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Techniques of Water-Resources Investigations
of the United States Geological Survey

Chapter A15

COMPUTATION OF WATER-SURFACE PROFILES IN OPEN CHANNELS

By Jacob Davidian

Book 3
APPLICATIONS OF HYDRAULICS

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PREFACE

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CONTENTS

FIGURES

UNIT CONVERSION

Multiply inch-pound unit	By	To obtain SI unit
ft (foot)	3.048×10^{-1}	m (meter)
mi (mile)	1.609	km (kilometer)
ft^3/s (cubic foot per second)	0.028	m^3/s (cubic meter per second)

SYMBOLS AND UNITS

Symbol	*Definition*	*Unit*
A	Area of cross section	ft^2
A	Drainage area of basin	ft^2, mi^2
a	Area of subsection	ft^2
B	Subscript denoting bypass channel	
C	Chezy discharge coefficient	$ft^{1/2}/s$
C	Subscript referring to a culvert	
C	Type of backwater profile for critical-flow conditions	
c	Subscript denoting composite section	
D_{max}	Maximum depth of flow in cross section	ft
d	Subscript denoting downstream cross section	
d_b	Depth of flow on flood plain	ft
d_m	Mean depth	ft
d_{50}	Median diameter of bed material	ft
$\mathbf{F}$	Froude number	
FRDN	Index Froude number	
f	Darcy-Weisbach resistance coefficient	
g	Gravitation constant (acceleration)	ft/s^2
h	Static or piezometric head above an arbitrary datum	ft
h_e	Energy loss due to channel expansion or contraction	ft
h_f	Head loss due to friction	ft
h_v	Velocity head at a section	ft
i	Subscript referring to individual subsection	
K	Conveyance of a section	ft^3/s
K_q	Conveyance of the subsection containing the discharge that is not contracted to enter a single contracted opening	ft^3/s
k	Part of the total conveyance	ft^3/s
k	Coefficient for energy loss	
L	Length of reach	ft
L	Distance from left edge of water to a point in the cross section	ft
L	Subscript denoting left subsection or channel	
L	Subscript denoting the subsection having the largest conveyance	
M	Subscript denoting middle channel or main channel	
M	Type of backwater profile for subcritical-flow conditions	
n	Manning roughness coefficient	$ft^{1/6}$
P	Wetted perimeter of cross section of flow	ft
Q	Total discharge	ft^3/s
q	Part of the total discharge	ft^3/s
R	Hydraulic radius	ft
R	Distance from right edge of water to a point in the cross section	ft
R	Subscript denoting right subsection or channel	
R	Subscript referring to road embankment	

Symbol	*Definition*	*Unit*
S	Water-surface slope	
S	Subscript denoting a surcharge	
S	Type of backwater profile for supercritical-flow conditions	
S_0	Channel bed slope	
T	Width of a section at the water surface	ft
T	Subscript referring to total quantity for cross section	
t	Subscript referring to a tributary stream	
u	Subscript denoting upstream cross section	
V	Mean velocity of flow in a section	ft/s
v	Velocity in a subsection	ft/s
W	Length of an embankment between bridge openings	ft
y	Depth of flow	ft
y_c	Critical depth of flow	ft
y_n	Normal depth of flow	ft
1,2,3,4	Subscripts which denote relative order of cross sections or section properties; or which denote subsections of a cross section	
α	Velocity-head coefficient	
Δ	Difference in values, as Δh is the difference in head	
θ	Central angle at bed of triangular cross section	
Σ	Summation of values	
ϕ	Channel slope angle	
$=$	Equal to	
$\neq$	Not equal to	
$\cong$	Approximately equal to	
$>$	Greater than	
$<$	Less than	
$\int$	Integral	

COMPUTATION OF WATER-SURFACE PROFILES IN OPEN CHANNELS

By Jacob Davidian

Abstract

The standard step-backwater method of computing water-surface profiles is described in this chapter. The hydraulic principles and assumptions are reviewed, and the field data requirements are described. Certain special cases of backwater curves and certain special field conditions are discussed in detail. The technique is used to establish or extend stage-discharge ratings; to define areas which will be innundated by flood flows of a given frequency; and to compute profiles through various reaches, including multichannel flows, and past control structures such as bridges, culverts, and road embankments. A brief description of analysis of floodways and effects of encroachments is also presented.

Introduction

Water-surface profiles along stream channels can be computed quickly when electronic computers are applied to the commonly used step-backwater method. The method requires the evaluation of the energy losses between any two points on the water-surface profile.

Water-surface profile computations by the step-backwater method are a major part of most studies leading to the delineation of flood plains in urban and suburban areas. Flood plains must be delineated before they can be properly zoned to reduce flood damages.

The method is also applied to floodway analysis. Given a stream profile for a flood of a certain magnitude, a surcharge, or increase in stage, is chosen for the entire profile. Then the encroachments on the flood plains are determined by the step-backwater method such that the total stream conveyance remains unchanged. This application is useful in planning and in flood-insurance studies.

The method is also used in establishing or extending stage-discharge relations at gaging stations or at other sites along a stream. This information is valuable in the design of structures.

A survey of the geometry of the stream channel along the reach for which the profiles are needed produces the data required—principally, cross sections of the channel drawn to a common datum at intervals along the reach and the value of the roughness coefficient. From these data, the water-surface profile for any known or assumed discharge may be computed.

This manual describes the standard step-backwater method of computing water-surface profiles and the channel geometry data needed. Certain sources of error and some special field conditions are discussed in detail. Frequent reference is made to computer program E431 of the U.S. Geological Survey. Because computer programs are frequently changed, details of that program are not discussed here. Program E431 is described by Shearman (1976) in detail, with respect to data handling and preparation, computed results, error messages, and assumptions.

Hydraulic Principles

Steady, uniform flow

Almost all open-channel flows are both unsteady (depth at a point varies with time) and nonuniform (depth changes from point to point along a channel). Because these flows are difficult to analyze, a steady, uniform flow is frequently assumed. The assumption of steadiness is justified by the fact that at peak flows the discharge hydrograph flattens out and flow approximates steady conditions. High-water marks are left along the channel by these relatively steady, peak flows. Uniformity of flow is achieved by dividing channels into shorter lengths that are considered reasonably uniform between cross sections, both on the basis of high-water marks and on channel geometry.

When a steady discharge flows in a long channel of uniform hydraulic characteristics, it flows at a constant depth, called the normal depth. But adjacent subreaches are not identical in channel dimensions, roughness, or bed slope, and so the normal depth in each is different. A natural water-surface profile, therefore, is a series of curves relating the normal depth in one subreach to the normal depth in the next. The normal depth line in a long stretch of river is, thus, rarely a long, smooth curve.

Backwater curves

The water-surface profiles resulting from nonuniform-flow conditions are called backwater curves. Various types of such profiles for gradually varied flows are described by Chow (1959) and Woodward and Posey (1941). Of the many backwater curves possible, those for subcritical flows on mild slopes (fig. 1), and those for supercritical flows on steep slopes (fig. 2), are generally of most concern. In figures 1 and 2, the dashed lines represent the critical depth, y_c, and the straight, solid lines represent the normal depth, y_n. The heavy, solid, curves lines represent the water-surface profiles from a control point to the normal-depth profile. (The expression "control point" or "control section" is defined as any section at which the depth of flow is known or can be controlled to a required stage.)

Curve M1 in figure 1*A* could be the result of a dam or a constriction downstream, or it could represent the backwater in a tributary which is flowing into a flooding main stream. Curve M2 could result from a dropoff (falls or riffles) farther downstream where the water surface drops below the critical-depth line. The flow then passes into the supercritical regime. For both the M1 and M2 curves, the control is

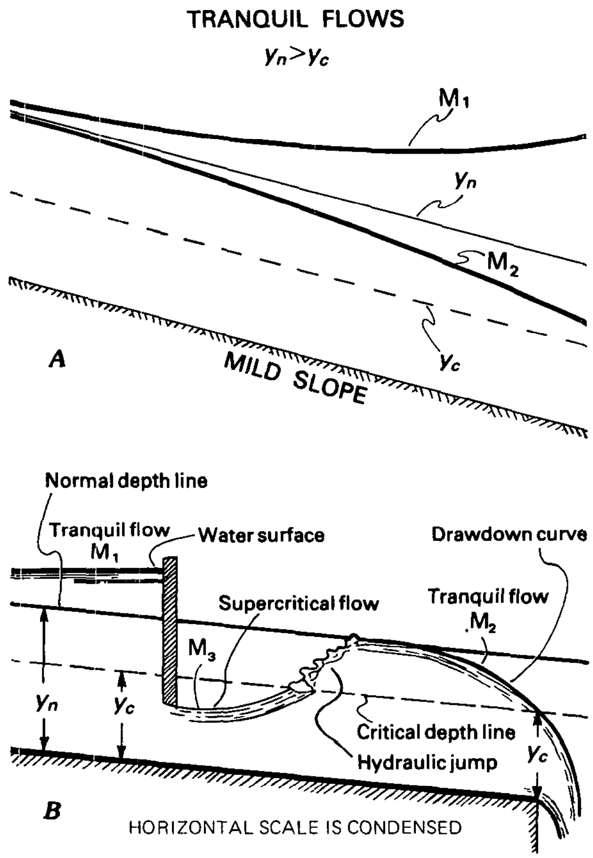


Figure 1.—Water-surface profiles on mild slopes. *A*, Tranquil flows, showing relations among M1, y_n, and y_c curves; *B*, Sketch showing typical instances of M curves.

downstream; if the channel were long enough, both curves would asymptotically approach the normal-depth line upstream. Typical examples M curves are shown in figure 1*B*.

In figure 2*A*, curve S3 could represent the profile of flow downstream from a sluice gate. Curve S2 could be the profile of flow which has just passed into the critical regime from a milder slope upstream, as might occur at a riffle. The control point for both these curves is upstream; if the channel were long enough, both curves would asymptotically converge toward the normal-depth line downstream. Typical examples of the S curves are shown in figure 2*B*.

Examination of figures 1*A* and 2*A* will show that the lowest possible M1 curve and the highest possible M2 curve will each coincide with the normal-depth curve, which represents

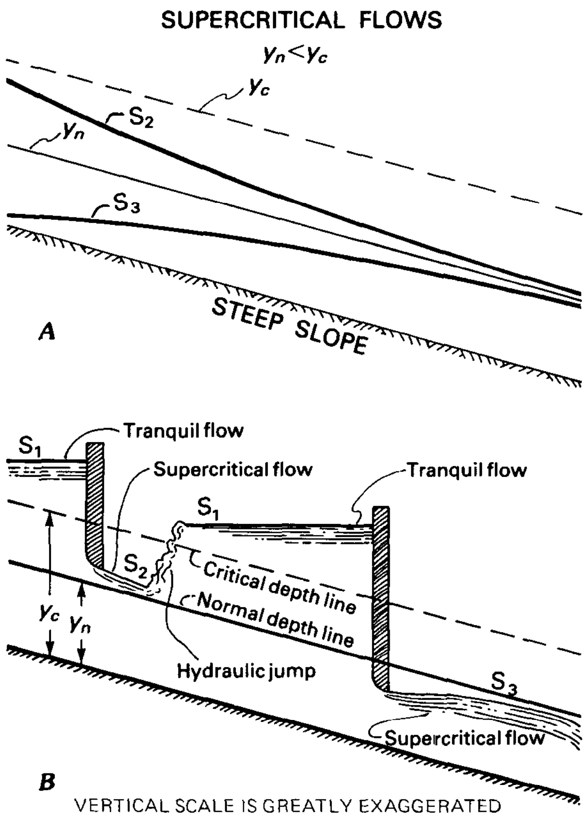


Figure 2.—Water-surface profiles on steep slopes. *A*, Supercritical flows, showing relations among S2, S3, y_n and y_c curves; *B*, Sketch showing typical instances of S curves.

the no-backwater condition on the mild slope; and that the lowest S2 and highest S3 curves will each similarly coincide with the no-backwater, normal-depth line on the steep slope.

Because most natural streamflow is subcritical, with only occasional stretches of supercritical-flow conditions, this manual emphasizes M1 and M2 curves, refers to the S2 and S3 curves where appropriate, and briefly mentions the rare M3 and S1 curves.

Water-surface elevations along M1 and M2 curves should be computed only in an upstream direction, away from a control, to ensure that successive points on the computed curve will asymptotically approach, or converge toward, the normal-depth line. Similarly, water-surface elevations along S2 and S3 curves should be computed only in a downstream direction, away from a control, to ensure that the computed curves will converge toward the normal-depth line. Computations in the wrong direction will define profiles that diverge from the normal-depth line, and they are, therefore, erroneous. It is possible, however, to make a few computations in the wrong direction without introducing large errors if the velocity head is small. The small errors locally introduced and reflected in the computed profiles in such instances are quickly dissipated if further computations of the water-surface profiles are in the correct direction for the remainder of the reach being investigated.

It should be noted that the description of a reach as mild or steep, or as subcritical, or critical, or supercritical refers not to the numerical value of the slope of the reach, but, rather, to the regime of the discharge flowing through the reach. Whether the numerical value of the slope is small or large, a reach could be considered "mild" because of the subcritical flow it carries, but it could later be considered "steep" because a different discharge is supercritical during the passage of a flood wave through the reach.

Transition curves

Figures 3–8 show some backwater transition curves for flow passing from one reach into another. In figure 3, the normal-depth lines are at different depths; the M1 curve of the mild slope, therefore, smoothly joins the normal-depth line of the milder slope.

In figure 4, a change in roughness (n) on a mild slope will result in different normal depths. The transition curve is, therefore, very similar to that of figure 3.

In figures 5 and 6, the normal depths on mild slopes are high in the upstream reach for a very mild slope or a very high roughness, and they are low in the downstream reach for a steeper slope or a lower roughness. The water-surface profile in the transition is, therefore, an M2 curve which smoothly joins the downstream normal-depth line.

Figure 7 shows a break in channel bed slope from mild to steep. The tranquil flow upstream from the control point has a normal depth higher than the critical depth; the supercritical flow downstream from the control point has a

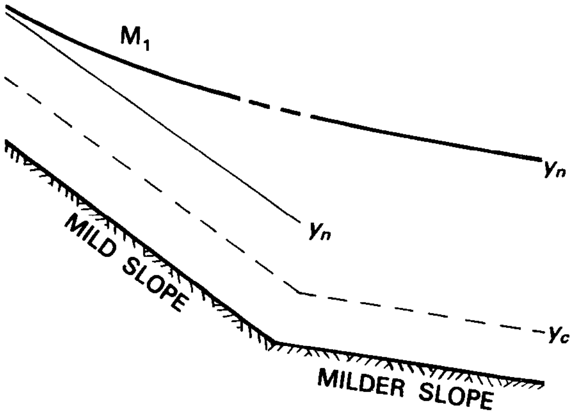


Figure 3.—Backwater-curve transition for mild slope to milder slope.

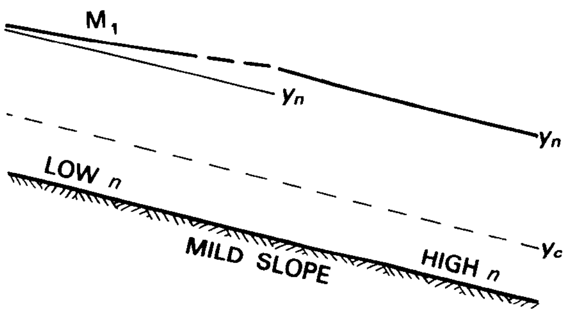


Figure 4.—Backwater-curve transition from small n to large n on mild slope.

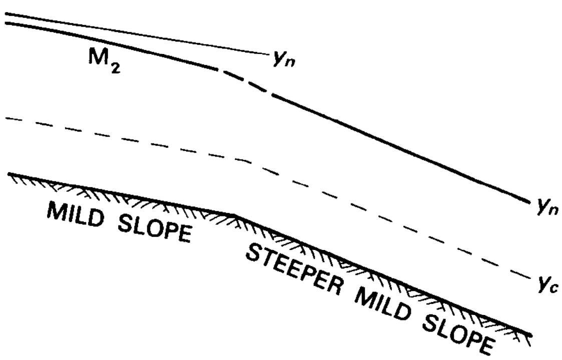


Figure 5.—Backwater-curve transition for mild slope to steeper mild slope.

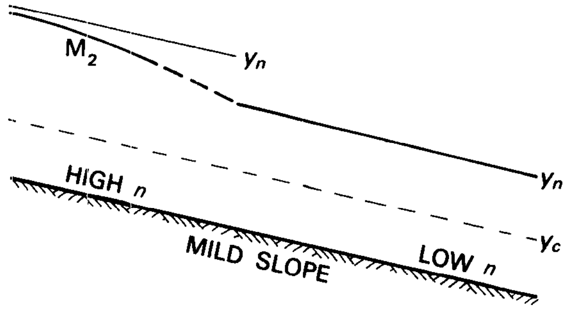


Figure 6.—Backwater-curve transition from large n to small n on mild slope.

normal depth lower than the critical depth. The water-surface profile must, in the transition between the two normal-depth lines, pass through critical depth by means of M2 and S2 curves. Such a transition is known as a drawdown curve or a hydraulic drop.

The transition from a steep slope ($y_n < y_c$) to a mild slope ($y_n > y_c$) in figure 8 requires a hydraulic jump in order for the water surface to pass through critical depth. The location of the jump depends on the relative elevations between the upstream and downstream normal depths. Associated with the hydraulic jump could be a short segment of an S1 or an M3 curve, which is not ordinarily of concern because of its proximity to the jump.

Determination of normal depth

Average profile in a long reach

The simple transitions described in figures 3–8 are only a few of many possible combinations that could result from changes in channel

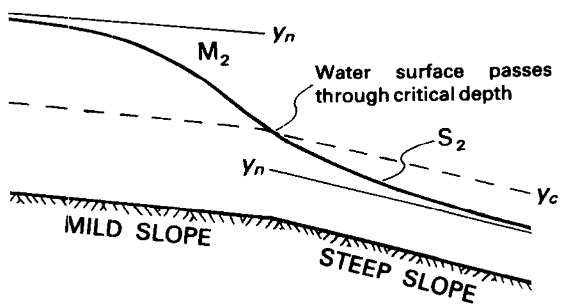


Figure 7.—Backwater-curve transition for mild slope to steep slope.

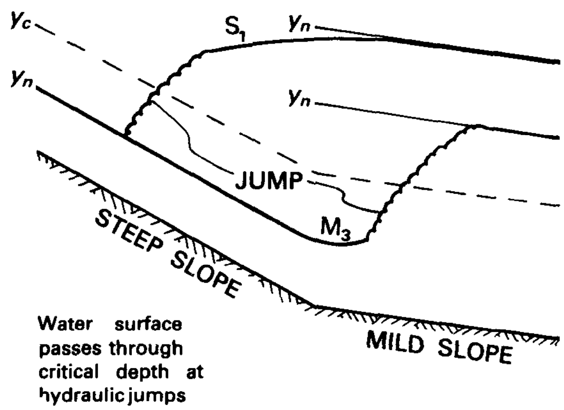


Figure 8.—Backwater-curve transitions for steep slope to mild slope.

characteristics such as slopes, roughnesses, widths, depths, or any combination of these between adjacent subreaches in a long reach. Obviously, in a long stretch of river that has been divided into shorter subreaches of "uniform" characteristics, the water-surface profile is a series of transition curves from the normal-depth line in one subreach to the normal-depth line in the adjacent subreach. Unless there are radical changes in characteristics, as for example at control points and hydraulic jumps, one could speak of the uniform-flow profile in a long reach as an average of the numerous transition curves that are reflecting local nonuniformities in the channel.

The normal depth line is determined using the backwater curves for channels of mild or steep slopes.

Use of M1 and M2 curves

A characteristic of the tranquil-flow M1 and M2 backwater curves is that one can start at any point on either of them and, by solving the energy equation, determine the elevation of the water surface at another point farther upstream. The intervening geometry and roughness, as well as the discharge, have to be known. As may be seen in figure 1*A*, if the procedure were carried far enough upstream in increments, this step-by-step computed profile would asymptotically approach the normal-depth line. This characteristic of the backwater curves is used in determining normal depth in a channel of mild slope.

Computed profiles that start with an elevation higher than normal depth would be M1 curves; those starting with elevations lower than normal depth would be M2 curves. When computed values of water-surface elevations along two profiles converge mathematically, it is generally assumed that the normal-depth profile has been reached. Computations along either profile continued farther upstream would be identical, regardless of whether the profiles had been two M1's, two M2's, or one of each, at the start of computations at the downstream end of the reach. Upstream from the point of convergence, the computed profile would define a locus of normal depth at each cross section. It would be the expected profile because the nonuniformities in the channel geometry and roughness would be translated to a series of minor, transitional backwater curves.

Determination of normal depth for subcritical flow in a channel by the procedure described above thus has two distinct phases. First, two starting backwater curves are assumed to exist in the channel, caused by different, assumed, control conditions downstream. Normal depth at a point (gaging station, bridge, or other place of interest) is determined when the two profiles have converged. The two computed profiles up to this point are imaginary and become useless after serving as a ploy for determining the normal-depth profile. The second phase begins with the point of convergence. All subsequent computations of the profile farther upstream represent the expected, or normal, water-surface elevation in this channel, to be used for inundation studies, flood-insurance studies, bridge-backwater studies, and so on.

