Techniques of Water-Resources Investigations of the United States Geological Survey

Chapter A10

DISCHARGE RATINGS AT GAGING STATIONS

By E. J. Kennedy

Book 3

APPLICATIONS OF HYDRAULICS

DEPARTMENT OF THE INTERIOR

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PREFACE

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TWI 8-B2.	Calibration and maintenance of vertical-axis type current meters, by G. F. Smoot and C. E. Novak. 1968. 15 pages.

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GLOSSARY

Term	*Definition*
ADP	Automatic data processing used to compute the discharge records for stations equipped with digital recorders
Bankfull Stage	Stage below which all discharge is confined to the main channel and above which part of the flow occurs in overbank areas of the flood plain
Complex rating	Discharge rating that relates discharge to stage plus some other independent variable such as rate of change in stage or fall in a reach between two gages
Control	Closest section or reach of a channel downstream from a gage, usually a natural constriction or artificial weir, where the channel is shallower, narrower, or rougher than it is elsewhere and where the water-surface slope is significantly steeper
Digital descriptors	Set of coordinates (usually gage heights and dependent variables), or the coefficients and exponents for some type of equation, that describes a curve of relation digitally for convenient use with a computer or calculator
Fall	Difference between the water-surface elevations of two locations on a stream, usually base and auxiliary gage sites for a slope station
Gage height	Water-surface elevation referred to some arbitrary gage datum; gage height is often used interchangeably with the more general term "stage," although gage height is more appropriately used for reading on a gage
Gage height of zero flow	Gage reading corresponding to infinitesimal discharge at a gaging station; the gage height of zero flow is often used interchangeably with the "point of zero flow," which is more appropriately used for a physical location in the streambed near the gage
Index-velocity rating	Complex rating in which a point velocity in a cross section is used as an indicator of mean velocity in the section
Permanent control	Natural or artificial control, the location and dimensions of which remain unchanged for very long periods
Point of zero flow	*See* Gage height of zero flow
Scalloping	Undesirable discharge rating characteristic in which the straight-line segments of a logarithmic rating curve, plotted by using rectangular coordinates, billow upward between nodes at the descriptor points; corresponding rating table shows erratic differences in discharge for each 0.10-ft difference in gage height
Shape curve	Curve similar in shape to that of a rating curve being developed, usually a previous rating or, for a new site, one derived form weir formulas or channel measurements
Shift adjustment	Adjustment, usually varying with time and stage, applied to gage heights to compensate for a change in the rating shape or position
Shifting-control method	Systematic use of shift adjustments as a substitute for revised ratings
Simple rating	Discharge rating that relates discharge to stage only
Slope rating	Complex rating that relates discharge to gage height at one gage (base gage) and to the fall in water-surface elevation between the base gage and an auxiliary gage at another site

Stable channel	Channel whose discharge rating remains unchanged for relatively long periods of time, generally between major floods
Stage	Gage height
Unstable channel	Channel whose discharge rating is changed frequently by minor rises or, in alluvial channels, continually during all flow conditions
V diagram	Graphic representation of the relation between shift adjustment and time or stage
WATSTORE User's Guide	Volumes 1 and 5 of a set of instruction manuals regarding the format of data input to the ADP system used for discharge-record computation (Hutchison and others, 1975, 1980)

