological Survey computer program (Shearman, 1976) also prints out the value of $\sqrt{g}\,A^{3/2}/(\alpha T)^{1/2}$ for each of these elevations. This quantity is equivalent to that discharge whose water surface would be at the indicated elevation if the flow were critical. A simple plot of elevation versus discharge represented by the quantity $\sqrt{g}\,A^{3/2}/(\alpha T)^{1/2}$ can be prepared by the engineer quickly for each cross section at which supercritical-flow problems are suspected, as illustrated in figure 12. From elevations for selected discharges the profiles can be plotted.

Control sections

Examination of a plot of bed profile and of the loci of critical-depth points for several cross sections will generally reveal a section which can be taken to be the control—a convex break in slope, with a steep downstream leg and a flatter upstream leg. The control is at that point at which continuous computations upstream for the subcritical flow reach must begin again. In figures 11*A*-11*C*, profile computations in the upstream direction along the downstream mild slope could be invalid and should be halted in the steep-sloped reach where flow is supercritical. The control point for the upstream mild reach would be identified as the junction between the upper mild slope and the steep slope, where the M2 curve meets the critical-depth line. The step-backwater computations of the water-surface profile could be started again at this control point and continued upstream for as long as the upstream reach remains at a mild slope. The starting elevation for the new computation would be the critical-depth elevation at the control cross section, and the M2 profile computed would represent the water surface, which will ultimately coincide with the normal—depth line farther upstream. There would be no need to use two or more starting elevations, representing various M1 or M2 curves, to effect a convergence; this M2 curve is the profile for the discharge being considered.

Transitions between tranquil and rapid flows

The water-surface profile can take several forms between the control point and the tranquil-flow profile on the mild slope downstream. In figure 11*C* the flow is just at critical depth downstream from the control, and the transition to the tranquil-flow profile farther down-

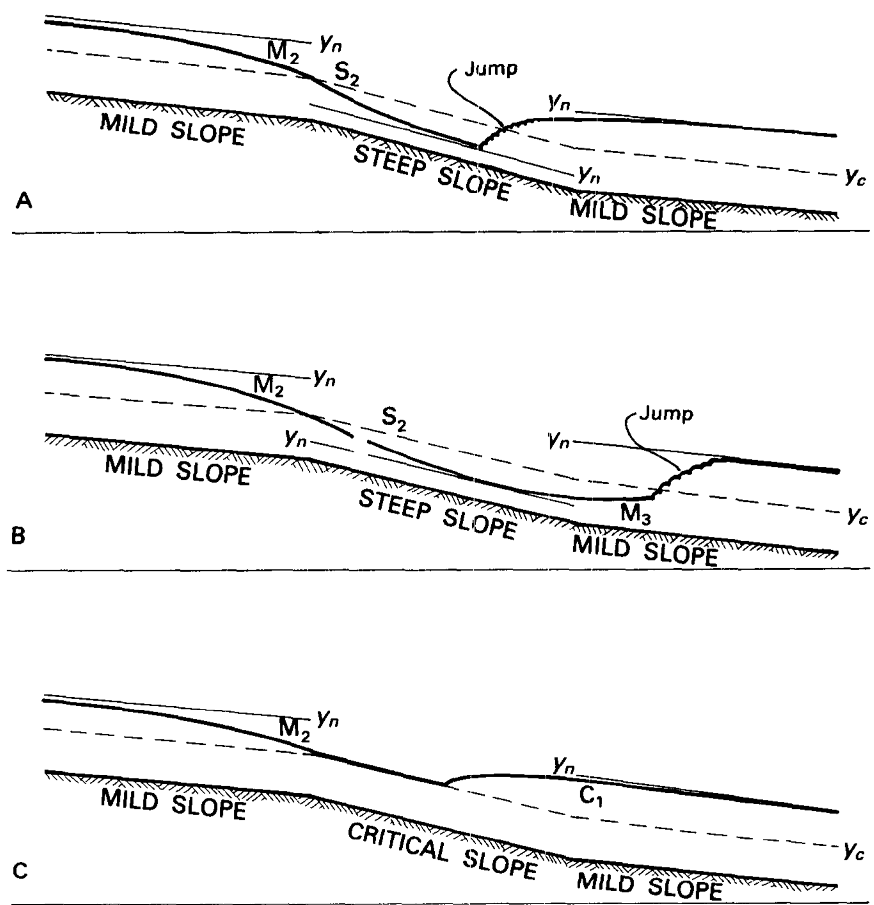

Figure 11.—Water-surface profiles involving critical- or supercritical-flow transitions.