Use of S2 and S3 curves

To determine normal depth in a steep channel, the supercritical-flow S2 and S3 backwater curves are used. The procedures correspond to those described for M1 and M2 curves, but in the downstream direction. Starting at any point on the S2 or S3 curve, the elevation of the water surface at any point farther downstream can be determined by solving the energy equation. The intervening geometry and roughness, as well as the discharge, have to be known. As may be seen in figure 2*A*, if the procedure is carried out far enough downstream, in increments, this step-by-step computed profile asymptotically approaches the normal-depth line.

Local effects on profiles

The importance of the effects of local channel nonuniformities on the computed profiles is a relative matter. In the first phase of computations along an M1 or an M2 curve, the profile usually has more slope and the effect of a local disturbance will show in the profile at that point, but it will probably diminish within a short distance upstream. For example, a bridge in the channel through which an M1 or an M2 curve is being computed will cause a local jump in computed elevations as in figure 9. An M1 curve will continue along another, higher M1 curve. An M2 curve could jump up to an M1 curve, or it could continue along another, higher M2 curve. The effect of the bridge is quickly dissipated.

If, however, the channel bed has a very small slope or if the bridge is located in the reach where backwater curves have already converged and for which the expected water surfaces are being computed, the local effects will be reflected farther upstream. In such instances, the effects of several bridges in a series would be additive. The resultant M1 curve could require a long distance before converging with the average normal-depth line for the channel.

All M1 curves where the computed profile or the channel bed have very little slope should be examined and interpreted with care.

Convergence of backwater curves

There are several factors to consider in the technique of using converging backwater curves to determine normal depth. The work "converge" is not the proper word to use in describing the relation of a backwater curve to the normal-depth profile. Backwater curves approach the normal-depth profile asymptotically, and the relation between two M1 curves or two M2 curves is an asymptotic convergence. During computation for two adjacent profiles, results may be identical or within an allowable difference or tolerance. Then the profiles are considered to have converged for all practical purposes.

If an M1 and an M2 curve are used as a pair and if the two profiles converge, normal depth is assured. It is not always feasible, however, to work with this ideal pair. An M1 curve lies above the normal-depth line (see figure 1*A*). Cross-sectional properties, therefore, must be available for elevations higher than normal depth at all of the cross sections in the reach. To calculate such properties is frequently impossible, particularly for large discharges where normal depths may be near bankfull stage; there simply may not be any ground points available above bankfull stages to which the cross sections could be vertically extended. This problem is more acute, of course, for the most downstream cross sections, where the M1 curve would be at its highest elevations with reference to the normal-depth line.

Another characteristic of the M1 curve is that it requires a longer reach to converge with the normal-depth line than does an M2 curve, both having started an equal vertical distance from the normal depth at the starting cross section downstream. A longer reach of channel must, therefore, be surveyed whenever an M1 curve is used and an even longer one if a pair of them are used.

The use of two M2 curves to determine normal depth is just as effective, but there are a few precautions to consider. The starting elevations preferably should be a foot or more apart. They should not be near the channel bed because the length of reach required before the backwater curve converges with the normal-depth line will be longer. The starting elevation of an M2 curve to be used for convergence purposes should never be taken below the critical-depth elevation. The starting elevation for computing an M2 profile upstream from a control would, on the other hand, have to start at critical depth. The difference in this instance is that the computations are defining the actual water surface in the reach, rather than a profile to be used to converge with the normal-depth profile.

Convergence of any two or more M1 curves or any two or more M2 curves is no guarantee that normal depth has been reached. Although they may seem to converge mathematically,

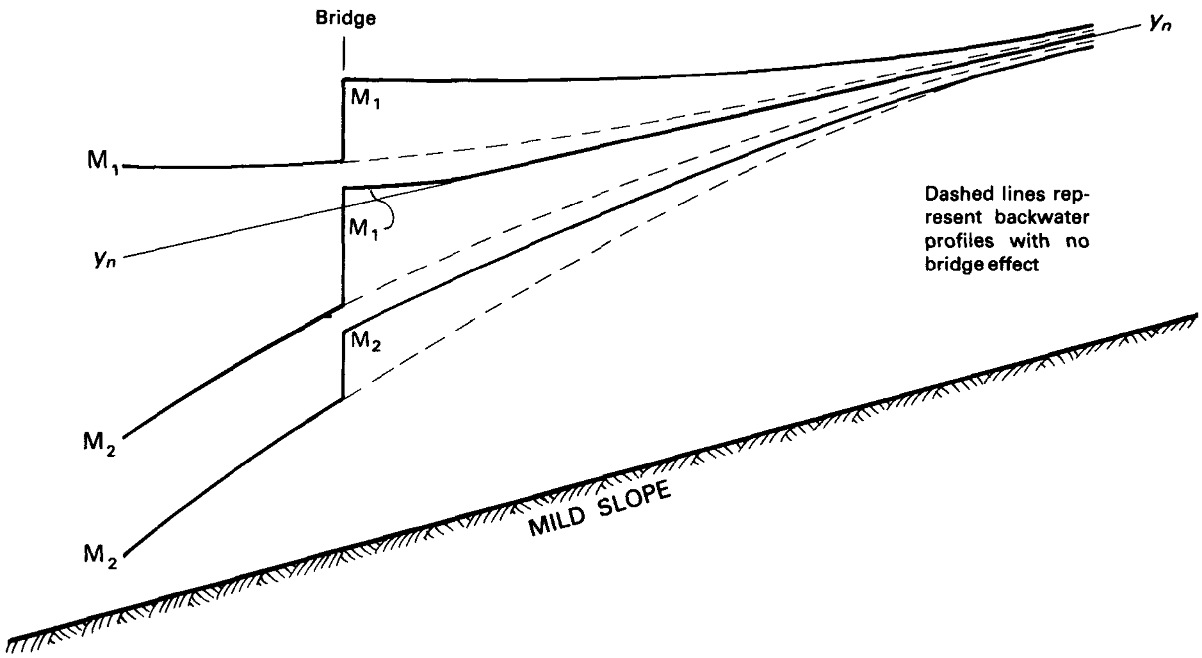


Figure 9.—Local effect of a bridge on an M1 or an M2 backwater curve.

the true normal-depth profile could still be some distance away in the vertical. For example, in figure 10, several M1 curves have apparently converged, and several M2 curves have apparently converged. None of them, however, have reached the normal-depth profile. Regardless of how many M1 curves have converged, therefore, the one starting with the lowest elevation is the most nearly correct. Similarly, the M2 curve starting with the highest elevation is the most nearly correct of the M2 curves. Figure 10 illustrates graphically that when an M1 and an M2 curve converge, the normal-depth profile has been reached. The starting elevations of all M1 and M2 curves should preferably be within the range 0.75–1.25 of the estimated normal depth.

Special Cases of Backwater Curves

Flows on very small slopes

Sometimes a stream will have so small a slope that the rate of convergence between two M2 curves is negligible. Two curves starting a foot apart might, after 10,000–20,000 feet, still be 0.75 foot apart at the upper end of the reach, and there may be no more stream channel available for extending the reach downstream; for example, the stream might flow into a lake or large river.

In such a case, a technique that can produce satisfactory results is the artificial extension of the reach. First, the average streambed slope is determined from a plot of the thalweg and extended downstream on the plot. An average cross-sectional shape can then be determined. This may be done by superposing all of the cross sections on a plot and averaging them, not only as to geometric shape, but as to roughness values as well.

Extension of the reach length and placement of an average cross section at a few intervals along it, should provide enough length for two M2 backwater curves to converge downstream from the reach under study. If a suitable computer program is used, an extended reach could be as long as 2,000 miles (Shearman, 1976) to try for convergence.

The procedure described might not prove satisfactory in streams having extremely small slopes. It should be kept in mind that the nor-

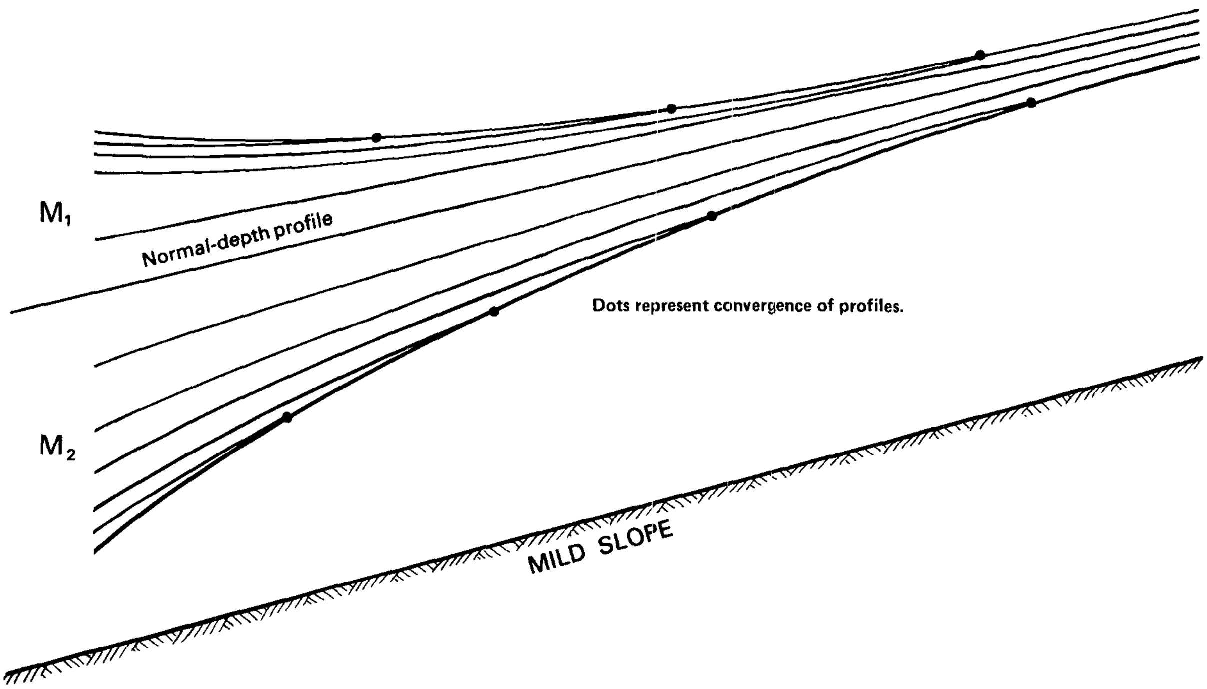


Figure 10.—Sketch of a family of M1 and M2 backwater curves. Dots indicate points of convergence.

mal depth for a slope of zero is infinity. If the slope is extremely small, even a 2,000-mile artificial extension of channel might prove inadequate in length, or the channel cross sections might not extend high enough to contain a flow that requires a very large normal depth. Flow could spill out over banks into adjacent drainages such that large discharges would have no distinguishable channels in which normal depths could be computed.

The use of an artificially extended reach could force the use of negative numbers, not only for section reference distances, but for elevations as well. Most computer programs will handle negative reference distances. But, to avoid the use of negative numbers and to lessen the likelihood of errors resulting from them, the datum can be adjusted by several hundred, or a thousand feet, throughout the entire reach.

A factor to consider in interpreting computed profiles on very small flat slopes is that relatively minor local departures from the general uniformity of the reach, both with respect to cross-sectional geometry and roughness and to bed slope, will affect the profile markedly. The local effects will extend far upstream (farther the flatter the slope), and they will more likely be additive than compensatory. In contrast, local disturbances in a reach that has a steeper bed slope but is otherwise the same will not affect the profile as far upstream.

Flows on Steep Slopes

Water-surface profiles on slopes, where supercritical flows are possible, should be carefully examined. Not only is it possible to get a variety of incorrect profiles, but, if the water surface breaks, the usual computations become inapplicable and no solution is possible. Sections of a stream that are likely to cause problems are at falls and riffles and at places where chute flows and hydraulic jumps are likely. A convex break in bed slope, where the slope is mild or flat upstream and steeper downstream, is particularly suspect, as are extreme contractions of area, owing either to smaller widths, to smaller depths, or to a combination of smaller widths and depths. Figure 11 shows several possible water-surface profile transitions involving critical or supercritical flow. These are the most common types. Other examples of flow profiles and transition curves at control sections are shown by Chow (1959, p. 229–236).

Locus of critical-depth stages

When critical-flow conditions are suspected for a given discharge in a reach, it is advisable to compute the critical depth at each cross section in the general vicinity and to connect these points on a plot to show the locus of critical-flow stages through that part of the reach. The procedure to follow is straightforward.

At each cross section investigated, let the Froude number, **F**, equal 1.0:

$$\mathbf{F}=1=V\sqrt{\alpha}\,/\,\sqrt{gd_m\cos\phi}\,.$$

The value of $\cos\phi$, where ϕ represents the channel slope angle, is generally close enough to unity to be ignored. Let velocity be equal to discharge, Φ, divided by total area, A, and let mean depth, d_m, be equal to total area divided by T, the top width. The velocity-head coefficient is represented by α and g is the gravitation constant in feet per second per second. Then,

$$\mathbf{F}=1=\frac{\sqrt{\alpha}\;Q/A}{\sqrt{g}\;\sqrt{(A/T)}}\,.$$

Now solve for Q:

$$Q=\sqrt{g}\;A^{3/2}/\,(\alpha T)^{1/2}.$$

Find the elevation at each cross section for which this holds true.

The computations are greatly simplified by the fact that cross-section properties, including area, velocity-head coefficient, and top width, are tabulated by computer at many elevations for each cross section. The Geological Survey computer program (Shearman, 1976) also prints out the value of $\sqrt{g}\,A^{3/2}/(\alpha T)^{1/2}$ for each of these elevations. This quantity is equivalent to that discharge whose water surface would be at the indicated elevation if the flow were critical. A simple plot of elevation versus discharge represented by the quantity $\sqrt{g}\;A^{3/2}/(\alpha T)^{1/2}$ can be prepared by the engineer quickly for each cross section at which supercritical-flow problems are suspected, as illustrated in figure 12. From elevations for selected discharges the profiles can be plotted.

Control sections

Examination of a plot of bed profile and of the loci of critical-depth points for several cross sections will generally reveal a section which can be taken to be the control—a convex break in slope, with a steep downstream leg and a flatter upstream leg. The control is at that point at which continuous computations upstream for the subcritical flow reach must begin again. In figures 11*A*–11*C*, profile computations in the upstream direction along the downstream mild slope could be invalid and should be halted in the steep-sloped reach where flow is supercritical. The control point for the upstream mild reach would be identified as the junction between the upper mild slope and the steep slope, where the M2 curve meets the critical-depth line. The step-backwater computations of the water-surface profile could be started again at this control point and continued upstream for as long as the upstream reach remains at a mild slope. The starting elevation for the new computation would be the critical-depth elevation at the control cross section, and the M2 profile computed would represent the water surface, which will ultimately coincide with the normal—depth line farther upstream. There would be no need to use two or more starting elevations, representing various M1 or M2 curves, to effect a convergence; this M2 curve is the profile for the discharge being considered.

Transitions between tranquil and rapid flows

The water-surface profile can take several forms between the control point and the tranquil-flow profile on the mild slope downstream. In figure 11*C* the flow is just at critical depth downstream from the control, and the transition to the tranquil-flow profile farther down-

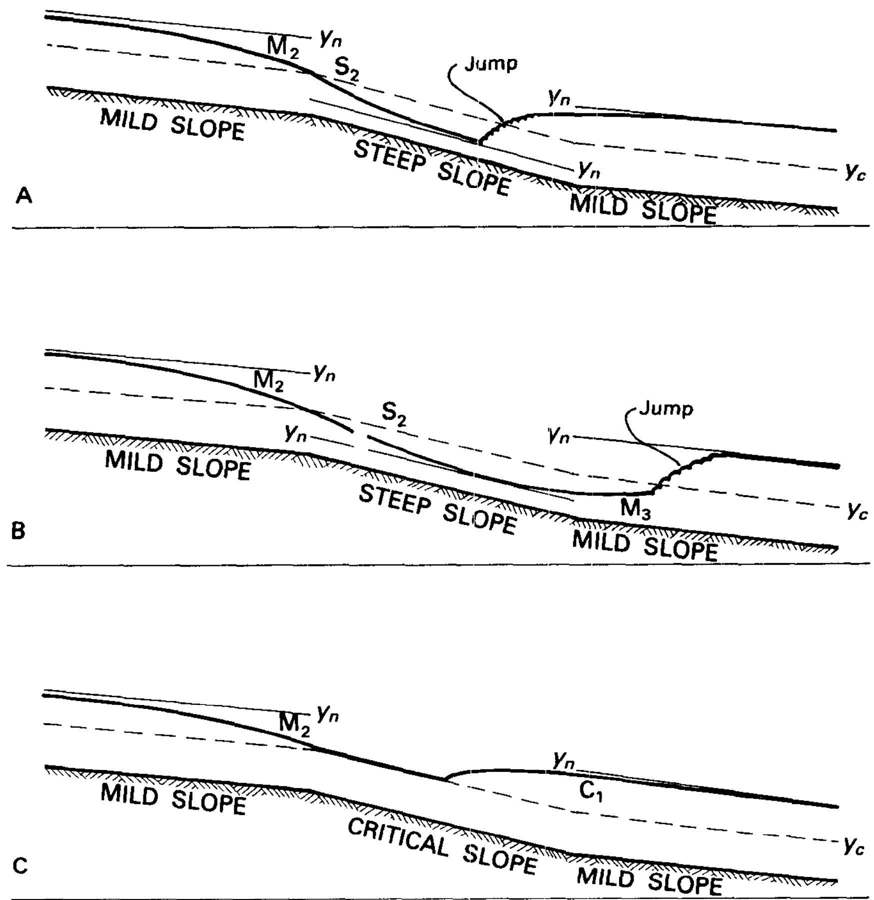


Figure 11.—Water-surface profiles involving critical- or supercritical-flow transitions.

stream is made by a C1 curve. This is an uncommon backwater transition curve and it will not be discussed in detail hereafter. It occurs generally on reservoirs.

If the flow on the steep slope were slightly supercritical (Froude number between 1.0 and about 1.7), the transition could be an undulatory curve as it approaches the normal-depth line on the mild slope downstream. A distinct hydraulic jump is formed for Froude numbers larger than 1.7, as the water surface passes from supercritical elevations, through the critical-depth line, and up to the subcritical normal-depth elevation. In figure 11*A*, the jump has formed on the steep slope and an S1 curve completes the transition to the subcritical normal-depth line downstream. In flows having higher Froude numbers, the jump could form on the mild slope, as in figure 11*B*. An M3 curve would accomplish the transition between the normal-depth line for the steep slope and the hydraulic jump.

In each of figures 11*A*-11*C*, the water-surface profile in the steep-slope transition can be computed in a downstream direction, beginning with the control point. The starting elevation will correspond to critical depth. If flow is just critical, the computed profile will coincide with the critical-depth line, as in figure 11*C*, and thence to the most upstream cross section for which an elevation was computed on the mild downstream slope. If the flow is supercritical, the water surface on the steep slope will follow an S2 curve, and the computation can be terminated just upstream from the last computed water-surface elevation on the

COMPUTED CROSS-SECTION PROPERTIES

Water-surface elevation, in feet	Area, in square feet	Top width, in feet	Alpha	$\sqrt{g}A^{3/2}/(\alpha T)^{1/2}$
292.0	226	65	1.42	2006
293.0	293	68	1.49	2826
294.0	363	71	1.57	3715
295.0	435	74	1.59	4744
296.0	511	78	1.63	5811
297.0	590	81	1.68	6968
298.0	673	84	1.84	7965

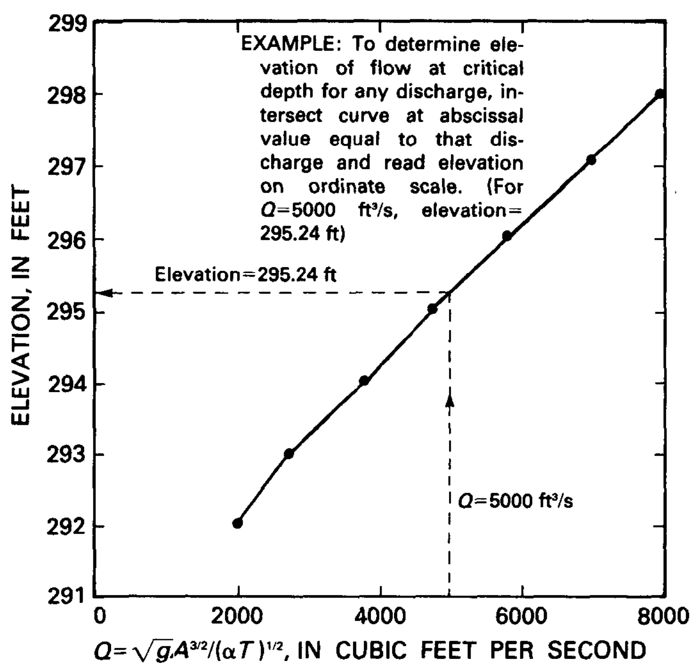


Figure 12.—Determination of elevation of critical flow in a cross section for any discharge.

mild slope, with a jump assumed between the two profiles, as in figures 11*A* and 11*B*.