SYMBOLS

Symbol	***Definition***
a	Constant
a_n	Coefficients for polynomial rating equations where the subscript n indicates the degree of the x term to which the coefficient applies ($y=a_0+a_1x+a_2x^2$...)
b	Constant used to indicate the slope of a log rating curve (ratio of horizontal distance to vertical distance)
C_c	Value of a_0, in a one-curve equation for index-velocity rating, that makes $Q_m=Q_r$
C_r	Stage-related coefficient (Q_m/Q_b) in a two-curve analysis of index-velocity rating
C_u	Value of a_0 in a one-curve index-velocity rating, usable in a variation of the shifting-control method for one-curve index-velocity ratings, that applies at the time of a specific discharge measurement
dh/dt	Rate of change in stage; also J
e	Gage-height scale offset used for a log rating curve plot; where the log curve is a straight line, e is the effective gage height of zero flow
F	Fall, or difference in water-surface elevation, between two points on a stream
F_m	Fall measured by gage readings
F_r	Rating-fall value from a rating curve or table
G, GH, Ght	Gage height or stage
GZF	Gage height of the point of zero flow *(PZF)*
J	Rate of change in stage (also dh/dt); J is a more convenient symbol to use in an equation, especially in the numerator or denominator of a fraction
K	Channel conveyance
L	Length of a channel reach
log	Base 10 logarithm
Meas	Discharge measurement or its serial number
n	Manning roughness coefficient or constant-fall value other than 1.00
n	Column number in a computation sheet
P	A constant in a rating equation equal to Q when $(G-e)$ is 1.0 ft
PZF	Point of zero flow; the lowest point on the controlling section of a stream channel
Q	Discharge in general
Q_{adj}	Discharge adjusted by a rating factor
Q_b	Discharge from a base rating
Q_i	Discharge flowing into a channel reach
Q_m	Measured discharge
Q_o	Discharge flowing out of a channel reach
Q_r	Discharge from a rating
R	Hydraulic radius (area/wetted perimeter) of a cross section
S	Slope of energy gradient
S_c	Energy-gradient slope for steady-flow conditions measured at the same stage as Q_m
U	Velocity of a wave front at a specific stage
V	Mean velocity
V_g	Meter reading of index velocity or vane deflection
W	Average stream width in a reach of channel
X, x	Value measured as an abscissa on a plotted curve
Y, y	Value measured as an ordinate on a plotted curve
$1/US_c$	Term in the "Boyer equation" used for one type of rating (rate of change in stage) and also used as the name of the rating method
ΔQ	Change in discharge (Q_i-Q_o) at the ends of a channel reach caused by storage in the reach
$\Delta Q/J$	Storage effect (Q_i-Q_o) in a channel reach, per unit rate of change in stage, used as the name of a rating method

DISCHARGE RATINGS AT GAGING STATIONS

By E. J. Kennedy

Abstract

A discharge rating is the relation of the discharge at a gaging station to stage and sometimes also to other variables. This chapter of "Techniques of Water-Resources Investigations" describes the procedures commonly used to develop simple ratings where discharge is related only to stage and the most frequently encountered types of complex ratings where additional factors such as rate of change in stage, water-surface slope, or index velocity are used. Fundamental techniques of logarithmic plotting and the applications of simple storage routing to rating development are demonstrated. Computer applications, especially for handheld programmable calculators, and data handling are stressed.

Introduction

Most of the factors that affect the quality of a streamflow record are either determined by natural conditions or costly to improve. However, the hydrographer can greatly improve the quality of records through skillful use of proper procedures in the data analysis. The principal component of the data analysis—and the subject of this manual—is the discharge rating.

Discharge records for gaging stations are generally computed by applying a discharge rating for the site to a continuous or periodic record of stage. A discharge rating is the relation of discharge to stage and sometimes also to such variables as rate of change in stage, fall in a reach between gages, vane deflection, index velocity, gate opening, or turbine pressure differential.

A rating analysis is basically a process in which the data from a series of discharge measurements are plotted on graph paper, a curve defined by the measurements drawn, and a table prepared from the curve. Other data items might include bankfull stage, the dates of artificial changes to the channel or of floods that may have scoured or filled the channel, notes describing the presence or absence of backwater sources, and field surveys or other information defining the general shape of the rating curve.

Most ratings relate discharge to gage height only and are called simple ratings. A simple rating may be only one curve but is more often a compound curve consisting of three segments, one each for the low-, medium-, and high-water (or overbank) ranges. These segments of a compound curve may be connected by short transition curves. A complex rating is one that relates discharge to stage plus some other independent variable, usually either the rate of change in stage at one gage or the fall in a reach between two gages. Complex ratings usually have a stage-discharge relation curve plus one or more supplementary curves.

The purpose of this chapter of "Techniques of Water-Resources Investigations" is to enable the hydrographer to develop, with the least amount of effort, a discharge rating whose quality matches that of the available data. Logarithmic plotting, the channel-storage effect, the relation of various types of data to the rating, and the use of computers are stressed. Some complex rating situations that are rarely encountered are well described in published reports or manuals. Rating procedures for these special methods are mentioned only in general terms in this manual, and the detailed reports are given as references. Ratings for such universally used equipment as flumes and weirs, so well covered in handbooks by Brater and King (1976) and others, have been omitted.