stream is made by a C1 curve. This is an uncommon backwater transition curve and it will not be discussed in detail hereafter. It occurs generally on reservoirs.

If the flow on the steep slope were slightly supercritical (Froude number between 1.0 and about 1.7), the transition could be an undulatory curve as it approaches the normal-depth line on the mild slope downstream. A distinct hydraulic jump is formed for Froude numbers larger than 1.7, as the water surface passes from supercritical elevations, through the critical-depth line, and up to the subcritical normal-depth elevation. In figure 11*A*, the jump has formed on the steep slope and an S1 curve completes the transition to the subcritical normal-depth line downstream. In flows having higher Froude numbers, the jump could form on the mild slope, as in figure 11*B*. An M3 curve would accomplish the transition between the normal-depth line for the steep slope and the hydraulic jump.

In each of figures 11*A*–11*C*, the water-surface profile in the steep-slope transition can be computed in a downstream direction, beginning with the control point. The starting elevation will correspond to critical depth. If flow is just critical, the computed profile will coincide with the critical-depth line, as in figure 11*C*, and thence to the most upstream cross section for which an elevation was computed on the mild downstream slope. If the flow is supercritical, the water surface on the steep slope will follow an S2 curve, and the computation can be terminated just upstream from the last computed water-surface elevation on the

COMPUTED CROSS-SECTION PROPERTIES

Water-surface elevation, in feet	Area, in square feet	Top width, in feet	Alpha	$\frac{\sqrt{g}A^{3/2}}{(\alpha T)^{1/2}}$
292.0	226	65	1.42	2006
293.0	293	68	1.49	2826
294.0	363	71	1.57	3715
295.0	435	74	1.59	4744
296.0	511	78	1.63	5811
297.0	590	81	1.68	6968
298.0	673	84	1.84	7965

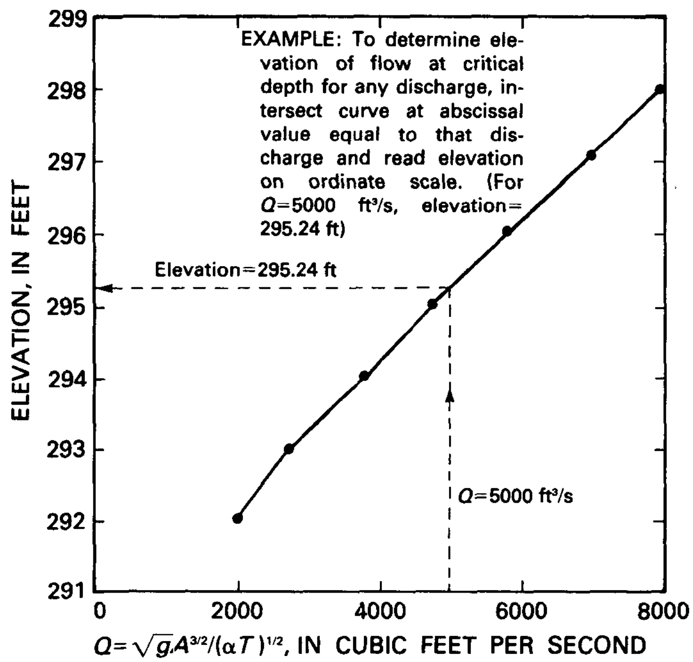

Figure 12.—Determination of elevation of critical flow in a cross section for any discharge.

mild slope, with a jump assumed between the two profiles, as in figures 11*A* and 11*B*.