In a situation such as one of those shown in figure 11, the temptation is usually to reduce the strict criteria necessary in the solution of the energy equation by (a) raising the Froude number limit far higher than the ordinarily tolerated value of 1.5, (b) raising the tolerance within which a solution would be acceptable, or (c) both of these. The effect is to force a computed profile upstream from the mild-slope region (on the right in figure 11), into and through the steep-slope region, and continuing uninterruptedly into the mild-slope region (on the left in figure 11). The computed values in the steep-slope region will be erroneous not only because they have been computed in the wrong direction, but also because supercritical flows are associated with large velocity heads, which have not fully been taken into account.

Any computations using less stringent criteria should be carefully examined before being accepted. Generally, it is preferable to break the solution at the control point and to start a new computation. Under no circumstances should a computation be accepted if there is reason to suspect the existence of a distinct hydraulic jump, as in figures 11*A* and 11*B*, where Froude number would be about 1.5 or 1.7, and higher. A solution forced through a transition depicted in figure 11*C*, where the Froude number is less than 1.5 and where only one or two cross sections are involved might, after close examination, prove to be acceptable.

Alternate depths

Once the water-surface profile has been computed in a reach involving a control section, a steep slope, and a hydraulic jump, such as the profiles in figure 11, it is pertinent to investigate other possible water-surface profiles in the same reach for the same discharge. Each supercritical-flow condition has an alternate subcritical depth at which the same discharge can flow. A tree or any other large object could lodge in the channel and trigger subcritical flow, or the location of the hydraulic jump could be shifted upstream. The result, in terms of figure 11, could be the elimination or drowning out of the critical or supercritical elevations through the steep-slope middle subreach. Those profiles could be superseded by a completely subcritical transition between the M2 curve on the upstream mild slope to the normal-depth line on the downstream mild slope, somewhat akin to the transition curve shown in figure 5.

The approach and getaway depths associated with hydraulic jumps are called conjugate depths. The depth after the jump is called the sequent depth. Determination of these depths requires analysis of the hydrostatic pressure and the momentum of the flow at cross sections before and after the jump. Such analyses, involving study of specific force diagrams, are explained thoroughly in hydraulics texts such as Chow (1959) and Woodward and Posey (1941).

The use of the specific energy curve rather than specific force offers a simpler approach that sacrifices little in accuracy as far as computation of water-surface profiles by the step-backwater method is concerned. It is relatively easy to develop and apply the specific energy curve. The method is described below and shown in figure 13. When this method is used, the depth before and after the hydraulic jump are called alternate depths.

The specific-energy curve at each cross section in a steep subreach is developed manually but the procedure is greatly simplified by having the bulk of the computations available in the computer output for cross-section properties. As was described in the discussion of figure 12, the computer provides for each cross section a tabulation of values for many different elevations at a user-predetermined interval, including for each elevation the cross-sectional area, A, and the velocity-head coefficient, α. These data are used in figure 13 to compute velocity heads ($\alpha Q^2/2gA^2$) at various depths of flow for a discharge of 5,000 ft^3/s. The specific energy diagram has water-surface elevation for its ordinate and the sum of this elevation and the velocity head for its abscissa. The point of minimum energy corresponds to critical-flow conditions, which were computed, in figure 12, to be at an elevation of 295.24 feet. If flow for this cross section and discharge were to be computed at an elevation of, say, 293.45 feet, which is a supercritical-flow condition, the corresponding subcritical-flow elevation would be about 297.31 feet.

The locus of subcritical-flow elevations, as computed above, for the various cross sections in a reach having supercritical flow (as in figure 11), would represent the highest elevations for the discharge in that reach. Such a "worst-condition" profile might be the preferable computation under certain circumstances.

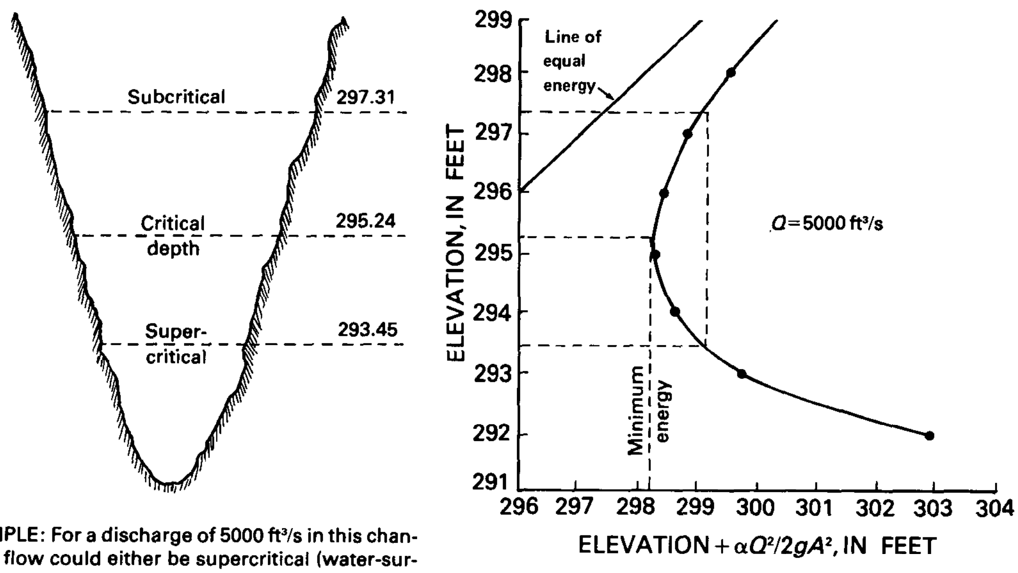
A simpler, and probably not greatly erroneous, substitute for the above computations is the extension of a straight line from the critical-depth elevation at the control to the last (most upstream) subcritical-flow water-surface elevation computed on the downstream mild slope.

Energy Equation

The determination of a water-surface profile by the step-backwater method involves the solution of the energy equation in a series of subreaches. In the solution of the energy equation for open-channel flow conditions, as described by Benson and Dalrymple (1967), all the criteria that apply to computations of discharge by the slope-area method apply as well

COMPUTATION OF ENERGY FOR A KNOWN DISCHARGE
(Q=5000 ft³/s)

Water-surface elevation, in feet	Area, in square feet	Alpha	$\frac{\alpha Q^2}{2gA^2}$	Elevation + $\frac{\alpha Q^2}{2gA^2}$
292.0	226	1.42	10.81	302.81
293.0	293	1.49	6.75	299.75
294.0	363	1.57	4.63	298.63
295.0	435	1.59	3.27	298.27
296.0	511	1.63	2.43	298.43
297.0	590	1.68	1.88	298.88
298.0	673	1.84	1.58	299.58



EXAMPLE: For a discharge of 5000 ft³/s in this channel, flow could either be supercritical (water-surface elevation=293.45 ft), or subcritical (water-surface elevation=297.31 ft)

Figure 13.—Determination of alternate depths from energy diagrams for a given cross section.

to the step-backwater method. Among these criteria and assumptions are the following, which refer to each subreach of a step-backwater reach:

1. The flow must be steady.
2. The flow at both end cross sections of the subreach, as well as through it, must be either all supercritical (**F** $\geqslant$ 1.0) or all subcritical (**F** $\leqslant$ 1.0). A change in the type of flow within a subreach negates the solution. An end cross section may be at critical flow (a control point, where **F** = 1.0) or it may be at a break in water surface, such as a hydraulic jump.
3. The slope must be small enough so that normal depths can be considered to be vertical depths.
4. The water surface across a cross section is level.
5. The effects of sediment and air-entrainment are negligible.
6. All losses are correctly evaluated.

Figure 14 is a definition sketch of an open-channel-flow reach. This reach may be considered one of the many subreaches used in a step-backwater computation of a water-surface profile. It has been chosen such that conditions of roughness (both amount of roughness and

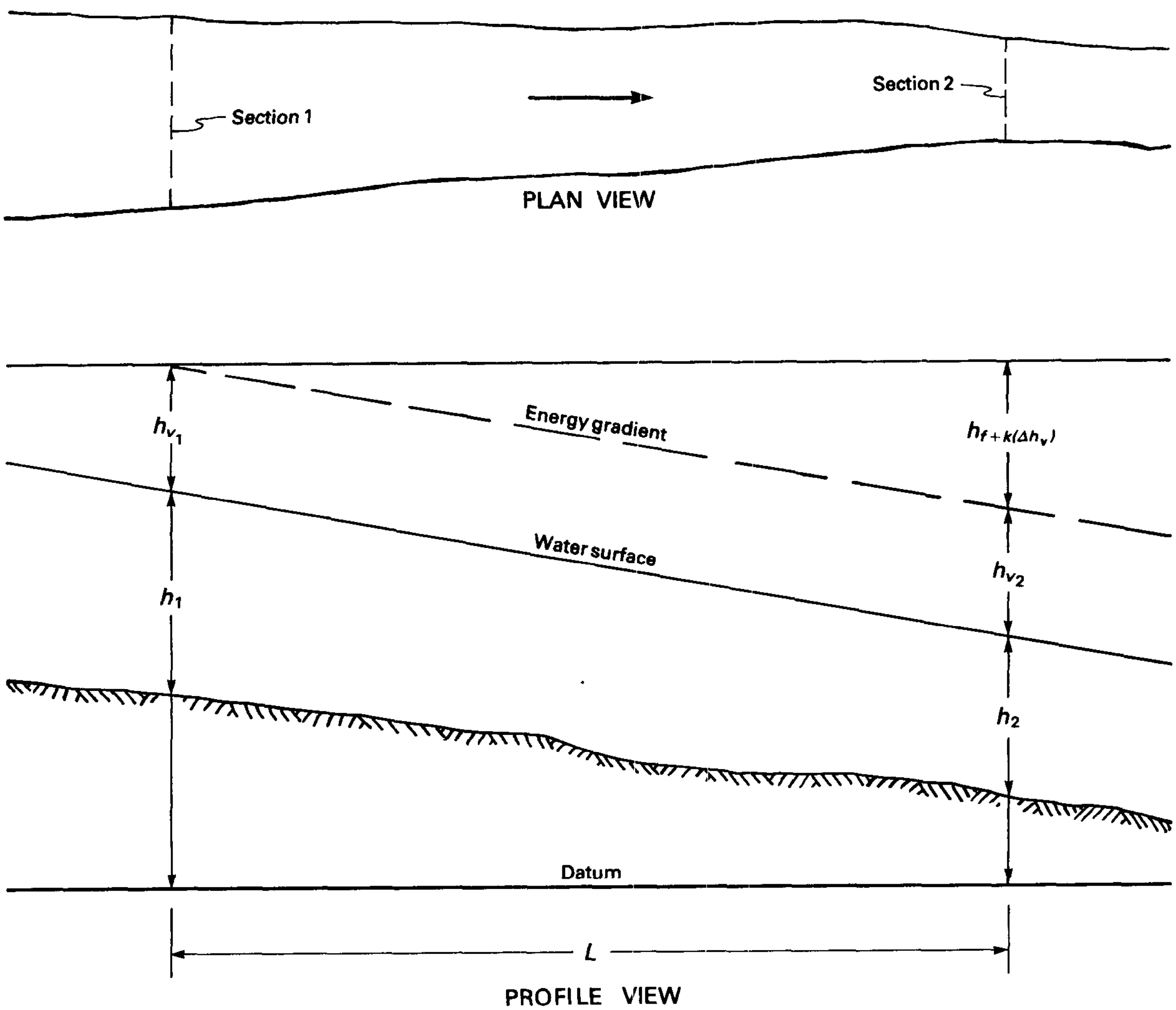


Figure 14.—Definition sketch of an open-channel flow reach.

the distribution of roughness) and channel geometry (area, hydraulic radius, and depth) all are as nearly constant as possible throughout the subreach. The uniformity of flow is measured by the degree to which the water-surface profile and the energy gradient are parallel to the streambed. The energy equation for this reach is:

$$(h+h_v)_1=(h+h_v)_2+(h_f)_{1\text{-}2}+k(\Delta h_v)_{1\text{-}2},$$

where

h = elevation of the water surface at the respective sections above a common datum,

h_v = velocity head at the respective section = $\alpha V^2/2g$,

h_f = energy loss due to boundary friction in the reach,

Δh_v = upstream velocity head minus the downstream velocity head,

$k(\Delta h_v)$ = energy loss due to acceleration or deceleration in a contracting or expanding reach, and

k = a coefficient, 0.5 for expanding reaches, and zero for contracting reaches.

The friction loss in the subreach is defined as

$$h_f=LQ^2/K_1K_2,$$

where Δh is the difference in water-surface elevation at the two sections, L is the flow distance through the subreach, Q is the total discharge, and K is the conveyance at the cross section. The mean conveyance through the subreach is computed as the geometric mean of the conveyance at the end sections. This procedure is based on the assumption that the conveyance varies uniformly through the reach.

The velocity head (h_v) at each section is computed as

$$h_v=\frac{\alpha V^2}{2g},$$

where V is the mean velocity in the section and α is the velocity-head coefficient. The value of α is assumed to be 1.0 if the section is not subdivided. The value of α in subdivided channels is computed as

$$\alpha=\frac{\Sigma\,(k_i^3/a_i^2)}{K_T^3/A_T^2},$$

where the subscript i refers to the conveyance or area of the individual subsections and subscript T refers to the area or conveyance of the entire cross section.

The energy loss, h_e, due to contraction or expansion of the channel in the reach is assumed to be equal to the difference in velocity heads at the two sections (Δh_v) multiplied by a coefficient k. The value of k is taken to be zero for contracting reaches and 0.5 for expanding reaches. Coefficient k may also be defined as follows.

If $[\alpha_2 - \alpha_1 (A_2/A_1)^2]$ is $\geqslant 0$, k=0; if < 0, k=0.5. Both the procedure and the coefficient are questionable for expanding reaches, however. Major expansions therefore should be avoided, if possible, in selecting locations of cross sections in a step-backwater reach. Where expansions are unavoidable, more frequently placed cross sections will tend to minimize the relative degree of expansion between them and leave the individual subreaches more nearly uniform within themselves.

The value of Δh_v is computed as the difference between upstream and the downstream velocity head; thus, the friction loss term is computed algebraically as

$$h_f=\Delta h+(\Delta h_v/2) \qquad \text{(when } \Delta h_v \text{ is positive)},$$

and

$$h_f=\Delta h+\Delta h_v \qquad \text{(when } \Delta h_v \text{ is negative)}.$$

The effect of α on coefficient k and the friction loss term should be noted. A geometrically contracting reach would have a larger velocity at the downstream end, and Δh_v would be expected to be negative. If the distributions of conveyance at each cross section are such, however, that α_1 is larger than α_2, then the term Δh_v may become positive and affect the value of h_f. The opposite change may occur if α_2 is larger than α_1.

The role and importance of α is described in more detail in a subsequent section entitled "Velocity-Head Coefficient, α."

Standard step method

Subcritical flows

The individual steps in the solution of the energy equation for tranquil flow by the step-backwater method are listed below. Reference is made to figure 14.

1. A discharge, for which the water-surface profile is to be determined, is chosen.
2. All necessary channel geometry and roughness information in the lateral, longitudinal, and vertical directions is obtained. Subdivisions are chosen and subreach lengths are computed.
3. The water-surface elevation, h_2, at the downstream end is chosen.
4. For the value of h_2 chosen in step 3, the corresponding area, conveyance, velocity head, and α values are computed for the downstream section.
5. A water-surface elevation, h_1, is assumed for the upstream cross section.
6. For the value of h_1 chosen in step 5, the corresponding area, conveyance, velocity head, and α values are computed for the upstream cross section.
7. The friction loss between sections 1 and 2 is computed, $(h_f)_{1\text{-}2}=LQ^2/K_1K_2$.
8. The coefficient k is determined; k is 0.5 if Δh_v is positive, and zero if Δh_v is negative.

9. The energy equation is solved. If the equation is acceptably balanced, the next operation is step 12.
10. If the energy equation is not balanced within an acceptable predetermined tolerance, a new value of h_1 is chosen for the upstream water-surface elevation.
11. Steps 5 through 10 are repeated until the energy equation is satisfactorily balanced.
12. The solution moves one step, or subreach, farther upstream. The value of h_1 at the upstream end of the first subreach is now equivalent to the value of h_2 at the downstream end of the new subreach. This operation is equivalent to step 3, above.
13. Steps 4–12 are repeated subreach by subreach until the water-surface profile throughout the entire reach has been computed.

If the first value of h_2 in step 3 for the most downstream cross section is above the normal-depth line, the profile computed will follow an M1 curve; if h_2 is started originally at an elevation below normal depth, the computed profile will follow an M2 curve. To determine the normal-depth line in a channel, the procedure is to choose two or more starting values of h_2 at the most downstream cross section, and, for the same discharge, compute the resultant profiles until these profiles all converge farther upstream, and thereafter give identical values of water-surface elevation at succeeding cross sections. Limitations to this method of determining convergence are discussed in the section entitled "Convergence of Backwater Curves."

Because of the trial-and-error nature of the solution of the energy equation, manually determining water-surface profiles is extremely tedious. Computer programs for the determination of water-surface profiles by the step-backwater method are available for subcritical-flow conditions (Shearman, 1976).

Supercritical flows

For supercritical-flow conditions, the standard step method of computing water-surface profiles, as described above for subcritical flows, is applied similarly, but in a downstream direction. With reference to figure 14, the first step is to choose the upstream elevation, h_1, and then to balance the energy equation by choosing an appropriate value of h_2 for the downstream cross section. The solution progresses subreach by subreach in the direction of the flow until the water-surface profile is determined throughout the entire length of reach in which the flow is supercritical. It is advantageous to choose the upstream elevation of the first subreach at critical depth, because generally, supercritical-flow computations would begin at a control point in natural channels.

Much of the tediousness of a manual computation of a supercritical-flow water-surface profile is alleviated by partial use of an electronic computer. Computer programs for subcritical flow provide, as part of their output, tables of cross-section properties at numerous elevations for all cross sections in a reach. For each of the elevations, values of cross-sectional area, conveyance, velocity-head coefficient (α), top width, stations at left and right edges of water, and wetted perimeter are given. If a sufficiently small elevation increment is specified, it is a relatively easy matter to prepare plots or to interpolate directly from these computer tables, so that the appropriate values of area, conveyance, and α can be quickly determined for any elevation. The trial-and-error procedure of balancing the energy equation is thus considerably simplified. Supercritical-flow conditions usually exist for only a few subreaches; therefore, the manual procedure described above should be used from a control point to a cross section downstream from it, at which a subcritical-flow profile solution has indicated the possibility of a hydraulic jump.

Field Data

All of the channel-geometry considerations that go into the selection of sites for slope-area and n-verification measurements apply as well for each subreach of a step-backwater reach. Some of these are discussed in the following paragraphs, together with the special requirements of the step-backwater method.

A stadia survey or equivalent for the entire stream channel is required for each study site. The surveys are run using the same basic techniques described by Benson and Dalrymple (1967) for indirect discharge measurements. A common datum must be established by levels throughout the length of the reach. Gage datum should be used in the vicinity of gaging stations.

Maps and ground elevations from photogrammetric methods or from topographic maps with contours at close intervals are practical alternatives to field surveys. Horizontal and vertical control points throughout the reach must be established.

Total reach length

Limiting total length of reach to be surveyed to the shortest useful distance is important in keeping costs of field surveys or photogrammetry reasonable. The length of reach needed to ensure convergence of computed backwater curves depends on the slope, the roughness, and the mean depth for the largest discharge for which the normal-depth profile is desired. Because the length depends on the depth, and the depth itself is the unknown which must ultimately be determined, the total reach length must be computed by estimating the normal depth.

Bresse's equations (Woodward and Posey, 1941) for backwater curves may be used to determine the distances required for M1 and M2 backwater curves to converge to the normal-depth profile. Figure 15 represents the equations in graphic form for steady, uniform, tranquil flow in a wide, rectangular channel, where the initial elevation of the M2 curve is 0.75 times the normal depth, and the initial elevation of the M1 curve is 1.25 times the normal depth. Profile convergence is accepted when the computed M2 depth is 0.97 times the normal depth, or the computed M1 depth is 1.03 times the normal depth.

The equations for the curves in figure 15 are:

$$\frac{LS_0}{Y_n}=0.86-0.64\ \mathbf{F}^2 \qquad \text{(M1 curve)},$$

and

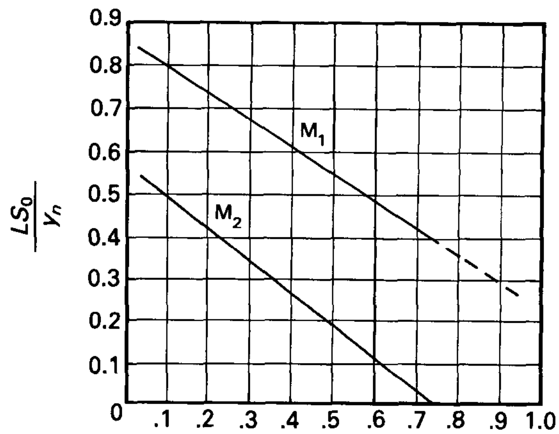


Figure 15.—Graphic determination of distances required for convergence of M1 and M2 backwater curves in rectangular channels.

$$\frac{LS_0}{Y_n}=0.57-0.79\ \mathbf{F}^2 \qquad \text{(M2 curve)},$$

where L is the required total reach length, S_0 is the bed slope, y_n is the normal depth, and $\mathbf{F}$ is the Froude number.