Development of discharge ratings requires relatively minor applications of hydraulic theory, a greater amount of "handed-down" lore that applies mainly to streams in the general area, and a considerable amount of data manipulation. The data-handling aspect is emphasized in this manual. The procedures described are generally the simplest that apply to each type of rating but are not necessarily the only approaches. The methods used are mostly computer-based variations of standard, time-honored procedures. They apply to or can be modified to fit nearly all stream sites that are likely to be gaged. No attempt is made to credit the many hydrographers who first described the techniques that are demonstrated or those who subsequently added refinements.

Basic Concepts

A discharge rating is analyzed by applying some elementary arithmetic and algebraic processes and certain basic concepts of open-channel flow to the available field data. Familiarity with the basic concepts will allow a hydrographer to plan the rating analysis in logical steps and proceed with the least possible effort to develop a rating that makes the best use of all the data.

Controls

The relation of stage to discharge is usually controlled by a section or reach of channel below the gage, known as the station control, which eliminates the effect of all other downstream conditions on the velocity of flow at the gage. Section controls may be either natural or constructed and may consist of a ledge of rock across the channel, a boulder-covered riffle, an overflow dam, or any other physical feature capable of maintaining a fairly stable relation between stage and discharge. Section controls are often effective only at low discharges and are completely submerged by channel control at medium and high discharges. Relatively flat alluvial channels may have no section control at any discharge. Channel control consists of all the physical features of the channel that determine the stage of the river at a given point for a certain rate of flow. These features include the size, slope, roughness, alinement, constrictions and expansions, and shape of the channel. The reach of channel acting as the control may lengthen as the discharge increases and thus may introduce new features affecting the stage-discharge relation.

Knowledge of the channel features that control the stage-discharge relation is important in developing stage-discharge curves. If more than one control is effective and if the number of measurements is limited, interpolation between measurements and extrapolation beyond the highest measurements will require much judgement, particularly if the controls are not permanent and if various discharge measurements represent different positions of the stage-discharge curve.

When a stream overflows its banks, the configuration of, and perhaps the vegetation on, the flood plain affects the discharge rating, and the control becomes a combination of these features and those of the main channel.

As an earlier discussion stated, a discharge rating is often a compound curve consisting of three segments—one for low flow (section control), one for medium flow (channel control),and one for overbank high-water flow (combined channel and flood-plain control).

Gage height of zero flow

The stage that would occur at a gaging station if the discharge were infinitesimal is the gage height of zero flow (*GZF*). It is also defined as the gage height of the point of zero flow (*PZF*), the highest point on the thalweg (the longitudinal thread of the stream that follows the deepest point in each cross section) downstream from the gage. *GZF* can be measured by levels where the section control is artificial or naturally well defined and permanent. Otherwise, *GZF* should be measured at the time of each low-water wading measurement at an unstable channel site and less frequently in a relatively stable channel. *GZF* is usually measured by taking soundings along the thalweg near the control and subtracting the minimum thalweg sounding from the gage height. An accuracy of *GZF* measurement within a tenth of the sounding depth is possible over a smooth control. Errors may be much greater over a rough, boulder-lined control or where flow is great enough to obscure the control's location. A rough estimate of *GZF* for an alluvial channel that has no evident controlling section can be made by subtracting the deepest sounding in a wading measurement from the gage height. All *GZF* determinations should include the hydrographer's estimate of probable error. The logarithmic scale to be used in a rating analysis and the shape of the low-water rating curve are closely related to *GZF*, and a rigorous analysis of an unstable channel rating cannot be made wtihout determining *GZF* for most of the low-water discharge measurements.

Channel-storage effect

The measuring section or control for a gaging station may be a considerable distance from the gage. If that distance is great enough, the effect of channel storage on the discharge measurements or on the rating must be accounted for. A long stream reach having no tributaries has almost the same discharge at all locations only during periods when the water-surface elevations in that reach remain constant. If the water surface is rising or falling, the discharges at various locations may differ significantly because some of the flow is going into or coming out of storage in the channel.