In a situation such as one of those shown in figure 11, the temptation is usually to reduce the strict criteria necessary in the solution of the energy equation by (a) raising the Froude number limit far higher than the ordinarily tolerated value of 1.5, (b) raising the tolerance within which a solution would be acceptable, or (c) both of these. The effect is to force a computed profile upstream from the mild-slope region (on the right in figure 11), into and through the steep-slope region, and continuing uninterruptedly into the mild-slope region (on the left in figure 11). The computed values in the steep-slope region will be erroneous not only because they have been computed in the wrong direction, but also because supercritical flows are associated with large velocity heads, which have not fully been taken into account.

Any computations using less stringent criteria should be carefully examined before being accepted. Generally, it is preferable to break the solution at the control point and to start a new computation. Under no circumstances should a computation be accepted if there is reason to suspect the existence of a distinct hydraulic jump, as in figures 11*A* and 11*B*, where Froude number would be about 1.5 or 1.7, and higher. A solution forced through a transition depicted in figure 11*C*, where the Froude number is less than 1.5 and where only one or two cross sections are involved might, after close examination, prove to be acceptable.

Alternate depths

Once the water-surface profile has been computed in a reach involving a control section, a steep slope, and a hydraulic jump, such as the profiles in figure 11, it is pertinent to investigate other possible water-surface profiles in the same reach for the same discharge. Each supercritical-flow condition has an alternate subcritical depth at which the same discharge can flow. A tree or any other large object could lodge in the channel and trigger subcritical flow, or the location of the hydraulic jump could be shifted upstream. The result, in terms of figure 11, could be the elimination or drowning out of the critical or supercritical elevations through the steep-slope middle subreach. Those profiles could be superseded by a completely subcritical transition between the M2 curve on the upstream mild slope to the normal-depth line on the downstream mild slope, somewhat akin to the transition curve shown in figure 5.

The approach and getaway depths associated with hydraulic jumps are called conjugate depths. The depth after the jump is called the sequent depth. Determination of these depths requires analysis of the hydrostatic pressure and the momentum of the flow at cross sections before and after the jump. Such analyses, involving study of specific force diagrams, are explained thoroughly in hydraulics texts such as Chow (1959) and Woodward and Posey (1941).

The use of the specific energy curve rather than specific force offers a simpler approach that sacrifices little in accuracy as far as computation of water-surface profiles by the step-backwater method is concerned. It is relatively easy to develop and apply the specific energy curve. The method is described below and shown in figure 13. When this method is used, the depth before and after the hydraulic jump are called alternate depths.

The specific-energy curve at each cross section in a steep subreach is developed manually but the procedure is greatly simplified by having the bulk of the computations available in the computer output for cross-section properties. As was described in the discussion of figure 12, the computer provides for each cross section a tabulation of values for many different elevations at a user-predetermined interval, including for each elevation the cross-sectional area, A, and the velocity-head coefficient, α. These data are used in figure 13 to compute velocity heads ($\alpha Q^2/2gA^2$) at various depths of flow for a discharge of 5,000 ft^3/s. The specific energy diagram has water-surface elevation for its ordinate and the sum of this elevation and the velocity head for its abscissa. The point of minimum energy corresponds to critical-flow conditions, which were computed, in figure 12, to be at an elevation of 295.24 feet. If flow for this cross section and discharge were to be computed at an elevation of, say, 293.45 feet, which is a supercritical-flow condition, the corresponding subcritical-flow elevation would be about 297.31 feet.

The locus of subcritical-flow elevations, as computed above, for the various cross sections in a reach having supercritical flow (as in figure 11), would represent the highest elevations for the discharge in that reach. Such a "worst-condition" profile might be the preferable computation under certain circumstances.

A simpler, and probably not greatly erroneous, substitute for the above computations is the extension of a straight line from the critical-depth elevation at the control to the last (most upstream) subcritical-flow water-surface elevation computed on the downstream mild slope.

Energy Equation

The determination of a water-surface profile by the step-backwater method involves the solution of the energy equation in a series of subreaches. In the solution of the energy equation for open-channel flow conditions, as described by Benson and Dalrymple (1967), all the criteria that apply to computations of discharge by the slope-area method apply as well