After an estimate of the depth is made, the Froude number for the maximum discharge to be considered can be computed for a typical cross section. By entering figure 15 along the ordinate, where $\mathbf{F}^2$ is equal to Q^2T/gA^3, corresponding values of LS_0/y_n can be determined for either an M1 or an M2 curve. A mean bed-slope value is chosen for the reach, and the total convergence length is computed.

It is evident from figure 15 that an M1 curve that has started an equivalent distance above the normal depth (1.25 y_n), as compared to an M2 curve starting the same relative distance below the normal depth (0.75 y_n), will require a much longer length of reach to converge to a comparable degree with the normal depth line.

An alternate estimate of the total length of reach required for an M2 backwater curve, starting at an elevation of about 0.75 y_n, to converge to within 3 percent of the normal-depth elevation (about 0.97 y_n), is given by the equation:

$$L=\frac{0.4\ y_n}{S_0},$$

where L, y_m, and S_0 are defined as for figure 15. This equation is equivalent to a value of $\mathbf{F}^2$ of about 0.2 on the M2 curve of figure 15, or a value of $\mathbf{F}$ of about 0.45, which is representative of most natural flows.

Because channel roughness has a minor effect on the rate of convergence of computed backwater curves, n is neglected in these approximations of total length.

Locations of cross sections

In natural stream channels, cross sections are placed at intervals which will divide a total reach into a series of subreaches each of which is as uniform in geometry and roughness as practical. Dividing a reach is a relatively easy matter in slope-area and n-verification studies, where reaches are chosen for their conformity to ideal conditions. In profile computations through long stretches of a river, one must work with conditions as they are. Frequently they are far from ideal. Fairly uniform channels will require fewer cross sections than those having many irregularities in size, shape, slope, or roughness. The cross sections should be representative of the reach between them and should have nearly the same characteristics. They should be located to enable proper evaluation of energy losses. With reference to figure 14, cross sections should be located at such intervals that the energy gradient, the water-surface slope, and the streambed slope are all as nearly parallel to each other as possible and as close to being straight lines as possible. If any channel feature causes one of these three profiles to curve, break, or run unparallel to the others locally, this is a clear indication that that particular subreach should be further subdivided. If cross sections are located according to the general criteria listed below, reasonable evaluations of energy losses can be made. Many of the criteria apply equally as well to slope-area and n-verification reaches.

Cross sections:

1. Should be located at all major breaks in bed profile. If old flood profiles are available, cross sections should also be placed at major breaks in the known water-surface profile.
2. Should be placed at points of minimum and maximum cross-sectional areas.
3. Should be placed at shorter intervals in expanding reaches and in bends to minimize errors, because areas with upstream flow, dead water, or flow at an angle cannot be evaluated quantitatively. To represent flow by the relation $Q=KS^{1/2}$, it is necessary for the distribution of discharge across any section to be similar to the distribution of conveyance in that cross section. In the computations, it is further assumed that all flow is downstream and perpendicular to the cross sections. These assumptions are violated at expansions, embayments, and bends, where eddies or dead water may exist.
4. Should be placed at shorter intervals in reaches where the conveyance changes greatly as a result of changes in width, depth, or roughness. Because friction losses within subreaches are computed with a conveyance equal to the geometric mean of the end conveyances, the relation between upstream conveyance, K_1, and downstream conveyance, K_2, should satisfy the criterion: $0.7 < (K_1/K_2) < 1.4$. Conveyance, if it varies between cross sections, should do so at a uniform rate.
5. Should be located at points where roughness changes abruptly, for example, where the flood plain is heavily vegetated or forested in one subreach, but has been cleared and cultivated by the land user at the adjacent subreach. In such an instance, the same cross section should be used twice, once as part of the rougher reach, and once again only a foot or two away, as part of the smoother reach. Because $h_f=LQ^2/K_1K_2$, and L is extremely small, the effects of the error in h_f are minimized. If flow from an upstream cross section with clear flood plains reaches a cross section where the overbanks are heavily vegetated, the condition is akin to a contracted opening. Similarly, if flow from an upstream cross section with heavily vegetated flood plains reaches a cross section where the flood plains

are clear and the roughness coefficient is relatively much smaller, the condition is akin to expanding flow at the downstream end of a constriction. There are no adequate guidelines in these two situations for properly determining friction losses, contraction losses, and expansion losses, nor for computing the water-surface profiles. The use of a cross section twice, in close proximity, and with different roughness values, must suffice for the present.

6. Should be placed between sections that change radically in shape, even if the two areas and the two conveyances are nearly the same. (Consider, for example, sections that change shape from just a main channel to a main channel with overbank flow, or from triangular to rectangular.)
7. Should be placed at shorter intervals in reaches where the lateral distribution of conveyance in a cross section changes radically from one end of the reach to the other, even though the total area, total conveyance, and cross-sectional shape do not change much. In general, the cross section having more subdivisions will have a larger α. A large value of α can have as much effect on the magnitude of a velocity head as can a change in cross-sectional area.
8. Should be placed at shorter intervals in streams of very low gradient which are significantly nonuniform, because the computations are very sensitive to the effects of local disturbances or irregularities. These effects can be reflected far upstream. Shorter subreaches may help to reduce these effects. See the section entitled "Local Effects on Profiles."
9. Should be located at and near control sections, and at shorter intervals immediately downstream from control sections, if supercritical-flow conditions exist.
10. Should be located at tributaries that contribute significantly to the main stem. The cross sections should be placed such that the tributary would enter the main stem in the middle of a subreach.
11. Should be located at bridges in the same locations as required for computations of discharge at width constrictions (see Matthai, 1967):
    - (a) just downstream of the bridge, across the entire valley,
    - (b) at the downstream end of the bridge, within the constriction,
    - (c) at the approach cross section, one bridge-opening width upstream,
    - (d) if there is road overflow, along the higher of either the crown or curb, including the road approaches to the bridge, and the bridge deck.

It would seem, from a perusal of the list of suggested cross-section locations above, that the effects of almost all the undesirable features of nonuniform, natural stream channels can be lessened by taking more cross sections. This is true, but consideration must also be given to the time, cost, and effort to locate and survey additional cross sections. A balance must be set between the number of cross sections deemed desirable, and the number that is practical. These criteria for cross-section locations serve, therefore, to call attention to the considerations behind the need for cross sections, and to help the engineer to understand anomalies in computed profiles if cross sections are omitted. Practice will provide the engineer with the experience necessary to anticipate the circumstances that would permit relaxation of, or elimination of, some of these criteria, and the probable nature and magnitude of resultant errors.

Individual subreach lengths

In addition to the criteria for locations of cross sections, there are a few considerations as to reach lengths and number of subreaches which will influence the selection of cross sections:

1. The total reach length should be divided into at least 10–12 subreaches to be reasonably sure of convergence.

2. No subreach should be longer than about 75–100 times the mean depth for the largest discharge to be considered, or about twice the width of the subreach. This is a maximum limit, and applies only if other considerations for cross-section locations do not control.
3. The fall in a subreach should be equal to or greater than the larger of 0.50 foot, or the velocity head, unless bed slope is so flat as to require defaulting to the second criterion in this list.
4. The subreach length should be equal to or less than the downstream depth (for the smallest discharge to be considered) divided by the bed slope.
5. If the bed profile is a convex or concave curve, the reach should be broken up into shorter subreaches, because the losses within the reach might not be properly accounted for. In this method of computation, straight-line variation of bed elevations between end cross sections is assumed.

Weighted length of a subreach

If water-surface profiles for several discharges are to be computed, the lengths between any two cross sections may have to be computed differently for different discharges. Small discharges would stay entirely within banks and follow the meanders of the main channel. The length for the subreach would be a maximum. Large discharges may have flood-plain flows, and their effective flow distances would be shorter.

For overbank flows, a weighted or effective subreach length must be used. The centroids of conveyance in the subsections of each cross section are determined and connected through the subreach by curvilinear or straight lines. One line will follow the main channel, and the others will be along the flood plains. The length of the main channel is multiplied by the conveyance for the main channel, and the lengths along the flood plain are multiplied by the corresponding overbank conveyances. The sum of these products is divided by the total conveyance to obtain the weighted subreach length.

Profile computations for a range of discharges may require one set of subreach lengths for all discharges within banks and another set of subreach lengths for discharges with overbank flow. Extra cross sections may be necessary within the main channel for the lower discharges to satisfy some of the criteria listed in the section entitled "Locations of Cross Sections."

Cross-section attributes

Up to 200 points may be used to define the shape of each cross section in the U.S. Geological Survey's computer solution (Shearman, 1976). Data for each point are a ground elevation and a transverse station number (distance from a reference point on the left bank) which increases in magnitude toward the right bank. At cross sections that start with negative station numbers, the stationing would be less negative toward the right bank. The ends of the cross section must be extended higher than the expected water-surface elevation of the largest flood that is to be considered in the subreach.

All cross sections must be perpendicular to the flow direction throughout the entire width of the cross section. If the flow pattern warrants a broken, or dog-legged cross section, each subsection of the cross section should be perpendicular to the direction of flow through that subsection. All subsections of a cross section should be straight lines.

Cross sections should not cross each other within the live-flow boundaries of the channel, nor should they ever be drawn so as to share a common subsection on one end. There must be a measurable longitudinal distance between each subsection of one cross section, and a corresponding subsection of the adjacent cross sections, upstream and downstream.

Cross sections at bridges and at road-overflows are described in detail in the section entitled "Bridges."

Subdvisions of cross sections

Criteria for subdividing cross sections are the same as those described by Benson and Dalrymple (1967) for indirect measurements of discharge. Subdivision should be done primarily for major breaks in cross-sectional geometry. Besides these, major changes in roughness may call for additional subdivisions. The roughness coefficients verified by the Geological Survey (Barnes, 1967) are based on unit cross sections that have complete or nearly complete wetted perimeters. Basic shapes that are approximately rectangular, trapezoidal, semicircular, or triangular are unit sections having complete wetted perimeters. Subdivisions for major breaks in geometry or for major changes in roughness should, therefore, maintain these approximate basic shapes so that the distribution of flow or conveyance is nearly uniform in a subsection.

The importance of proper subdivision, as well as the effects of improper subdivision, can be illustrated very dramatically. In figure 16, a trapezoidal cross section having heavy brush and trees on the banks has been subdivided near the bottom of each bank because of the abrupt change of roughness there. A large percentage of the wetted perimeters (P) of the triangular subareas (A_1 and A_3) and possibly of the main channel (A_2) is eliminated. A smaller wetted perimeter abnormally increases the hydraulic radius ($R=A/P$), and this in turn results in a computed conveyance different from the conveyance determined for a section with a complete wetted perimeter. In figure 16, a conveyance (K_T) has been computed for the cross section that would require a composite n value of 0.034. This is less than the n values of 0.035 and 0.10 that describe the roughness for the various parts of the basic trapezoidal shape. The basic shape should be left unsubdivided, and an effective value of n somewhat higher than 0.035 should be assigned to this cross section, to account for the additional drag imposed by the larger roughness on the banks.

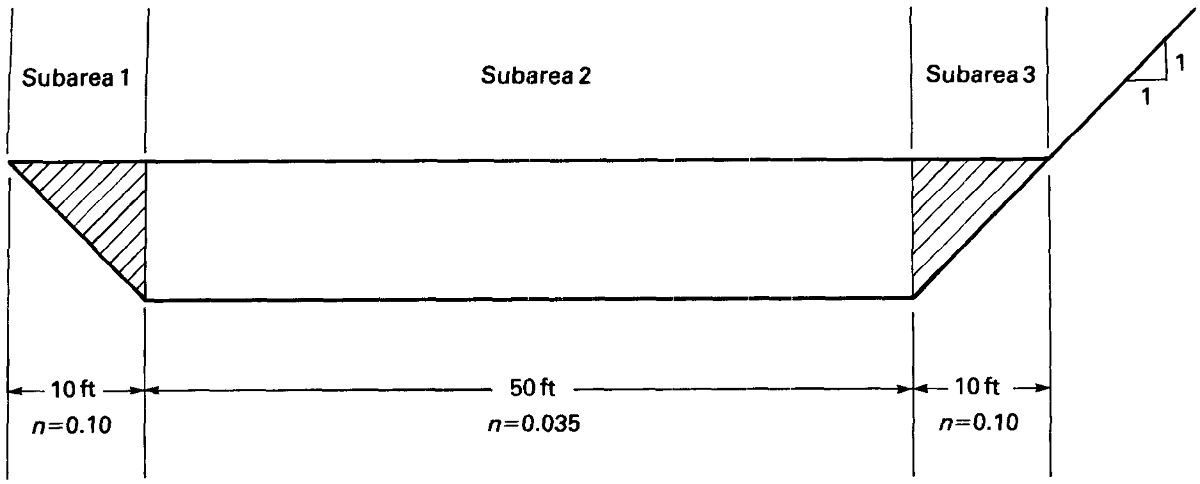
At the other extreme, the panhandle section in figure 17, having a main channel and an overflow plain, must be subdivided into two parts having nearly complete wetted perimeters. The value of n is 0.040 throughout the section. If the section is not subdivided, the increase in wetted perimeter of the flood plain is relatively large with respect to the increase in area. The hydraulic radius is abnormally reduced, therefore, and a fictitious, lower value of 0.028 for n is needed to obtain the conveyance equivalent to that of a unit section. It is clear that irregular cross sections such as that in figure 17 should be subdivided to create individual basic shapes.

The cross section shapes in figures 16 and 17 represent two extremes of the problems associated with improper subdivision. Between these two are shapes with benches or terraces, as shown at the top of figure 18. R. H. Tice, 1973, "Subdivision and the Hydraulic Radius Term in Flood-Flow Formulas:" (written communication) has suggested criteria which are generally quite satisfactory for subdivision of bench panhandle shapes. He recommends subdivision if the value of the ratio L/y is 5 or greater (see figure 18). He also recommends subdivision of a large, flat triangular shape with a central angle of 150° or more, because L/y would be about 5 or greater. If the value of L/y is almost 5, he recommends that the subdivision be made at a distance of about $L/4$ from the edge of water. For L/y equal to 20 or greater, Tice recommends several subdivisions. For very large values of L/y, the cross-sectional slope would become flatter, the depth would become more uniform, and, consequently, the distribution of velocities would be more uniform; no subdivisions would be required on the basis of shape alone, but subdivisions on the basis of roughness distribution would be permissible.

Another shape criterion for subdivision has been proposed by Matthai (oral communication, 1973) and is shown in figure 19. If the main-channel depth is more than twice the depth at the stream edge of the overbank area, Matthai recommends subdivision.

Subdivision on the basis of geometry should be coordinated with the expected range in depths. For example, the cross section of figure 20 should be subdivided differently for different stages, as shown in the figure.

There are borderline cases in which the decision to subdivide could go either way. Subdivisions in adjacent sections should be similar to avoid large differences in α. Therefore, if a borderline case is between sections that do not require subdivision, do not subdivide; if between sections that must be subdivided, subdivide the borderline case. Where the section in question is between one that



$A_1=A_3=50$
$P_1=P_3=14.14$
$R_1=R_3=3.54$
$K_1=K_3=1730$

$A_2=$ 500
$P_2=$ 50
$R_2=$ 10
$K_2=98{,}500$

$K_T=K_1+K_2+K_3$
$=102{,}000$

Composite solution: $A_c=600$
$P_c=$ 78.3
$R_c=$ 7.66

$$n_c=\frac{1.486\,A_c R_c^{2/3}}{K_T}=0.034$$

Figure 16.—Effects of subdivision on a trapezoidal section.

must be subdivided and one that should not be subdivided, proceed so that the sections are the best representation of a uniform reach.

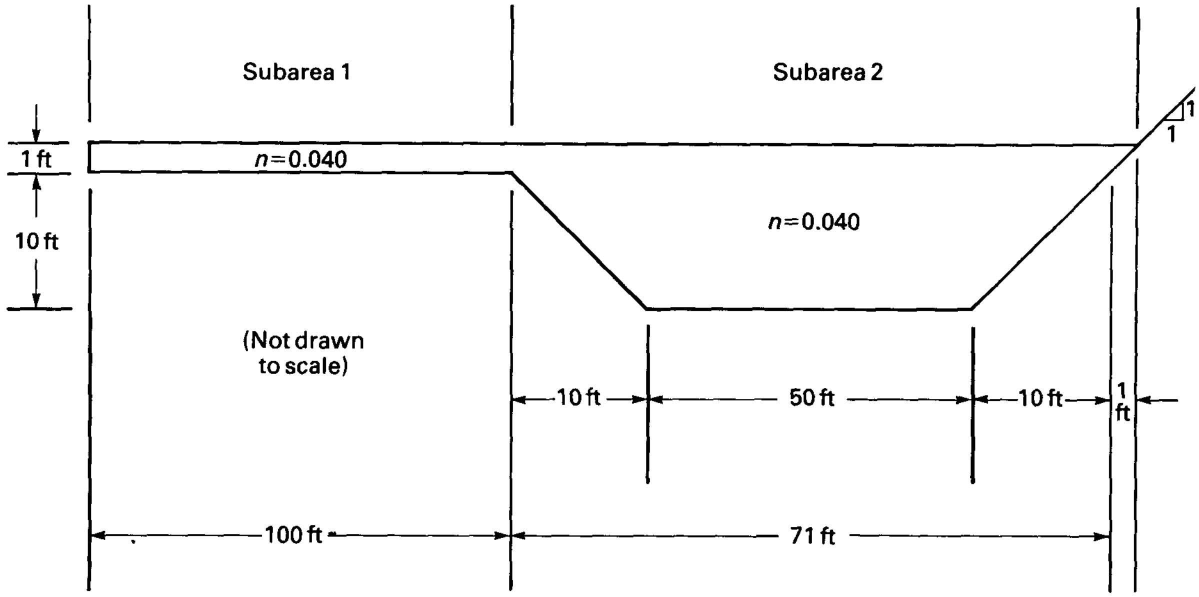
Besides the shape and roughness criteria for subdivision described here, the step-backwater computer program used by the Geological Survey requires additional subdivision points.

One example of computer-required subdivision of cross sections is associated with the Froude number test limit. The Froude number, as defined in the section on "Locus of Critical-Depth Stages," is expressed by the equation

$$F=\frac{\sqrt{\alpha}\,Q/A}{\sqrt{g}\sqrt{(A/T)}},$$

in which the value of α is frequently close enough to unity to be ignored. The Froude number does not directly enter computations in the step-backwater method; instead, it serves as a mechanism to warn of the possibility of supercritical-flow conditions. The Froude number for a cross section with relatively shallow overflow is not a reasonable index of either subcritical or supercritical flow. It is a reasonable index for a unit section such as a trapezoidal main channel. The Froude number for the main channel is considered a more reasonable index of flow conditions than Froude numbers for overbank subsections. Therefore, a cross section with overbank flow should be subdivided so that the main-channel subsection will have the largest conveyance throughout the range of water-surface elevations expected. To be certain that this relation holds, extra subdivisions may be necessary on wide overbank areas or those having very low roughness coefficients. Then the index Froude number for the main channel can be computed from the equation:

$$FRDN=\frac{Q_T\,K_L}{K_T\,A_L\sqrt{g\,A_L/T_L}},$$



$A_1 = 100$	$A_2 = 670.5$	$K_T = K_1 + K_2$
$P_1 = 101$	$P_2 = 79.7$	$= 107{,}000$
$R_1 = 0.990$	$R_2 = 8.41$	
$K_1 = 3700$	$K_2 = 103{,}000$	

Composite solution: $A_c = 770.5$
$P_c = 108.7$
$R_c = 4.26$

$$n_c = \frac{1.486\,A_c R_c^{2/3}}{K_T} = 0.028$$

Figure 17.—Effects of subdivision on a panhandle section.

Subscripts T and L designate values for the total cross section and the subsection having the largest conveyance, respectively.

Other subdivisions are required where there are bridges within the reach. Approach cross sections at bridges should be subdivided on the basis of the location of the K_q subsection (Matthai, 1967), in addition to all the other considerations mentioned above.

The Geological Survey computer program provides for up to 20 subdivisions per cross section. As many of these should be used as necessary to adequately define α for the cross section.

Figure 21 illustrates the criteria for subdivision of an approach cross section at a bridge. The solid lines represent subdivision on the basis of shape or roughness. In addition, the left overbank is subdivided (dashed line) to ensure that the subsection with the largest conveyance will be the main channel. Within the main channel section, another two subsections are created by the limits of the K_q subsection, representing the upstream projection of the boundaries of the constriction.

Velocity-head coefficient, α

Effects of the velocity-head coefficient have been mentioned in the discussion of figure 14 and the energy equation. It is important to understand the role of α and the method of computing it in open-channel flow computations.