If the inflow to the stream reach shown in figure 1 is greater than the outflow, the difference between inflow and outflow must be stored in the channel. The water surface must rise sufficiently during the period of imbalance to provide for the storage. Conversely, when the water surface is falling, the outflow must include the water coming out of storage and must be greater than the inflow. For a given reach, the relation between inflow and outflow depends on the rate of change in water-surface elevation and the average water-surface area in the reach. The general storage equation is

$$Q_i - Q_o = \frac{L \times W \times J}{3{,}600}$$

where Q_i is inflow in cubic feet per second, Q_o is outflow in cubic feet per second, L is the length of reach in feet, W is the average width of reach in feet, J is the average rate of change in water-surface elevation (positive for rising stage and negative for falling stage) in feet per hour, and $Q_i - Q_o$ is sometimes called ΔQ.

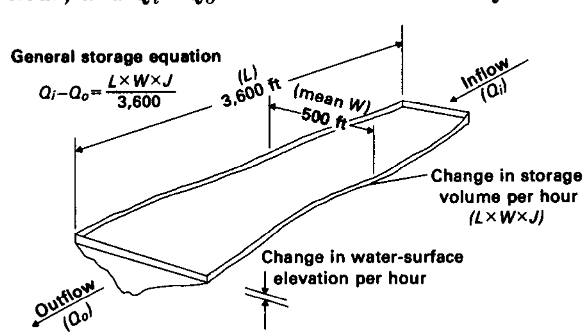


FIGURE 1.—Hypothetical stream-channel reach illustrating changing channel storage effect.

For the reach shown in figure 1, an inflow of 1,500 ft³/s, and an outflow of 1,000 ft³/s, the rate of change in water-surface elevation is computed as

$$1{,}500 - 1{,}000 = \frac{3{,}600 \times 500 \times J}{3{,}600}$$

Therefore, $J = +1.0$ ft/hr.

For the same reach, an inflow of 1,000 ft³/s, and a water surface falling at 0.5 ft/hr, the outflow is computed as

$$1{,}000 - Q_o = \frac{3{,}600 \times 500 \times (-0.5)}{3{,}600}$$

Therefore, $Q_o = 1{,}250$ ft³/s.

If the discharge Q_o of a stream whose average width W is 180 ft is measured as 1,000 ft³/s at a site 10,000 ft downstream from the gage while the water surface is falling 0.33 ft/hr at the gage and 0.27 ft/hr at the measuring site (average 0.30=J), the discharge Q_i at the gage is computed as

$$Q_i - 1{,}000 = \frac{10{,}000 \times 180 \times (-0.30)}{3{,}600}$$

Therefore, $Q_i = 850$ ft³/s.

The storage equation is used most frequently to adjust high-water discharge measurements made at a remote location while the stage is changing. The effect of storage on low-flow discharge measurements may be significant even where the rate of change in stage and the distance involved are surprisingly small, particularly for any large gage pool whose inflow was measured while the gage height was still falling after the control had been cleaned.

Channel storage is the dominant factor in defining rate of change in stage ratings covered in a subsequent section of this manual. For that type of rating, the storage reach falls between the gage and a control whose location may vary with stage. Graphic trial-and-error methods are used instead of the equation to evaluate storage, but the basic concept is the same.

Data limitations

Some rating curves would seem to fit the data more closely if one or two of the discharge measurements were eliminated from the analysis. Sometimes an outlier measurement is obviously and seriously in error, but, often, the measurement is satisfactory and important to the rating analysis. Low- and medium-water measurements are normally made by using standard procedures, and their errors rarely exceed 5 percent. A flood measurement, on the other hand, may be made at night and may involve road overflow or concealed culvert flow, rapidly changing stage, improvised equipment, drift, or other conditions that reduce accuracy. Even these measurements are rarely in error by more than 10 percent. A stream may be so flashy that the best possible measurement may consist of only a few surface velocities, and the error may be as much as 15 percent. Indirect measurements are not usually made unless the conditions promise accuracy within about 20 percent. Such relatively large variations from the ratings are acceptable for measurements of these types. Summer flood measurements, made while inundated trees and brush are covered with leaves, tend to plot to the left of winter flood measurements, but this seasonal effect can be corrected by shifting-control adjustments and should not be confused with measuring error.