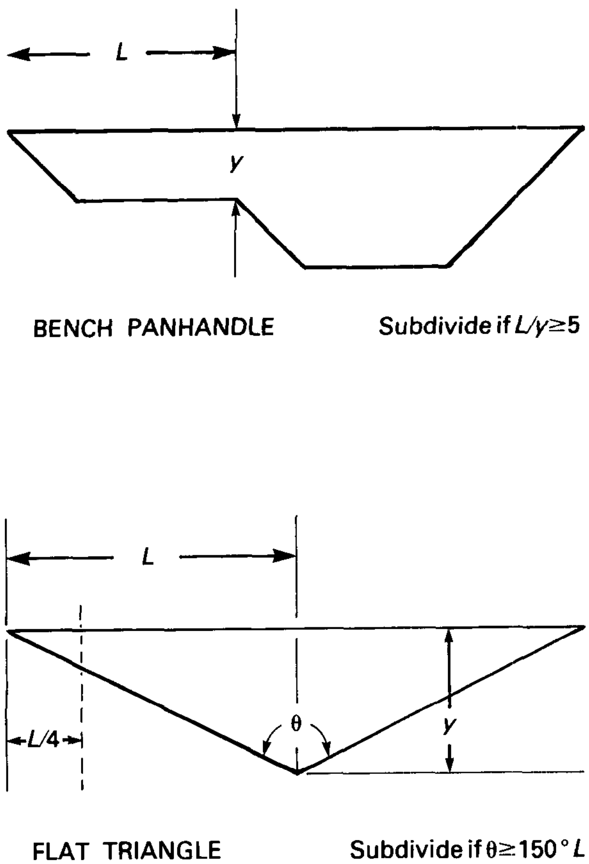


Figure 18.—Subdivision criteria of Tice (written communication, 1973).

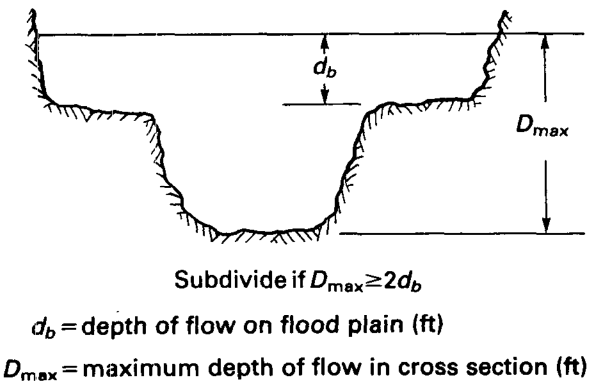


Figure 19.—Subdivision criterion of Matthai (oral communication, 1973).

Channel roughness, nonuniformities in channel geometry, bends, and obstructions upstream are some of the numerous factors that cause variations in velocity from point to point in a cross section. The true velocity head is greater than the value computed from the expression, $V^2/2g$, where V is the mean velocity in the cross section, because the square of the average velocity is less than the weighted average of the squares of the point velocities. The ratio of the true velocity head to the velocity head computed on the basis of the mean velocity is the velocity-head coefficient, α.

The average velocity head in a cross section is defined as the discharge-weighted mean of the velocity heads in its constituent subsections. Each subsection is taken to represent a zone of uniform velocity. The velocity-head coefficient is computed as the integral

$$\alpha=\frac{\int Q\,(v^2/2g)\,dQ}{Q\,(V^2/2g)},$$

where v represents the mean velocity in a subsection, V is the mean velocity in the cross section, and Q is the total discharge.

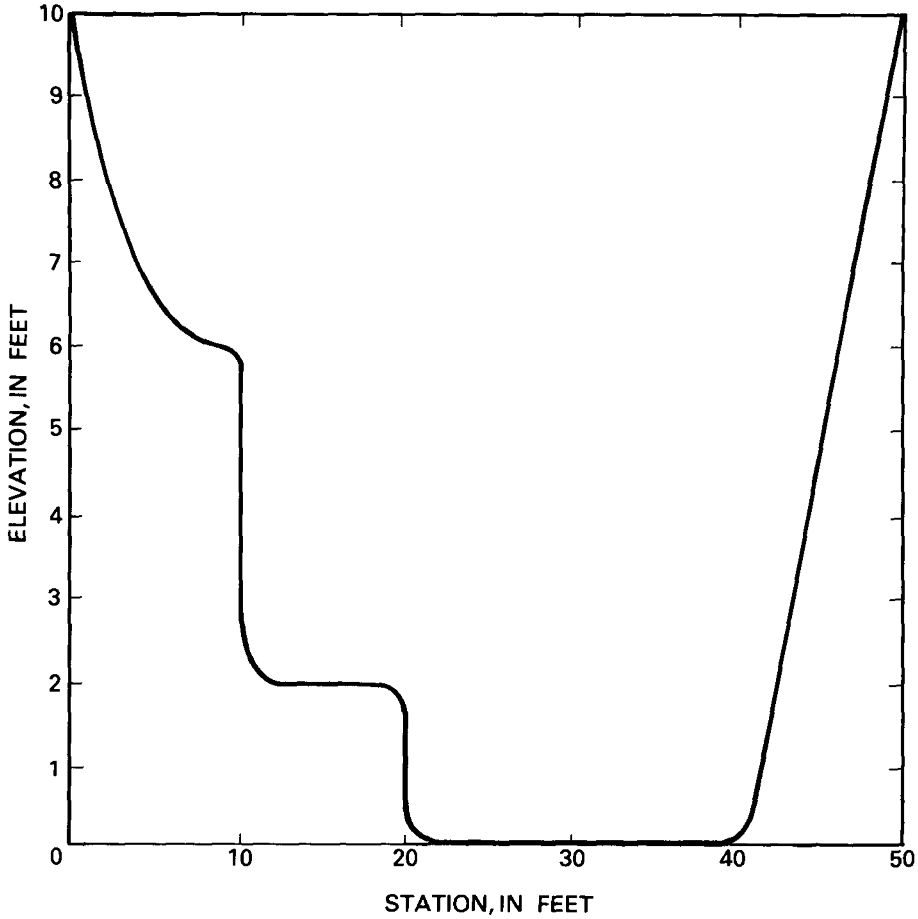
The distribution of velocity in a cross section is not known. Therefore, a cross section is divided into subsections on the basis of geometry and of roughness characteristics to approximate uniform velocities in each subsection. Then the assumption is made that the differential dQ in the equation above may be replaced by q, the discharge within the subsection, and that q in turn is equal to av, where a is the subsection area. The integral can be approximated by a summation across the cross section, such that

$$\alpha=\frac{1}{A}\Sigma\,[(v/V)^3a].$$

If the subdivisions of the total cross section do indeed create subsections of uniform velocity, then the distribution of discharge can be represented by the distribution of conveyance. By use of the Manning equation, the following substitutions are made:

in each subsection, $v=kS^{1/2}/a$, and
in the total cross section, $V=KS^{1/2}/A$,

where k and K represent subsection and total conveyances, and S is the slope of the channel.



Range of water-surface elevation, in feet	Substation: Station 20	Substation: Station 10
0–2	No	—
2–4	Yes	—
4–6	No	—
6–8	No	Yes
8–10	No	No

Figure 20.—Cross section in which subdivision could be dependent on expected elevations of water surface.

The latter is assumed to be identical in each subsection. The expression for the velocity-head coefficient thus reduces to the familiar form,

$$\alpha=\frac{\Sigma(k^3/a^2)}{K^3/A^2},$$

In general, the more subdivisions in a cross section, the larger α will become. Because of the effect α has in the velocity-head term, $\alpha V^2/2g$, the cross section should be subdivided, as needed, to validate as nearly as possible the assumptions mentioned.

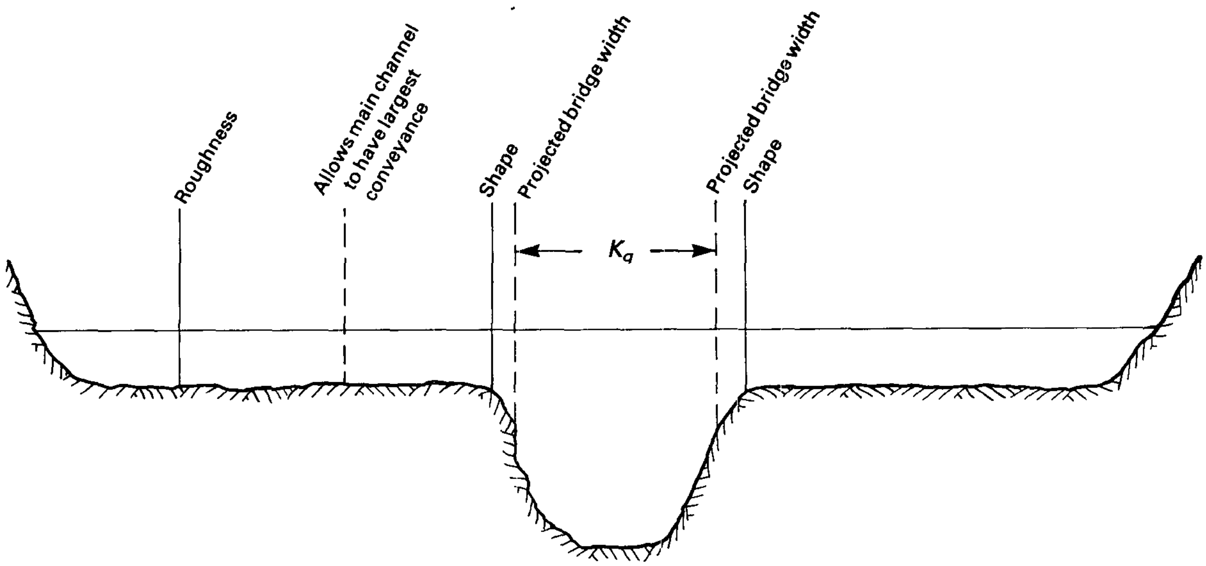


Figure 21.—Subdivision of an approach cross section at a bridge.

In manual computations, it is possible to account for dead water or negative flows in parts of a cross section by assigning values of zero or of negative numbers for the subsection conveyances. Coefficient α will, therefore, be properly computed. In machine computations, however, it is not easy to assign zero or negative values because of the implicit understanding that conveyance and discharge are similarly distributed across a cross section. This implicit understanding is particularly important at bends, embayments, and expansions, and at cross sections downstream from natural and manmade constrictions. If dead water or upstream flow is suspected, subdivisions should isolate these parts. Then, by omitting the subsections or assigning very large n's to them, a better α will be computed.

Instances have been reported of values of α in excess of 20, with no satisfactory explanations for the enormous magnitude of the coefficient. If adjacent cross sections have comparable values, or if the changes are not sudden between cross sections, such values can be accepted, but if the change is sudden, some attempt should be made to obtain uniformity. This could be done with more cross sections to achieve gradual change, or it can be done by resubdividing the cross section. Some subsections can be consolidated (as long as the main-channel subsection remains the largest one), but compensation should probably be made for the fewer subdivisions, possibly by increasing the roughness coefficients somewhat or by decreasing areas somewhat.

Roughness coefficients

Criteria for the selection of roughness coefficients for indirect discharge measurements are described in detail by Benson and Dalrymple (1967), and they all apply equally well to computations of water-surface profiles by the step-backwater method. In addition to these, Barnes (1967) provides color photographs and details of channel geometry for many slope-area reaches in which values of n, the roughness coefficient, were verified.

The Survey's computer program has provision for varying n with depth. (The depths referred to in this paragraph are mean hydraulic depths.) For each subsection, two key mean depths may be selected, with a value of n for each. At all flows for which the mean depth corresponding to the water-surface elevation is equal to or less than the lower key mean depth, the subsection roughness coefficient will have the value selected for that key depth. At all flows for which the mean depth corresponding to the water-surface elevation is equal to or greater than the higher key mean depth, the subsection roughness coefficient will have the value selected for that key depth. For a flow whose mean depth corresponding to the water-surface elevation lies between the two key mean depths, the value of the roughness coefficient is interpolated. The coefficient of the larger key mean depth can be set equal to, larger than, or smaller than that at the smaller key mean depth, thus providing for considerable flexibility in defining the roughness characteristics of the subsection.

Before any water-surface profiles are computed in some regions, a decision must be made as to whether the profile should be for a summer flood or for a winter flood, because of seasonal changes in vegetation. A summer flood, when vegetation is at its peak, will require larger values of roughness coefficients, which in turn will raise the elevation of the computed profile.

Special Field Conditions

Verified reaches

Where high-water marks can be found to define flood elevations at several locations for known or estimated discharges, profiles for these events should be computed. When the computed profiles match the high-water marks, the computations can be used to evaluate roughness coefficients selected, number and locations of cross sections, and adequacy of subdivisions. Then the final profiles for the selected discharges should be computed, and they should be more reliable.

Short reaches

The part of the total surveyed reach that is used in the "convergence" phase of backwater-profile computations is generally not used to establish the normal water-surface elevation within that part of the reach. The interest is usually in the profile at a point upstream or in a reach upstream from the point of convergence. Sometimes, however, the water-surface profile is desired for a reach that is short and that cannot be extended farther downstream for physical reasons. If the reach is long enough to enable any two curves from among the M1–M2 family to converge at the normal depth at the upper end of the surveyed reach, a closer estimate of the elevation of normal depth at the downstream end is possible (see figure 22). A new pair of M curves, closer to y_n, can be computed. These will converge in a shorter distance and will verify the previously computed normal depth at the upper end. In this way the normal-depth profile is established for a greater part of the reach, and more benefit accrues from the data collected.

A manual computation of the profile in the downstream end of a short reach is also possible. The individual steps in the solution of the energy equation by the standard step-backwater method are described in the section entitled "Subcritical Flows." Many of the otherwise tedious trial-and-error operations of a manual computation are reduced by the information from the initial computer run that has established the normal depth at the upstream end of the reach. All necessary cross-section properties will be available. Although step-backwater computations on a mild slope should progress in an upstream direction, if the normal depth is known at the upstream end of a reach, the solution for the normal-depth profile can progress in a downstream direction. Once the normal depth is established at the upper end of a subreach, the elevation computed at the downstream end of it will be for the normal depth. The reach must be reasonably uniform, however; otherwise, the solution will be erroneous.

Crossing profiles

Occasionally the profiles for several M1 or several M2 curves for a given discharge will cross each other in the reach in which they are being computed to establish convergence with the normal-depth profile. This occurs particularly where the cross-sectional area and α at one elevation in the cross section are considerably different from those at another elevation within a foot or two. For the same discharge, the velocity and, therefore, the velocity head

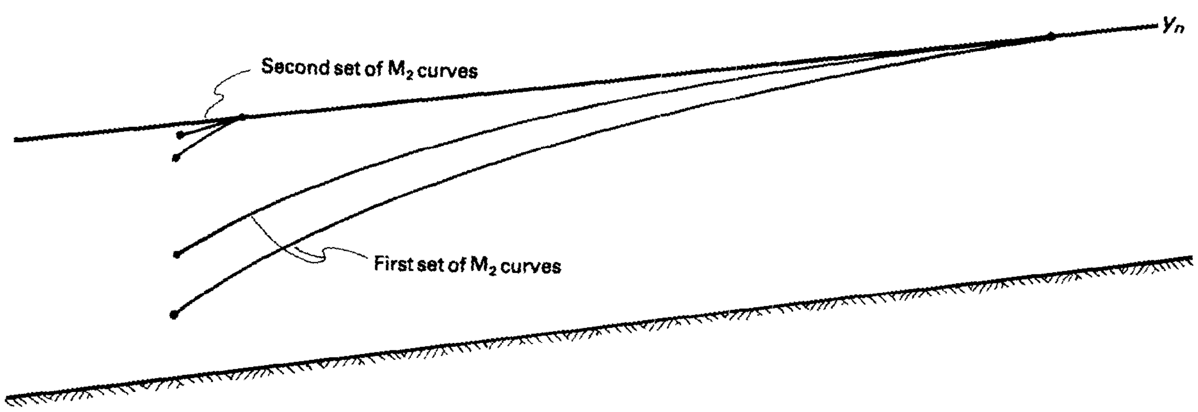


Figure 22.—Establishment of the normal-depth profile in a short reach.

may be sufficiently higher for the lower of two profiles such that the water-surface elevation computed for the next section upstream will be higher for what was the lower profile than for what was the higher profile.

Profiles will also cross in the first subreach if the starting elevation for one profile is less than the elevation of critical depth at the first cross section and the other starting elevation is above the critical-depth line. Because there is no Froude number check at the first cross section, care must be taken to ensure that starting elevations for M2 profiles are never below the critical-depth line.

Profiles can cross elsewhere in a reach if the Froude number limit is set so high as to accept otherwise supercritical solutions. Solutions involving Froude numbers larger than 1.5 should not be accepted, and computed profiles for any reaches with Froude numbers larger than unity should be closely examined.

Profiles that cross need not be more than a disconcerting problem if they occur in a steeply sloped stream, or if they occur on any M2 curves near the elevation of critical depth, where the M2 curve itself is naturally steep. Ordinarily, on steep bed slopes or on steep parts of M2 curves, the phenomenon shows up as a local perturbation that is quickly "righted" within a few subreaches. On flat slopes, however, the effects of such crossed profiles could extend far upstream. Unless crossed profiles either quickly converge or recross to their original relative positions, such a solution should be examined closely.

Transitions between inbank and overbank flow conditions

Solutions of water-surface profiles are rather straightforward either if the flow is confined to the main channel throughout the reach or if flow is a combination of main-channel and overbank flow throughout the reach. If flow conditions change from one of these to the other, there could be an interruption of the solution or an anomaly in computed values. Recognition of the circumstances under which these problems occur is essential to the proper handling or interpretation of the computations.

When flow breaks out over the banks or returns to the main channel between two cross sections, the very small change in elevation is associated with comparatively small changes in cross-sectional area and conveyance. There is a sudden change, however, in cross-sectional shape and in velocity-head coefficient, α. If there is a sudden, otherwise inexplicable jump in computed elevations both in manual and in machine computations, an abrupt change in α between the two cross sections may be the cause.

A sudden change in cross sectional shape might not create difficulties in manual computations of water-surface profiles. In machine computations, however, the solution might abort because of a Froude number problem. The explanation is that one of the overbank subsections may be larger in conveyance than the main-channel subsection; therefore, the Froude number is for the shallower depths of the overbank subsection.

If overbank subsections are further subdivided to avoid a Froude number problem, the increased number of subsections will increase the magnitude of the differences in α and the velocity-head term, $\alpha V^2/2g$. This will in turn create or compound the problem of a sudden, unreasonable change in computed water-surface elevations. On the other hand, a reduction in the amount of subdivision (and α) might induce the Froude-number problem.

Additional cross sections in the vicinity of the transition would improve the profile, but such a costly step might not be the ideal solution nor would it wholly solve the problem. Additional cross sections might be satisfactory for one discharge, but higher or lower discharges will simply translate the same problem to other points in the reach.

Either or both of the following methods should give satisfactory results for the determination of water-surface profiles in the region of transition:

A. Method of interrupting the computed profile.
   1. If the flow in the downstream reach is within the banks (over the banks) and if in the upstream reach it is overbanks (within the banks), stop the computation at the last cross section at which the flow is still inbank (overbank).
   2. Project the computed water-surface profile upstream to the next cross section where flow is out of banks (within banks) on the bases of the computed profile up to the downstream cross section and the local geometry and bed slope.
   3. Start a new profile computation at this upstream cross section, using the projected water-surface elevation in step 2 as the starting elevation.

B. Method of averaging computed profiles.
   1. Compute the water-surface profile for a discharge larger than the one under consideration so that the flow will be overbank throughout the transition reach.
   2. Compute the water-surface profile for a discharge smaller than the one under consideration so that the flow will be completely within banks in the transition reach.
   3. Estimate the profile for the given discharge through the transition reach from the profiles of steps 1 and 2.

Additional complications and uncertainties further compound the problem of sudden transitions between inbank and overbank conditions. These result from a lack of experience. For example, the sudden expansion of flow onto the flood plain from a completely inbank-flow situation is associated with tremendous expansion losses for which normally used computation guidelines may be inadequate; only 50 percent loss of energy is accounted for in an expanding reach. Conversely, the sudden drainage of overbank flows back into the main channel could be likened to a contracted opening—one for which present methods, and coefficients of contraction or of discharge for bridges, would not quite be applicable.

Flow at tributaries

As the computation of water-surface elevations progresses along the stream channel, the discharge must be known at each cross section so that the appropriate velocity heads and friction losses can be properly evaluated. At the mouth of a tributary, therefore, three discharges must be known:

1. Q_u, the main stem discharge, upstream from the confluence,
2. Q_t, the discharge in the tributary, and
3. Q_d, the main stem discharge, downstream from the confluence (sum of Q_u and Q_t).

The main stem discharge, Q_d, will be the one used up to the confluence. To continue the computations above the tributary, Q_u must be known. Unless the tributary discharge, Q_t, is known, some estimate of it must be made. Tice (oral communication, 1973) suggests the following approximation in the absence of a more reliable value:

$$\frac{Q_t}{Q_d}=\left(\frac{A_t}{A_d}\right)^{2/3},$$

where A_t is the drainage area of the tributary at its mouth, and A_d is the drainage area of the main stem just below the tributary mouth.

It is implicit that the arrivals of the peaks of discharges Q_u and Q_t at the confluence are simultaneous for the frequency of the main-stem discharge, Q_d. That is to say, the value of the 100-year discharge in the tributary should not necessarily be subtracted directly from the 100-year discharge on the main stem downstream from the confluence in order to determine the 100-year discharge on the main stem upstream. The engineer must adjust the discharges Q_u and Q_t for any significant lag time between the peaks along those channels.

When the discharges at the confluence are determined, the values of velocity heads and channel friction loss for the subreach into which the tributary flows, are computed as follows:

$$\text{downstream velocity head} = \alpha_d(Q_d/A_d)^2/2g,$$
$$\text{upstream velocity head} = \alpha_u(Q_u/A_u)^2/2g, \text{ and}$$
$$\text{friction loss, } h_f = \frac{L\,[1/2(Q_u+Q_d)]^2}{K_u\,K_d},$$

where subscripts d and u denote downstream and upstream cross sections, L is the subreach length, and K is conveyance.

Because of the averaging of discharges, where Q_u applies more nearly to the upstream part of the subreach, and Q_d applies to the downstream part, the cross sections should preferably be located at points equidistant from the tributary. The larger the tributary is, the more likely it is that K_u and K_d will be appreciably different, thus violating the criterion for proper evaluation of friction losses, $0.7<(K_u/K_d)<1.4$. Therefore, keep subreaches involving relatively large tributaries as short as practical, thereby confining uncertainties to the immediate locality.

The special case of a tributary in the immediate vicinity of a bridge is discussed in the section entitled "Bridges."

Flow past islands

If the channel in which water-surface profiles are being computed has an island so large that the paths around it are considerably different in length, slope, and roughness characteristics, each path around the island must be handled as a separate reach. For example, in figure 23, the total discharge, Q_t, is split into two unknown components, Q_L and Q_R. The computation progressing upstream has stopped at cross section A, at the downstream end of the island. The water-surface elevation at cross section U, just upstream of the island, must be computed to continue the profiles farther upstream. The problem is complicated because the division of the flow into components Q_L and Q_R is not known.

The junctions of the separate channels are considered to be similar to tributaries; and cross sections A, B_L, B_R, F_R, K_L, and U are located as described in the section entitled "Flow at Tributaries."

Each channel around the island is analyzed by establishing a stage-discharge relation for cross section U. For example, by beginning with cross section A, and working up the left channel, the water-surface profiles for various discharges, Q_L, are computed up to section U. To begin with, Q_L may be assumed to be equal to Q_T, and the water surface at U is computed. The same thing is done for several lower discharges, down to the other extreme, Q_L assumed to be zero. The stage-discharge relation is plotted as in figure 24 with water-surface elevation at U as ordinate, and Q_L as abscissa. A similar relation is plotted for the right channel as in figure 24. The intersection of the two curves determines the proper division of Q_T into components Q_R and Q_L, and it indicates the elevation of the water surface at cross section U. The computations would resume at cross section U with the starting elevation as determined from figure 24, and with the discharge, Q_T. The intersection of the two rating curves of figure

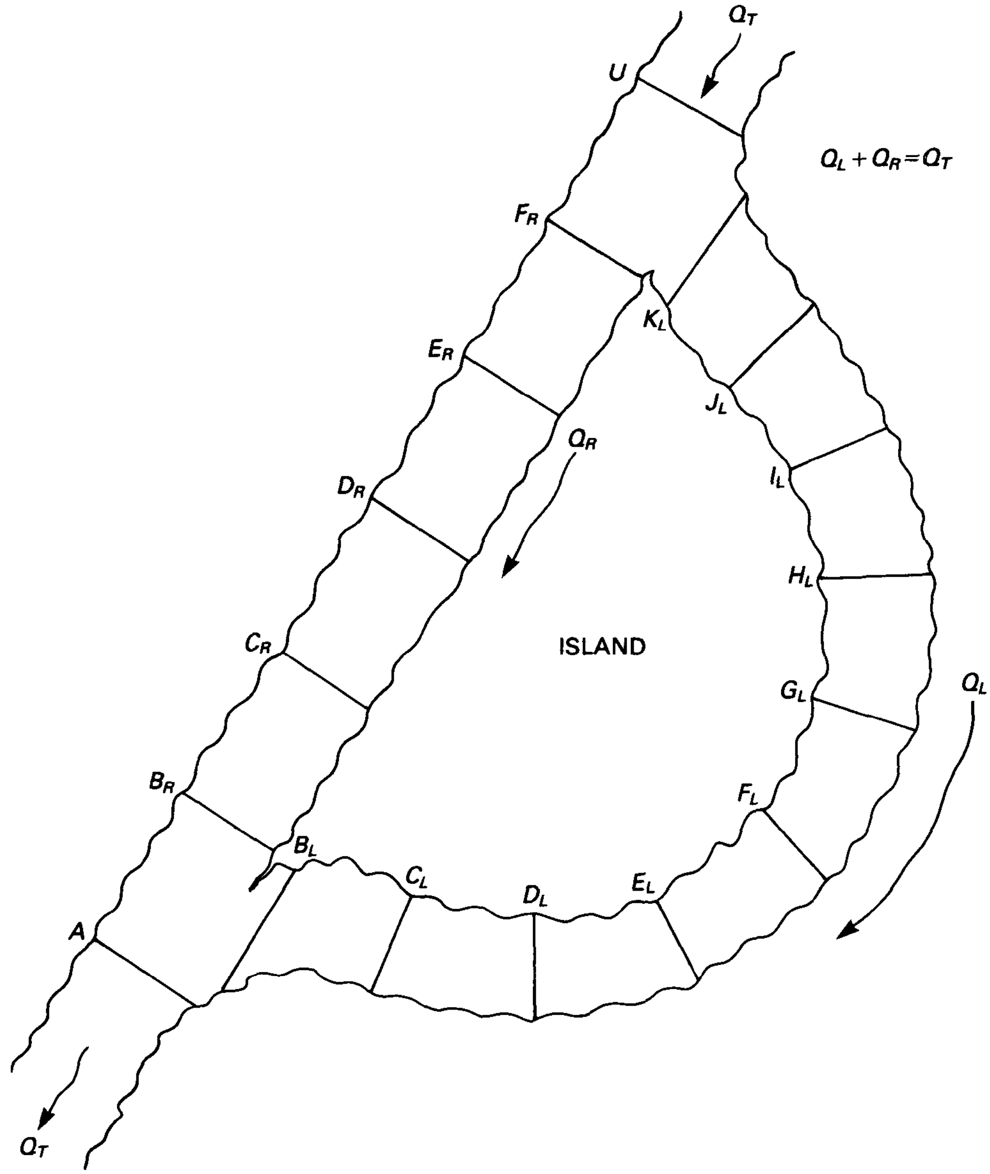


Figure 23.—Flow around an island.

24 may be defined with more precision by defining the curves with more trial runs in that vicinity.

If a known quantity of flow, Q_B, bypasses the main channel and later returns to it, the solution is greatly simplified because the division of discharge is known. There is no need to stop the computation below the point where Q_B returns to the main channel. Water-surface profile computations would progress up the main channel without interruption as follows: up to cross section A, the discharge Q_T would be used; discharge would change to $(Q_T - Q_B)$ at cross section B_R, and remain so up through

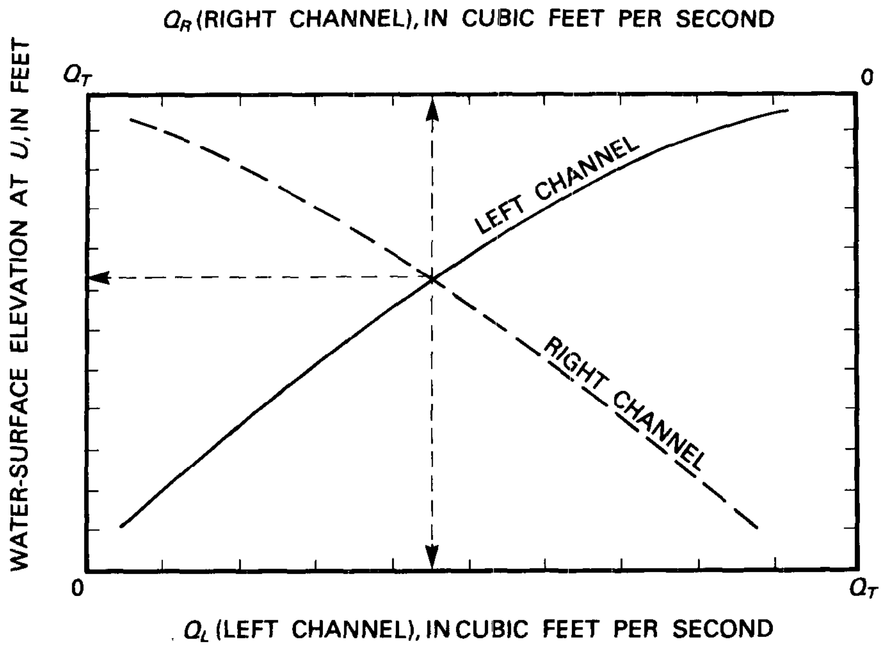


Figure 24.—Division of flow around an island.

cross section F_R; discharge would change to Q_T at cross section U, and remain so farther upstream.

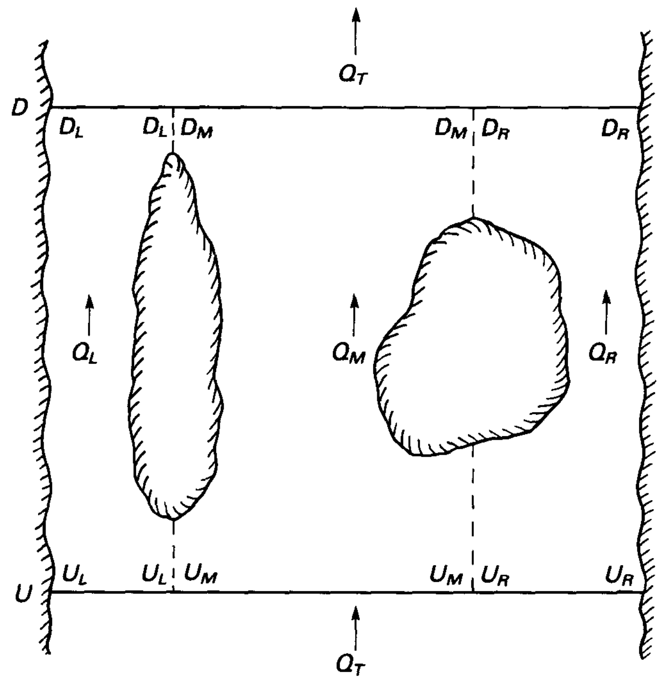
Multichannel flows

If the main-channel flow is divided into several branches rather than only two, as described for flow around an island, the following procedures are recommended. They will help to determine the discharge through each channel and the water-surface elevation upstream from the branches. The method is based on that described by Woodward and Posey (1941).

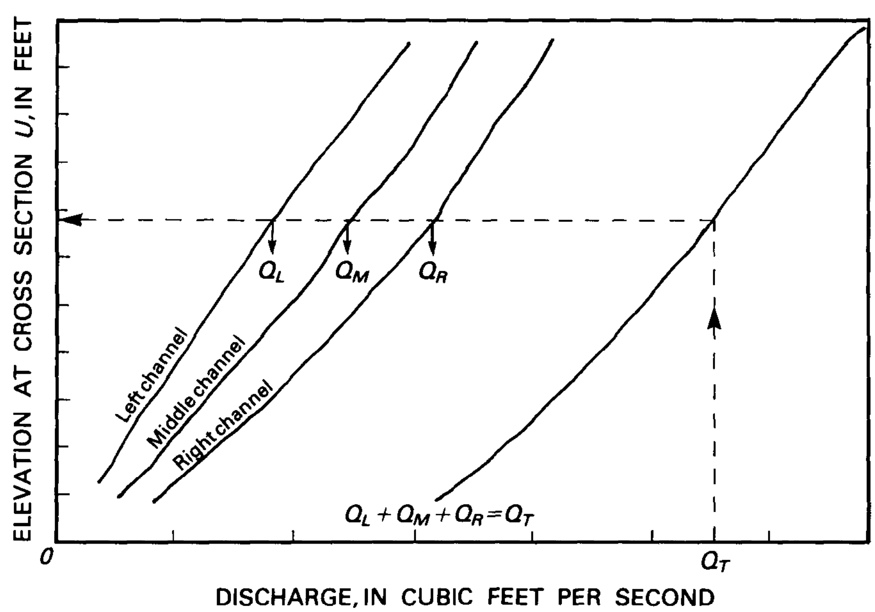
The main-channel cross section, D-D_R in figure 25*A*, is the last for which a water-surface elevation has been computed with the total discharge, Q_T. The elevation at the upstream cross section, U-U_R, must be determined as well as the division of Q_T into components Q_L, Q_M, Q_R, and any other branches, and the water-surface profiles in each branch.

To solve for the unknowns, an approximate division of flow is estimated, and each channel is analyzed by computing the profile for that channel's discharge from cross section D-D up to cross section U-U. In figure 25*B*, the elevation at U-U is plotted as ordinate, and the discharge producing it is plotted as abscissa. The steps are repeated for other estimated divisions of flow until a rating curve is defined for stage at section U-U corresponding to discharge in each channel. An additional rating curve is drawn to represent total discharge as abscissa by adding, for several elevations, the quantities Q_L, Q_M , and Q_R. This final curve gives the relation between total discharge and elevation at the upstream cross section, U-U. The known value of Q_T is used with figure 25*B* to determine the corresponding value of water-surface elevation at cross section U-U. The discharge in each branch for that elevation of U-U is determined from the individual branch rating curves. These discharges, Q_L, Q_M, and Q_R, are now used to compute water-surface elevations in each channel between cross sections D-D and U-U.

In the computations of profiles from section D-D to section U-U through any one of the branches, there will be a sudden and large change in magnitude of conveyance (a) between



A, PLAN VIEW OF MULTICHANNEL FLOW



B, RATINGS FOR INDIVIDUAL CHANNELS

Figure 25.—Division of flow in multichannel reach.

section D-D and the first cross section in the branch, and (b) between the last cross section in the branch and section U-U. To minimize errors in the computation of head losses which are due both to expansion and contraction, and to channel friction, a logical and consistent method of subdividing sections D-D and U-U must be used. Three possible methods are:

1. The shapes of sections D-D and U-U, as determined from plots of those cross sections, may reveal some obvious geometric basis for subdivision. If so, artificially extend these boundaries to the upstream and downstream ends of the islands or embankments, thereby dividing the stream into channels.
2. On the basis of the water surface at section D-D, make a reasonable estimate of the water surface at section U-U. For these elevations, plot the cumulative conveyance versus distance from left bank for sections D-D and U-U. The total conveyances are labeled K_D and K_U. Compute conveyances, K_L, K_M, and K_R, based on water-surface elevation at section D-D for the minimum cross section in each channel. Compute positions for pseudo-boundaries in sections D-D and U-U to simulate the actual boundaries of each branch by multiplying K_D and K_U by the ratio $K_L/(K_L+K_M+K_R)$ for the division between the left and middle channels, and by $(K_L+K_M)/(K_L+K_M+K_R)$ for the division between the middle and right channels. Extend pseudo-boundaries to the upstream and downstream ends of the islands or embankments.
3. On the basis of the water-surface elevation at section D-D, determine the gross cross-sectional areas for the most constricted cross section in each channel. Project the gross width of each island or embankment to the upstream end and divide it on the basis of these gross areas in the adjacent channels, as is illustrated in figure 29. This pseudo-boundary between channels is projected upstream to section U-U, and downstream to section D-D, as is shown in figure 25A.

Some engineering judgment must be used to select the best method. Boundaries determined by the first two methods might not yield similar divisions of section D-D and U-U, or they might be neither parallel to each other nor to extensions of the general axes of the dividing islands. The third method, which is suggested for multiple bridges, is least ambiguous, and should be used if there is not good reason to favor one of the other methods.

After Q_L, Q_M, and Q_R are determined, and the water-surface elevation at U-U is computed, the velocities at U-U should be checked to make certain that they are subcritical. If the main-channel flow at U-U is not tranquil, the proportion of flow going into each channel will depend upon flow conditions upstream from the point of division.

If the flow in any one channel is not tranquil, the steps described in the sections entitled "Steep Slopes," and "Supercritical Flows" are followed. The rating curve, as shown in Figure 25B for each channel, can still be defined in terms of discharge and the elevation at U-U.

Any one or all of the individual waterways of figure 25B could be natural stream channels around islands, bypass canals, or control structures such as bridges, culverts, or dams. Indeed, each path itself could have a series of such structures and(or) stretches of natural stream channel between cross sections U-U and D-D. The methods of computing water-surface elevations at bridges and at culverts are described in the appropriately entitled sections in this manual. Regardless of whether the flow through any one of the individual channels in figure 25B is subcritical or supercritical, or even whether the flow regime successively changes between sections U-U and D-D, the final relation to be plotted is between the discharge and the water-surface elevation at section U-U for each path.

Bridges

Water-surface profile computations may be carried through bridges and other constrictions providing that tranquil open-channel flow conditions exist and that no pressure flow is involved for the discharges being considered.

The effects of bridges or other constrictions on the computed M1 and M2 backwater curves were described in the section entitled "Local Effects on Profiles." Bridges do not present a serious problem if they are located in the reach downstream from the point of convergence of M1 or M2 curves. Bridges located in a channel for which the water-surface profile is being computed, also present no serious problems if the amounts of backwater are insignificant compared to the total fall in the approach reach. This would be true in a streambed having a fairly good slope and at sites where there is not much contraction involved.

The computation of water-surface profiles at bridges, including bridges with road overflow, has been incorporated into a computer program (Shearman, 1976). The methodology and coefficients outlined by the Bureau of Public Roads (Bradley, 1960, 1970) are used. Because of the methodology within the program and by Bradley, the computer solution is satisfactory only for the circumstances described in the preceding paragraph. At other bridges computer solutions should be stopped and backwater curves should be manually computed. Manual computations should be considered, in particular, at the following sites:

1. reaches having extremely flat streambed slopes,
2. two or more bridges in close proximity, longitudinally along the stream,
3. sites at which the flow is greatly constricted, and
4. sites at which the vegetation in the overbanks is extremely dense (n in excess of 0.10).

If the contraction causes critical- or supercritical-flow conditions, it is acting as a control section through which water-surface profiles cannot be computed without a break in computations. When the Froude number in the constricted cross section is 0.8 or greater, the manual methods of computing discharge (Matthai, 1967) or of backwater (Cragwall, 1958) are not reliable. Under such circumstances, terminate the profile at the downstream side of the bridge and attempt a manual routing of the flow through the constriction as if it were a culvert flowing as type 1, type 2, or type 5 (see Bodhaine, 1968).

Computations of water-surface profiles at constrictions having embankment or road overflows involve a trial-and-error solution. The division of flow must be estimated and the water-surface elevation at the approach cross section must be computed for each of the discharges until an acceptable approach-section elevation is found to satisfy the bridge-backwater and embankment-head requirements. Details of the iterative computer-program solution are discussed by Shearman (1976). Criteria for the hydraulics of the flow over highway embankments, including submerged-flow conditions, are discussed by Hulsing (1967).

Sometimes, the computer solutions will be disrupted if bridges have flow over very low embankments or the solution will result in an apparent discharge over the road larger than the discharge for which profiles are being computed. The main-channel subsection, as compared to the total cross-sectional area, could be quite small at such problem sites. If computations are interrupted or if the results appear to be unrealistic, the probable cause is that the bridge-with-road-overflow computation is unfeasible. Ignore the presence of the bridge and replace the bridge sections with a cross section running from the left bank along the crest of the road, down into the main channel, and up the other bank along the crest of the road. In addition to it and the approach cross section, add a third cross section across the whole valley at flood-plain level, one bridge-opening width downstream. Substitution of these three cross sections for the bridge-associated sections will generally provide satisfactory results. If differences between these cross sections are quite significant, additional full-valley cross sections may be required at the upstream and downstream faces of the embankment.

Tributaries are common in the immediate vicinity of bridges, but such flows (street runoff, drainage ditches, or very small tributaries) are generally small enough in comparison to the main-channel discharge to be ignored. If a large tributary enters the main stream immediately upstream of the bridge but below the approach cross section, manual computations of the bridge backwater present no particular problem. In the Survey's machine computation, however, the discharge cannot be changed in the subreach between the approach cross section and the constriction (Shearman, 1976). For computational purposes, therefore, such tributaries are assumed to enter immediately upstream from the approach cross section. As a consequence of this assumption, the next cross section above the approach cross section should be located at a distance equal to the width of the tributary mouth. Such a cross section need not be surveyed if channel conditions are almost identical with those at the approach cross section; it is sufficient to repeat and transpose the approach cross section and use an appropriate longitudinal station distance.

Tributaries entering the main channel immediately downstream from the bridge do not present such problems. The computer program will permit a change in discharge between the full-valley cross section at the exit of the bridge and the next downstream cross section. The full-valley bridge-exit cross section should be repeated and transposed downstream, using an appropriate longitudinal river-station distance, such that the tributary will enter the main channel at mid-subreach. In such a transposition of cross sections, consideration should be given to vertical adjustments of ground elevations if there is appreciable slope in the streambed.

Flow Through Culverts

Culvert flow has been classified into six types on the basis of the location of the control section and the relative heights of the headwater and tailwater elevations (Bodhaine, 1968). Of these types, only type 3 has tranquil flow throughout; therefore, it is only for type 3 flows that water-surface elevations may be computed by the step-backwater method through the culvert. All other types of flow through culverts involve either critical flows or pressure flows; the profile computations must be terminated at the downstream side of the culvert, and the elevation at the upstream side must be determined by other means.

If the culvert is one of the standard types described by Bodhaine, the following procedure is suggested. The U.S. Geological Survey computer program A526[1] will produce a stage-discharge relation for the culvert in terms of headwater elevation, tailwater elevation, and discharge. Inasmuch as the discharge is known and the tailwater elevation is that computed for a cross section located at the downstream end of the culvert, the headwater elevation can be determined easily. Begin the profile computations again at the approach cross section, using this headwater elevation and the total discharge.