Unsynchronized base and auxiliary gage timers at slope stations may make an outlier out of an otherwise good discharge measurement made while the stage was changing rapidly. Such discharge measurements can be corrected if the timing errors can be determined closely, but it is desirable to make measurements at a slope station only while the stage is relatively constant.

Base and auxiliary datum errors are present at most slope stations and may cause erratic plotting of the extreme low-water measurements. Even first-order levels that are run to establish datum differences between gages 10 mi apart may have errors as great as 0.06 ft. Datum agreement can be checked at a site where ponded conditions occur throughout a slope reach during periods of negligible flow, when base and auxiliary gages set to the same datum should read the same. A sluggish intake at one gage or the other can affect the measured fall and cause a discharge measurement to plot erratically.

Discharge measurements at gaging stations where rate of change in stage is a rating factor pose a special problem if the gage-height record is subject to surging or bubble-gage stepping. The plotting position of this type of measurement depends on a substantial changing-stage adjustment as well as on the measured discharge, and a measurement can plot as an outlier if its adjustment is based on gage readings distorted by surging or stepping. This factor can be eliminated by manually smoothing a graphic gage-height record or by using a smoothing option, described in the WATSTORE User's Guide (Hutchison and others, 1975, 1980) for the primary computation of records for stations with rate of change in stage rat-ings. If a smoothed gage-height record is used for daily discharge computation, the same smoothed record must be used to plot and adjust the discharge measurements.

No discharge measurement, made either by current meter or indirectly, should be disregarded (left with an unaccountably high percentage difference from the rating) without a good reason. Disagreement with other measurements is generally not reason enough. If an outlier measurement is truly in error, the reason for the disparity often can be discovered. The hydrographer should check the arithmetic of an outlier measurement, compare the mean gage height with recorded and outside gage heights, compare the plotted cross sections and velocity distributions for several measurements made at the same site, consider the possibility of backwater, and check the equipment used. If these checks and others indicate that the outlier is a valid discharge measurement, it should be given appropriate weight in the analysis.

Use of computers and calculators

The procedures demonstrated in this manual include logarithmic interpolation between curve coordinate points, fitting equations to curves, and some repetitive chains of arithmetic for trial-and-error solutions. The long manual computations required by these operations can be performed rapidly with a handheld card-programmable calculator and even faster with more elaborate computers.

Mathematical and statistical packages are available as accessories for most programmable calculators. These packages contain instructions and program cards or modules for standard operations, including linear and polynomial regression for curve fitting. Nearly all rating-related computations can be performed by using the U.S. Geological Survey's central computer facility through an appropriate terminal and the National Water Data Storage and Retrieval System (WATSTORE). Instructions for preparing input for logarithmic interpolation between rating coordinates (RATLIST) and the analysis of some complex ratings (for example, rate of change in stage and slope) are included in the WATSTORE User's Guide (Hutchison and others, 1975, 1980).

A relatively inexpensive, handheld, card-programmable calculator with a large capacity for storage is particularly helpful for rating analysis, used either alone or as a supplement to more elaborate computers. The most useful program, available from U.S. Geological Survey personnel, for several makes and models, stores coordinates for a logarithmic rating and displays the discharge for any gage height that is entered and, conversely, the gage height for any discharge that is entered.

Plotting

A major part of a discharge rating analysis consists of plotting the gage heights and measured flows from relevant discharge measurements, drawing average or weighted curves based on the measurements and related data, and preparing the discharge rating in a format suitable for use in processing streamflow records. The actual analysis for ratings extending to near-zero flow is usually done on worksheets, which may be discarded after the final results are plotted neatly on one master curve sheet for reproduction and permanent filing.

Gage height, the independent variable, is almost always plotted as the ordinate (Y axis) in hydraulic usage, contrary to standard convention. Because of this practice, the slope of the rating curve is the cotangent of the vertical angle (the ratio of the discharge increment to the gage-height increment, or X/Y) rather than the customary tangent (Y/X).