[1] Matthai, H. F., Stull, H. E., and Davidian, Jacob, 1970, Preparation of input data for automatic computation of stage-discharge relations at culverts: unpublished data.

Road overflow at culverts

Flow of water both through a culvert and over the road is not infrequent. Because culvert flows associated with road overflow are likely to involve pressure-flow conditions, the culverts and roads must be individually rated. Much of the work, however, can be done by computer, thereby simplifying the procedure.

Figure 26 depicts a culvert with road overflow. The total discharge, Q_T, is divided into unknown quantities Q_C, flowing through the culvert, and Q_R, flowing over the road. The tailwater elevation, H_4, is known. The water-surface elevation H_1 at the approach cross section must be determined.

The flow must be divided so that the headwater elevation computed for the flow through the culvert agrees within a selected tolerance with the headwater elevation computed for the flow over the road. The culvert itself can be calibrated by means of the Survey's computer program A526. Plot the rating, headwater, H_1, versus discharge, Q_C (fig. 27). As long as $Q_C=Q_T$, there will be no road overflow. The rating will have a family of curves if tailwater elevations, H_4, become a factor. A rating curve can also be established for the flow over the road, Q_R, in terms of H_1 and H_4. Criteria for the hydraulics of the flow over embankments, including submerged-flow conditions, are discussed by Hulsing (1967).

The two ratings are plotted in the same manner as was done for flow around an island (see fig. 24). A composite rating is shown in figure 27. The point at which the embankment, or road-overflow rating crosses the culvert rating at the known tailwater elevation, H_4, is shown with a filled circle. Lines through

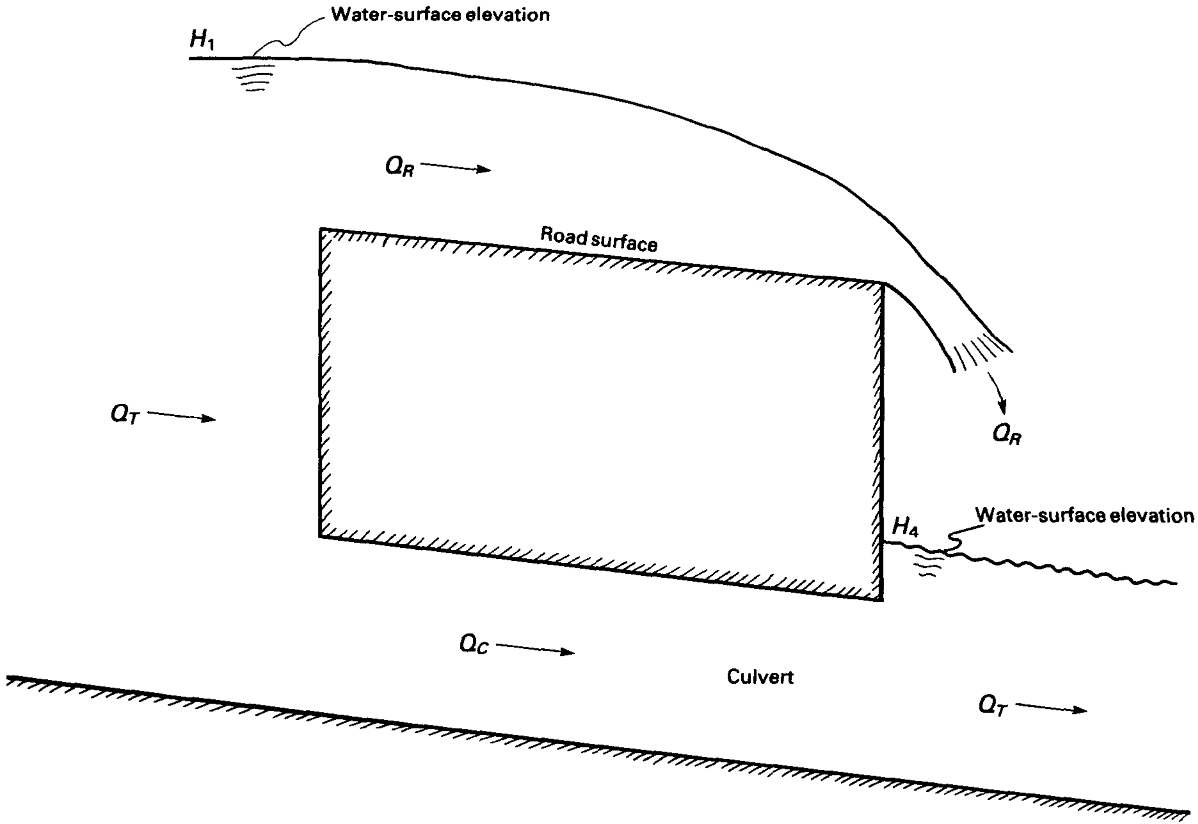


Figure 26.—Culvert with road overflow.

this point, extended to the upper and lower abscissas, and to the ordinate give the appropriate values of Q_R, Q_C, and H_1.

Once H_1 is determined, computations of water-surface elevations for Q_T can commence at the approach cross section and continue up the channel.

Storage at culverts

If headwater elevations are very high with respect to the elevation of the top of the culvert and if the size of the opening is very small with respect to the size of the approach cross section, reservoir-type storage effects are possible. The transition from an inflow hydrograph to an outflow hydrograph may be accompanied by attenuation in the peak rate and a time lag in the centroid. Figure 28 illustrates the effect of embankment-storage attenuation for a hypothetical hydrograph routed through so-called "linear" storage. The peak rates of discharge for inflow and outflow hydrographs and the pond elevations upstream of the culvert can be influenced considerably. The culvert peak attenuation problem has been discussed by Young (1971) and Bodhaine (1968). Jennings (1977) describes culvert hydrograph analysis by a reverse routing method. Mitchell (1962) developed techniques for correcting the outflow peak for the effects of embankment storage. His work is useful for culvert sites where only outflow peak is observed.

Multiple-Opening Constrictions

Multiple constrictions may be combinations of bridges or other constrictions spaced so that the embankments or even a small island be-

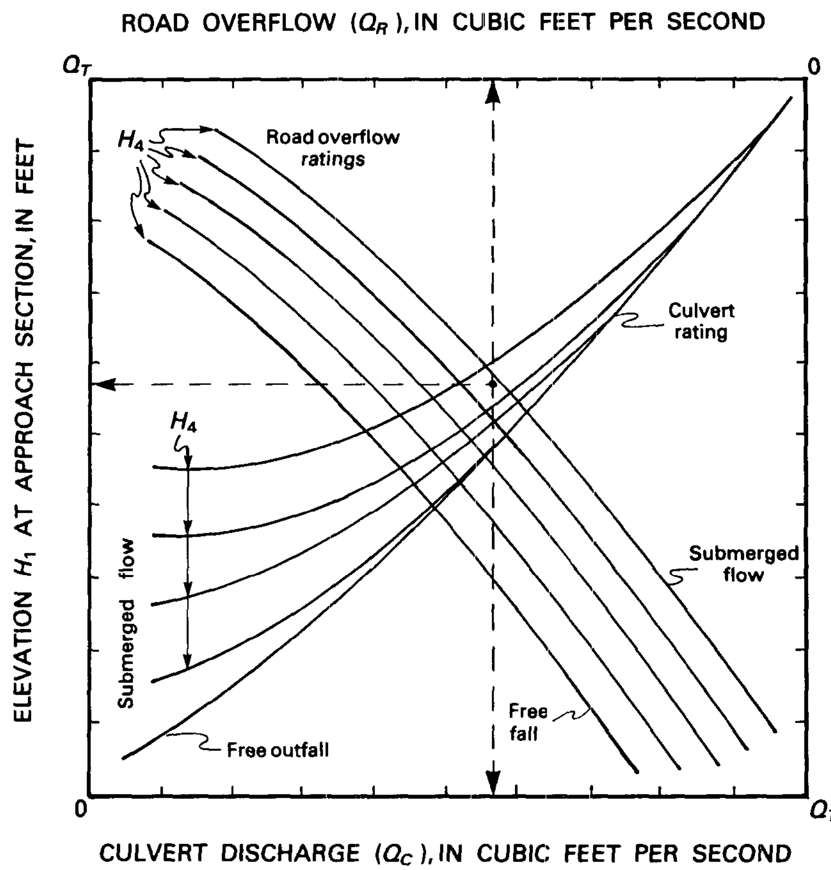


Figure 27.—Composite rating curve for culvert with road overflow.

tween them cannot be considered webs or piers in one very long bridge. The multiple-opening constriction is assumed to be a series of independent, single-opening constrictions, each geometrically and hydraulically distinct from the other (Davidian, Carrigan, and Shen, 1962). The discharge characteristics of the individual openings may then be defined in terms of those for single openings. This method requires that pseudo-boundaries be located in the reach upstream from each of the openings to simulate the actual upstream boundaries of a single-opening constriction. The boundaries may be extended downstream from each opening, also. The techniques are similar to those described in the sections entitled "Flow Past Islands" and "Multichannel Flows."

Division into single-opening units

The upstream flow boundaries may be located by first apportioning the width of each embankment in direct proportion to the gross flow areas of the openings on either side, the larger part of the embankment being assigned to the larger opening. The sketch in figure 29 illustrates the division of an embankment of length W_T into components W_L and W_R. The areas should be computed on the basis of the depths appropriate to the water-surface elevation at the downstream side of the embankment for the full-valley cross section.

After division of each embankment between two openings has been determined, lines parallel to the mean direction of flow are projected upstream from the points on the embankments thus determined. For computation, the lines are assumed to represent the fixed, solid upstream boundaries of an equivalent single-opening constriction. At the constriction embankments, they are reasonably close to the points at which the flow separates; elsewhere, they rarely coincide with the actual limits of the separate flow regions. They do, however, provide an adequate and unambiguous means of dividing the constriction into independent single-opening units.

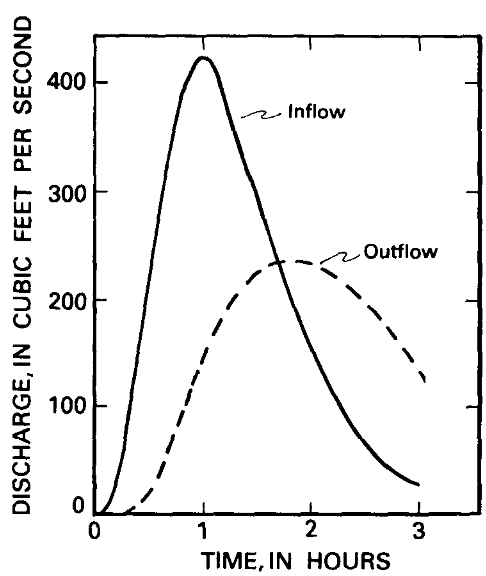
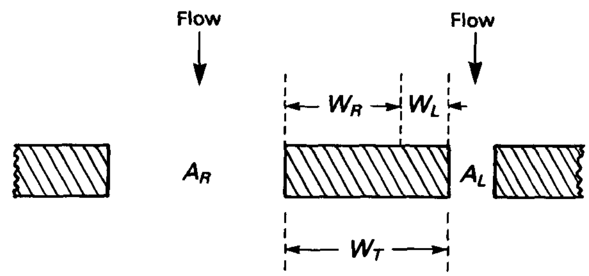


Figure 28.—Hypothetical culvert hydrographs illustrating the effects of embankment storage.

Two-bridge openings

Figure 30 is a sketch of two bridge openings in the main channel. The water-surface elevations will have been computed for the total discharge, Q_T, at a cross section D-D downstream from the constrictions and at V_L-V_R at the downstream face of the embankments. The latter elevation is used to determine the cross-sectional areas in the openings, and the center embankment is divided as shown in figure 29. The pseudo-boundary between the two openings, the dashed line in figure 30, is projected upstream to a full-valley cross section U-U, at



$$W_L = W_T \left(\frac{A_L}{A_L + A_R} \right)$$

$$W_R = W_T \left(\frac{A_R}{A_L + A_R} \right)$$

W —Embankment width
A —Gross area of cross section
L,R —Subscripts denoting left and right openings
T —Denotes total width

Figure 29.—Apportionment of width of embankment between two bridge openings.

the approach section to the larger opening. It is also projected downstream to full-valley cross section D-D.

Just as for flow around an island, the computations involve the determination of the water-surface elevation at section U-U and the proper subdivision of the total discharge into components Q_L and Q_R (see figure 24). In these computations for flow through each opening, an approach section is taken at one bridge-opening width upstream from each opening: section A_L-A_L for the left opening, and section A_R-A_R, which is part of section U-U, for the right opening. Section A_L-A_L can be estimated from sections U-U, V-V, and D-D, providing it is adequately representative of actual conditions one bridge width upstream from the smaller opening.

The computation of backwater through any one opening entails large changes in discharge and probably conveyance between adjacent sections. These sudden and large changes in magnitude are associated with improperly computed friction losses and large changes in velocity heads. To minimize errors and confine

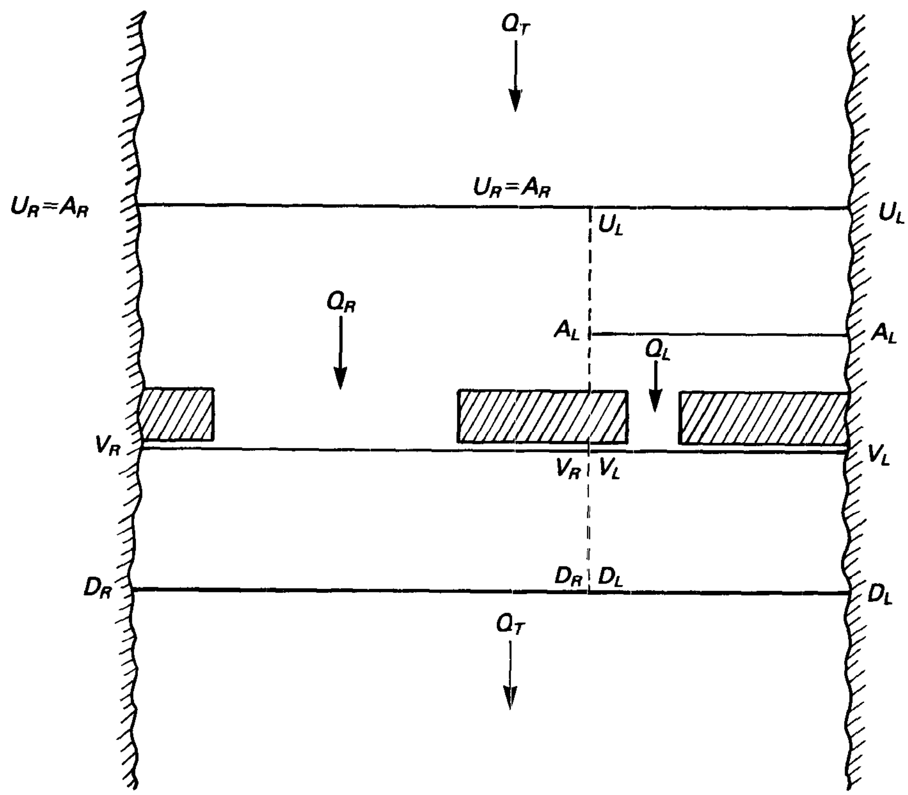


Figure 30.—Division of flow at multiple-bridge openings.

them to short subreaches, the following procedure is recommended. Cross sections *D-D* and *U-U* are divided on the basis of the pseudo-boundaries and each segment is considered to be a cross section. The computation of backwater through the left opening proceeds as follows:

1. Let D_L-D_L be the first cross section. Use the elevation already determined for section *D-D* with Q_T as the starting elevation. Assume any value of Q_L for computations.
2. Continue step-backwater computations at the valley cross section, V_L-V_L; the constriction; the approach section, A_L-A_L; and the left segment of the upstream cross section, U_L-U_L.
3. Plot the computed elevation at U_L-U_L versus Q_L as in figure 24.
4. Repeat steps 1 through 3 for other trial values of Q_L until a rating is developed for the left opening.

Perform the same operations for the right opening, up to approach section A_R-A_R, which is segment U_R-U_R of section *U-U*. After the elevation at *U-U* has been determined, make certain that flow conditions are subcritical at all cross sections in both the left and right channels for the appropriate values of Q_L and Q_R as determined from the composite stage-discharge relation. Flow conditions for the entire cross section, *U-U*, must also be subcritical for the elevation from the stage-discharge relation, and for Q_T. If these conditions are satisfied, full-valley computations can be resumed. Begin at section *U-U*, using Q_T, and start with the water-surface elevation for *U-U* as chosen from the composite bridge ratings.

Three or more bridges

Should there be three or more bridges, or combinations of bridges, culverts, and bypass channels, the computation procedures would be similar to those described earlier under "Multichannel Flows" and "Two-Bridge Openings." Pseudo-boundaries are located using the concepts shown in figure 29 and discussed in the sections mentioned above. Each bridge opening is considered to be a single opening and a rating is established for it in terms of elevation of water surface at an upstream cross section and discharge. The ratings are plotted as in figure 25*B*. The individual ratings are added horizontally to establish an additional rating representing the upstream stage versus the sum of the discharges passing through the individual openings, which correspond to that upstream discharge. Inasmuch as the true value of Q_T is known, the values of the individual discharges and of the water-surface elevation of the full-valley cross section upstream are all easily determined.

Before further computations are resumed, a check should be made of flow conditions at each cross section in the individual channels and at the full-valley section upstream. Supercritical-flow conditions at any one of them will require special consideration; refer to the section entitled "Bridges."

Alluvial Channels

The hydraulics of alluvial streams is complicated and not yet fully understood. The discharge, bed load, bed-material size, bed form, depth, and roughness coefficient are all interrelated in manners that are difficult to evaluate reliably. Scour, fill, and changes in configuration of the channel bed are continuous processes; therefore, the shape and position of the stage-discharge relation change with time and with changes in flow. The computation of water-surface profiles in such channels is, therefore, affected by such uncertainties. Even water temperature has been determined to be a factor in triggering a change in bed form in some streams and in laboratory studies. Familiarity with the results of research studies, such as Simons and Richardson, 1966, and the many references cited by Simons and Richardson, will assist the analyst with studies in alluvial streams.

Flow and bed forms in alluvial channels are classified into three major regimes:

A. Lower flow regime
   1. Ripples
   2. Dunes with ripples superposed
   3. Dunes
B. Transition zone (bed roughness ranges from dunes to plane bed or antidunes)
C. Upper flow regime
   1. Plane bed
   2. Antidunes
      a. Standing waves
      b. Breaking antidunes
   3. Chutes and pools

A relation which defines bed forms as a function of hydraulic radius, *R*, in feet, slope, *S*, mean velocity, *V*, in feet per second, and grain size, has been proposed by Simons and Richardson (1966). It is shown in figure 31. Another useful criterion for the classification of flow regimes is the ratio

$$\frac{V^4}{g^2\, d_m^{1/2}\, d_{50}^{3/2}},$$

in which g is the acceleration of gravity in feet per second per second, d_m is the mean depth in feet, and d_{50} is the median grain size in feet. For values of this ratio less than 1×10^3, the lower regime of bed forms will occur, and for values greater than 4×10^3, the upper regime will occur. Between these two values, the bed will be in the transition zone.

To compute depths or water-surface profiles in alluvial streams, the bed elevations and the channel roughness must be known. The bed forms and roughness coefficients for the bed depend on the regime of the flow, which in turn requires knowledge of the velocity and depth. Because water-surface elevations are more likely to be computed for high flows, it is probable that such computations will be for upper-regime flow conditions.

Flows in the higher ranges of the transition zone, and in the upper regime, frequently, but not necessarily, are critical or supercritical. In antidune flow, the fact that the water and bed surfaces are inphase is a positive indication that the flow is rapid ($\mathbf{F} > 1$). In many alluvial channels, the natural banks cannot withstand prolonged high-velocity flow without eroding. The erosion increases the cross-sectional area, and this reduces the average velocity and Froude number. Rarely does a Froude number,

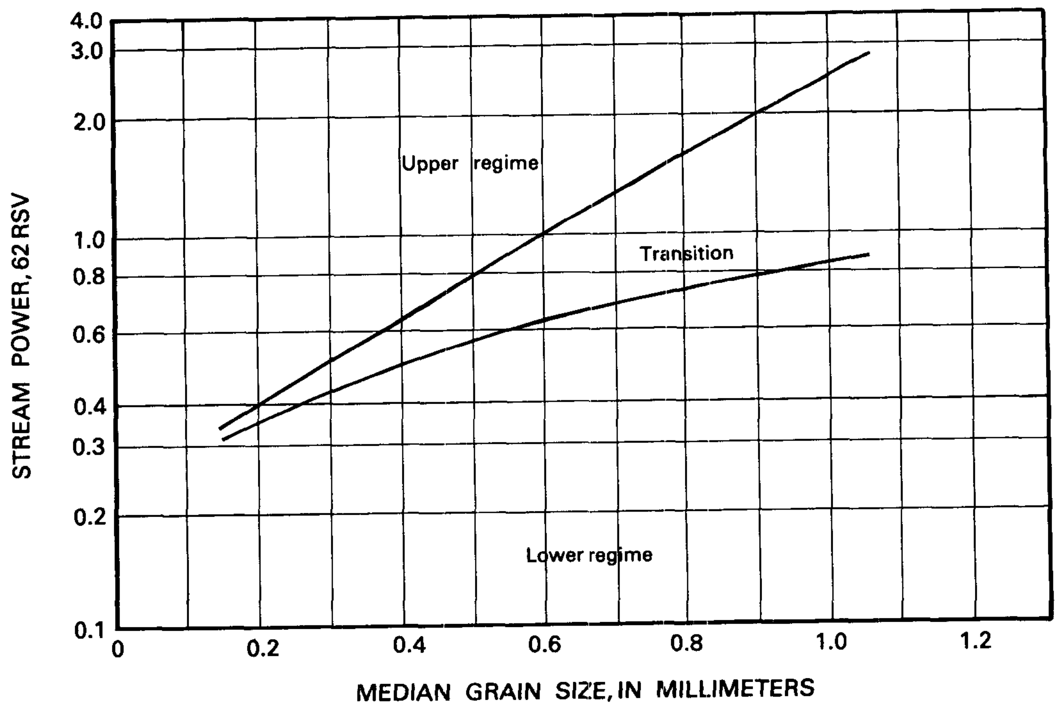


Figure 31.—Relation of form of bed roughness to stream power and median grain size (modified from Simons and Richardson, 1966).

based on average velocity and depth, exceed unity for any extended time period in a natural stream with erodible banks.

Values of Manning's n, for a hydraulic radius of 1.0 foot, were computed from the values of f given by Simons and Richardson (1966, p. 56) by the equation

$$n=\left[\frac{(1.486)^2 f (R^{1/6})^2}{8g}\right]^{1/2}=(0.00858\,f)^{1/2}$$

	Lower flow regime
1. Ripples	$0.021 \leqslant n \leqslant 0.033$
2. Dunes	$.019 \leqslant n \leqslant .037$

	Upper flow regime
1. Plane bed	$0.013 \leqslant n \leqslant 0.016$
2. Antidunes	
Standing waves	$.013 \leqslant n \leqslant .017$
Breaking waves	$.016 \leqslant n \leqslant .024$
3. Chutes and pools	$.024 \leqslant n \leqslant .028$

NOTE.—Multiply values tabulated by $R^{1/6}$ for correct value of n.

The smaller value of n for a given bed form goes with smaller sizes of bed material. For example, for antidunes-standing waves, the range of n is given as 0.013 to 0.017 times $R^{1/6}$. These data are based on laboratory tests for grain sizes (d_{50}) of 0.19 mm, 0.27–0.28 mm, 0.45–0.47 mm, and 0.93 mm.

Nordin (1964) reports on resistance coefficients measured in a reach of the Rio Grande near Bernalillo, New Mexico. For upper-regime flows, largely plane bed, and an average bed material size, d_{50}, of 0.29 mm, values of n range from 0.012 to 0.018 for mean depths ranging approximately between 2.0 and 4.5 feet.

In natural streams, standing and breaking waves associated with antidunes in upper-regime flow will generally be located in the middle of the cross section. The water surface at the banks might be relatively quiet. Therefore, flow at the sides may be in the lower regime or in the transition zone while the center of the stream is in the upper flow regime. Computations of depths, velocities, Froude numbers, and water-surface elevations will, however, be based on the bulk cross-sectional values.

The choice of a reasonable roughness coefficient is still a difficult problem. Therefore, a few studies of the resistance coefficients for alluvial channels, both in natural rivers and in laboratory flumes, are summarized below to indicate the relative magnitudes of the roughness factors to be expected. A considerable amount of judgment must be exercised by the analyst in choosing appropriate values.

Simons and Richardson (1966) offer some guidelines on resistance coefficients for alluvial channels in terms of the Darcy-Weisbach resistance coefficient, f, and the Chezy discharge coefficient, C. The relation between f, C, and Manning's n is:

$$\frac{8g}{\sqrt{f}} = C = \frac{(1.486\ R^{1/6})}{n},$$

in which

g = acceleration due to gravity, ft/s^2,
R = hydraulic radius, ft,
S = channel slope,
V = mean velocity, ft/s.

Benson and Dalrymple (1967) state that values of Manning's n for upper regime flow may be selected from the following table which shows the relation between median grain size and the roughness coefficient.

Median grain size	**Manning's n**
0.2 mm	0.012
.3	.017
.4	.020
.5	.022
.6	.023
.8	.025
1.0	.026

Culbertson and Dawdy (1964) made a study of hydraulic variables at several sites along the Rio Grande in New Mexico. Figure 32 shows Chezy's C as a function of d_{50} for upper regime flows. The relation between hydraulic radius, velocity, and C is shown for one station, Rio Grande at Cochiti, in figure 33. The median diameter of bed material at Cochiti is approximately 0.44 mm, and the mean depth for upper regime flow is between 3.6 and 4.8 feet.

The examples cited (Simons and Richardson, 1966; Nordin, 1964; and Culbertson and Dawdy, 1964) are primarily for relatively shallow depths and for discharges that probably are not as large as the design floods for which profiles are desired. Any information from past floods, such as measured profiles, bed forms, photographs, or eyewitness accounts, would be of great value in determining the probable regime of flow as well as in choosing appropriate values for the resistance coefficients.

Use of Step-Backwater Method for Indirect Discharge Measurements

The step-backwater technique can be applied to the determination of discharge by indirect means in a long, slope-area reach. The reach may be ideal in every respect for a slope-area measurement, having a uniform shape and roughness, and steep sides, but it may lack high-water marks except for an excellent mark or two at the upstream end. A stage-discharge relation can easily be established at the upstream end where the high-water marks are located.

Cross sections (at least 8–10) can be located through the reach, and two or more M2 profiles can be computed for each of a series of assumed discharges about the magnitude of the expected discharge. The reach should be long enough for the several M2 curves for each discharge to converge. In this manner, a stage-discharge relation is established for the cross section at which the high-water mark is located, as in figure 34.

The discharge corresponding to the elevation of the high-water mark, as determined from a well-defined rating as in figure 34, should be every bit as reliable as a slope-area measurement made in that reach with good high-water marks to define the water-surface profile.

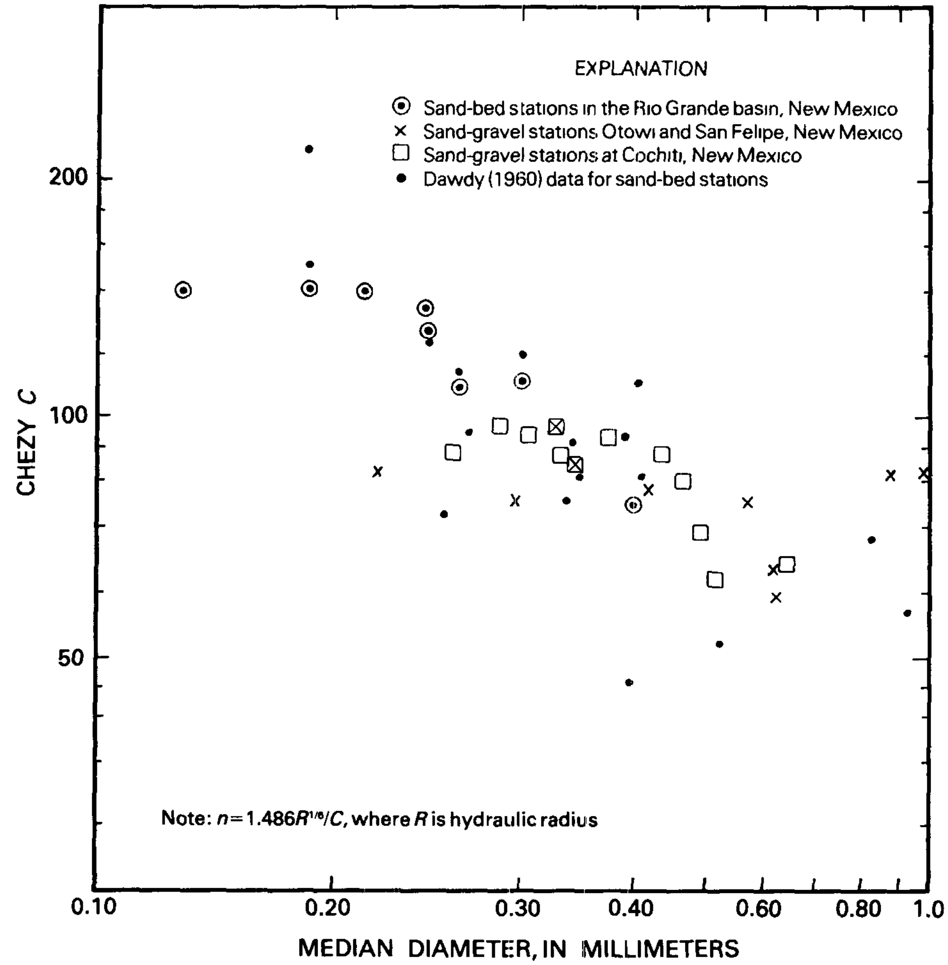


Figure 32.—Variation of Chezy C with median diameter of bed material for upper regime flows (modified from Culbertson and Dawdy, 1964).

Floodway Analysis

The material in the next two paragraphs is paraphrased from Shearman (1976). Floodway, as used in this manual, refers to a land use control measure widely used in the field of flood-plain management. In this context, a floodway may be defined as that portion of a watercourse required to convey a discharge of specified magnitude without exceeding a specified surcharge (fig. 35). The discharge magnitude and surcharge limit depend upon criteria established by the appropriate regulatory agency (which may be Federal, State, regional, or local).

Encroachment of cross sections

Ideally, floodway limits should be located such that the encroachments on both sides of the watercourse contribute equally to the surcharge. Encroachments could be based on equal area or equal horizontal distance. However, elimination of an area of open pasture on one overbank would contribute far more to the surcharge than would elimination of an area of dense forest on the other overbank. Likewise, encroachments of equal length on overbanks with unequal flow depths and(or) unequal roughness would also contribute unequally to the surcharge. Encroachments having equal

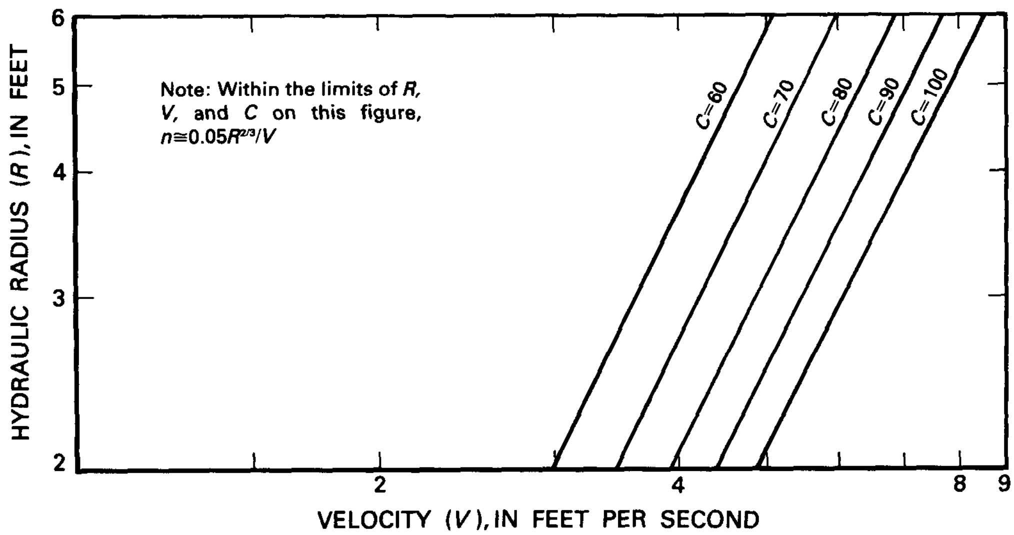


Figure 33.—Relation of roughness coefficient to hydraulic radius and velocity for Rio Grande at Cochiti, New Mexico (modified from Culbertson and Dawdy, 1964).

conveyance, which includes area (thereby length and depth) and roughness, would be more likely to contribute equally to surcharge. Therefore, conveyance is used in this manual as the basis for establishing floodway limits. Several problems may be posed as follows:

1. A surcharge, y, is acceptable if the side boundaries can be moved closer to the center. The conveyance to be removed from the left bank, K_L, is to be equal to its counterpart on the right bank, K_R, and their sum, K_L+K_R, is to be equal to the conveyance of the surcharged part, K_S, such that y is not exceeded. Where are the side walls, L and R, to be located?
2. The left boundary, L, is to be at a preselected location on the left flood plain. Where should the right boundary, R, be placed such that K_L+K_R are equal to K_S, and y is not to exceed a preselected value?
3. Move the left and right boundaries to any locations on their respective flood plains ($K_L \neq K_R$). At what depth will the discharge now flow in the constricted channel (y is not fixed)?

Many variations of these problems are possible. It may be desirable to do either 1, 2, or 3, as described above, at each cross section in the total reach. All the new left boundaries would be connected, and their loci would define a new left edge of water. After this is repeated for the other bank, it may be desirable to go back and readjust some of the boundaries to achieve a more nearly uniform constricted channel shape and alinement throughout the reach. In doing this, the relation $K_L=K_R$ must be preserved if that constraint had been selected; and the new depths must be checked so as not to violate the surcharge limit, y, if that constraint had been selected.

In another variation, a combination of problems 1, 2, and 3 may be used, with a different one at each cross section. It could be desirable to use none of these at some places, leaving the cross section unchanged.

In the manipulation of boundaries in floodway studies, care must be taken at bridges and culverts. If there is any possibility of road overflow, the reach between the approach cross section and the road embankment, and an equal distance downstream, should be examined carefully before and after any encroachments are made on any cross sections within this subreach. Any computed road overflow must be able to reenter the live stream again on the downstream side of the embankment.

Floodway analyses are made after the normal water-surface profiles are determined as described in the section entitled "Standard

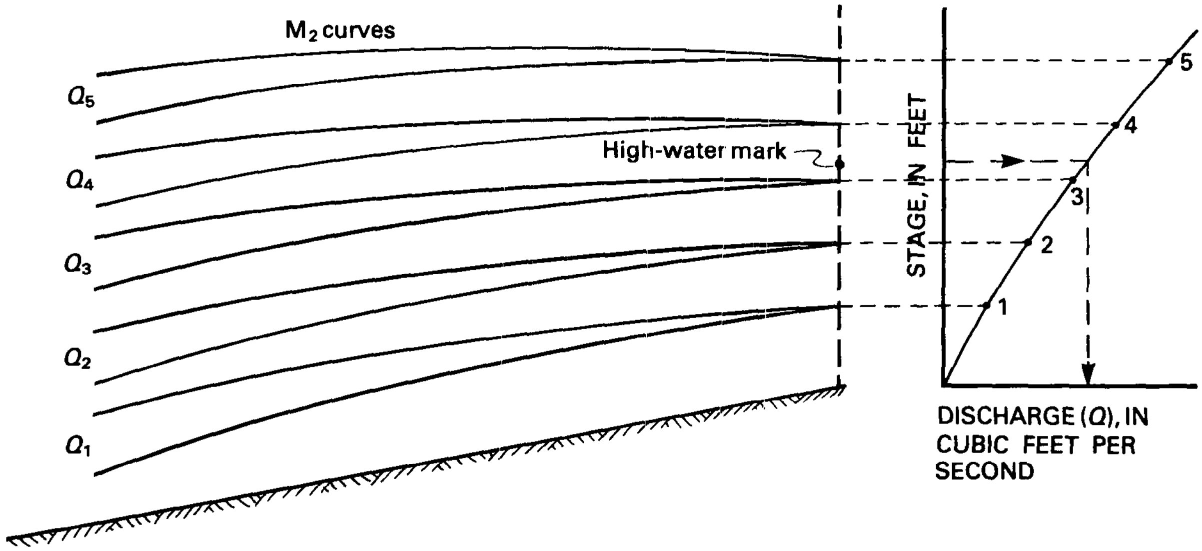


Figure 34.—Definition of a rating curve at the upper end of a long reach by means of the step-backwater method, using convergence of M2 curves.

Step Method, Subcritical Flows." Manual computations for a floodway analysis are impractical. However, a solution by computer is not difficult. Shearman (1976) describes in detail the documentation of the Geological Survey computer program, E431, and the various options available for solution of problems similar to those mentioned above (1, 2, and 3). Several of these options are described briefly below.

VER option

In this floodway option the surcharge, y, is specified. The locations of boundaries L and R are not fixed, but they are positioned so that equal conveyances are removed from each bank. With reference to figure 35, the following requirements are satisfied: (1) $K_L=K_R$; (2) $K_S=K_L+K_R$; and (3) $K_M+K_S=K_L+K_M+K_R$.

The VER option should be used preferably at cross sections having wide flood plains of roughly equal widths and(or) conveyances, and the reasonableness of the computed results should be evaluated. It is possible to obtain unsatisfactory solutions which would place both the L and the R boundaries on the same bank, or one of these boundaries in the main channel. Should either of these unacceptable solutions be obtained, some other option must be used, some constraints must be imposed, or some requirements must be relaxed. For example, it may be necessary to accept a solution from another option, one in which K_L and K_R are not necessarily equal, but their sum is still made equal to K_S by preventing either boundary from being located anywhere but on its own flood plain.

VSA option

This option specifies the surcharge limit, y, and also imposes a subsection constraint. The requirement that K_L equal K_R is removed, but their sum is still to be equal to K_S, and the quantity K_M+K_S is to be equal to $K_L+K_M+K_R$.

The subsection constraint is exercised by dedicating a certain subsection, usually the main channel, or a group of adjacent subsections including the main channel, as part of the floodway. If the main channel subsection is not to be encroached upon by boundaries L or R, the computer will manipulate locations for them from the edge of the flood plain up to the demarcation of the dedicated subsection, but will not go beyond. If the computer finds that L

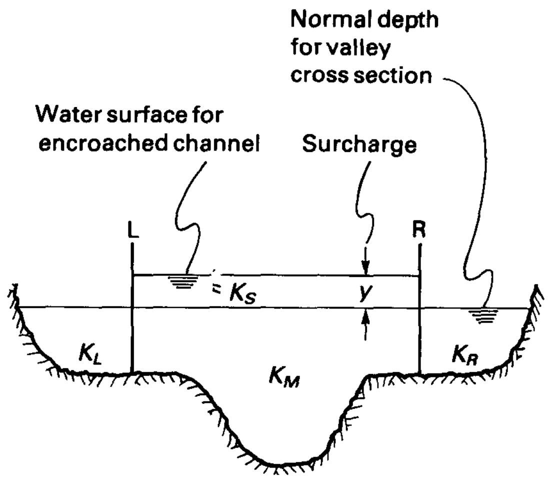


Figure 35.—Effect of encroachment of flood plains on normal valley cross section.

should stop at the edge of such a subsection, it will compute K_L up to that point. Then it will move R until it gets $K_L+K_R=K_S$, providing R will not encroach into the dedicated subsection(s) also. If both L and R must stop at the edges of the dedicated subsection, the criteria for the VSA option mentioned above will not be fulfilled. Because the sum of K_L and K_R is less than it would be at the surcharge limit, y, the surcharge in the floodway channel between L and R is less than the limit allowed.

VHD option

In the VHD option, the maximum allowable surcharge, y, is specified. A horizontal distance or limit constraint is also imposed on the locations of L and R, beyond which they may not be placed. It is thus possible to preserve an unencroachable part of the cross section by specifying the stationing of its edges. In all other respects this option is similar to the VSA option. K_L and K_R need not be equal, but their sum is equal to K_S, and the quantity K_M+K_S is equal to the sum of $K_L+K_M+K_R$. Because of the horizontal distance constraints on the locations of L and R, the sum of K_L and K_R may yield a value of K_S which corresponds to a smaller surcharge than that allowed; therefore, the constraint on the magnitude of y will not be violated.

In specifying the limiting stations for the locations of L and R, the analyst should not try to create new subdivisions of the cross section. Such a step would unnecessarily affect the velocity-head coefficient, α. The specified stations serve only as limits.

Despite the specification of limits for L and R, sometimes the computed water-surface elevation at that cross section may be so low that all the flow is confined entirely within the restricted area. The computer printout will, therefore, show the stations of the left and right edges of water not to be at the limiting values of L and R, but within the restricted area. The criteria of the VHD option will not, however, have been violated. In this case, limits for L and R are not applicable. For higher discharges, the water-surface elevation will be higher, and the left and right edges of water will coincide with the locations of L and R if these boundaries are at their limiting stations and if the surcharge, y, is not exceeded.

HOR option

The HOR option has specified locations for L and R in figure 35. These are not variable locations with limiting values for the station or distance; they are fixed locations for an encroached cross section. There is no constraint on the surcharge, y. The effects of the encroachment are, therefore, reflected in the elevation of the computed water surface.

As is described for the VHD option, the specification of limits, or the designation of specific locations, for boundaries L and R does not necessarily mean that the computed water surface will be high enough for the left and right edges of water to reach these stations for all discharges.

An example of an advantageous use of the HOR option is a study of "before and after" water-surface profiles for a given discharge in a reach that is to have a part of its flood plain removed from the available cross section. If a highway were to be placed along the flood plain at L in figure 35, and parallel to the main stream, the highway would be the effective new left bank boundary. The location of L would be known for each cross section. The right boundary would remain on the right edge of the valley. The water-surface profile throughout the reach for this encroached channel would be determined by computing the resulting surcharge, y, at each cross section.

The HOR option could also be applied to a study of levee heights and locations along the flood plains for various flood discharges.

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