Rectangular grids

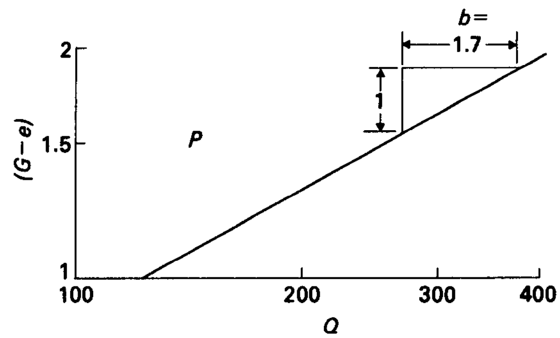
A curve of rectangular coordinates is the simplest and clearest graphic illustration of the relation between two variables. Rectangular plotting is usually appropriate for the low-water part of a master rating sheet and for some or all of the master curves in a complex slope rating. A curve that is developed on a logarithmic grid should always be replotted on rectangular paper and checked for reasonable shape before use. In general, however, a stage-discharge relation curve should not be plotted exclusively on a rectangular-grid work curve sheet.

Logarithmic grids

Logarithmic (log) plotting paper has some advantages over other types of grids that make it the principal tool for graphic rating analysis. A log rating curve has curvature, slope, and an intercept that are related to channel characteristics. Parts of a log-log curve or, in some instances, the entire curve can be straightened by adjusting the gage-height scale. A straight line on log paper represents a curve that can be described exactly by very few numbers or as a simple equation, convenient for computeror calculator use, and that is extended more readily than a curved line. Techniques for the use of log plotting are relatively simple and are essential for developing all types of discharge rating curves.

Straight-line rating curves

Figure 2 illustrates a rating curve plotted on a logarithmic grid. The gage-height values have been reduced before plotting by a constant value e, and, thus, the curve plots as a straight line. The magnitude of e, called the gage-height scale offset, is determined by one of several methods explained in the next section but usually approximates the gage height of zero flow. The slope of the straight-line rating can be determined graphically, as figure 2 shows, by drawing a vertical line one unit long starting at the rating curve and scaling the horizontal distance from the end of that line to the rating curve. The horizontal distance is the



Where:
b is the slope of the straight line.
e is the constant which when subtracted from G will result in a straight line on logarithmic paper for the plot Q vs $(G-e)$. The value of "e" is usually the gage height of zero flow or the effective GZF. The value of "e" used, or any other constant that is subtracted from G is called the "Scale Offset."
G is the gage height.
P is the intercept equal to Q when $(G-e)$ is equal to 1.0.
Q is the discharge.

The general equation for curves of this type is

$$Q=P\ (G-e)^b$$

The equation of the curve shown is $Q=125\ (G-e)^{1.7}$

FIGURE 2.—Hypothetical discharge rating plotted as a straight line on a logarithmic grid.

slope of the line, the exponent of the rating equation, and an indicator of the type of control. A slope greater than 2.0 generally indicates a section control, and a slope less than 2.0 indicates that a channel control is likely. A straight-line log rating usually can be extended upward but not beyond the stage where the channel changes shape (for example, at a terrace or at bankfull stage).

Adjustment of logarithmic scales

Figure 3A illustrates a "normal" logarithmic scale. Gage heights plotted to this scale first must be adjusted by subtracting the scale offset e to make the logarithmic plot of the rating curve a straight line. Conversely, this scale offset must be added to any value picked off the "normal" scale to convert that figure to a gage height.

Adjustment of the scale so that gage heights can be plotted directly is the general practice, universally followed where the scale offset can be rounded to the nearest foot or half foot without causing undue curvature of the plotted rating. Where the scale offset must be carried to a 0.01-ft refinement, gage-height adjustment is likely to be more convenient than scale adjustment. Figure 3B illustrates the difficulty of adjusting the scale by adding an offset carried to 0.01 ft. Only the lower cycle graduations can have useful labels. Figure 3C illustrates the effect of rounding the offset to 0.1 ft. Two cycles of graduations then can be labeled. Figures 3D and 3E illustrate how three cycles of useful labels result from rounding the scale offset to the nearest foot. Such rounding will yield a useful gage-height scale but may cause unsatisfactory curvature of the low-water rating.

If the stream depth at the control is more than about 3 ft when gage height is at the lower end of the rating, curvature of the log rating plot is relatively insensitive to changes of less than a foot in offset. The gage-height scale for a rating at substantial minimum depth can be adjusted by an offset rounded to the nearest foot, and the adjusted labels will fit the grid without significantly affecting the curvature of the rating. An artificially controlled stream whose rating must extend down to zero

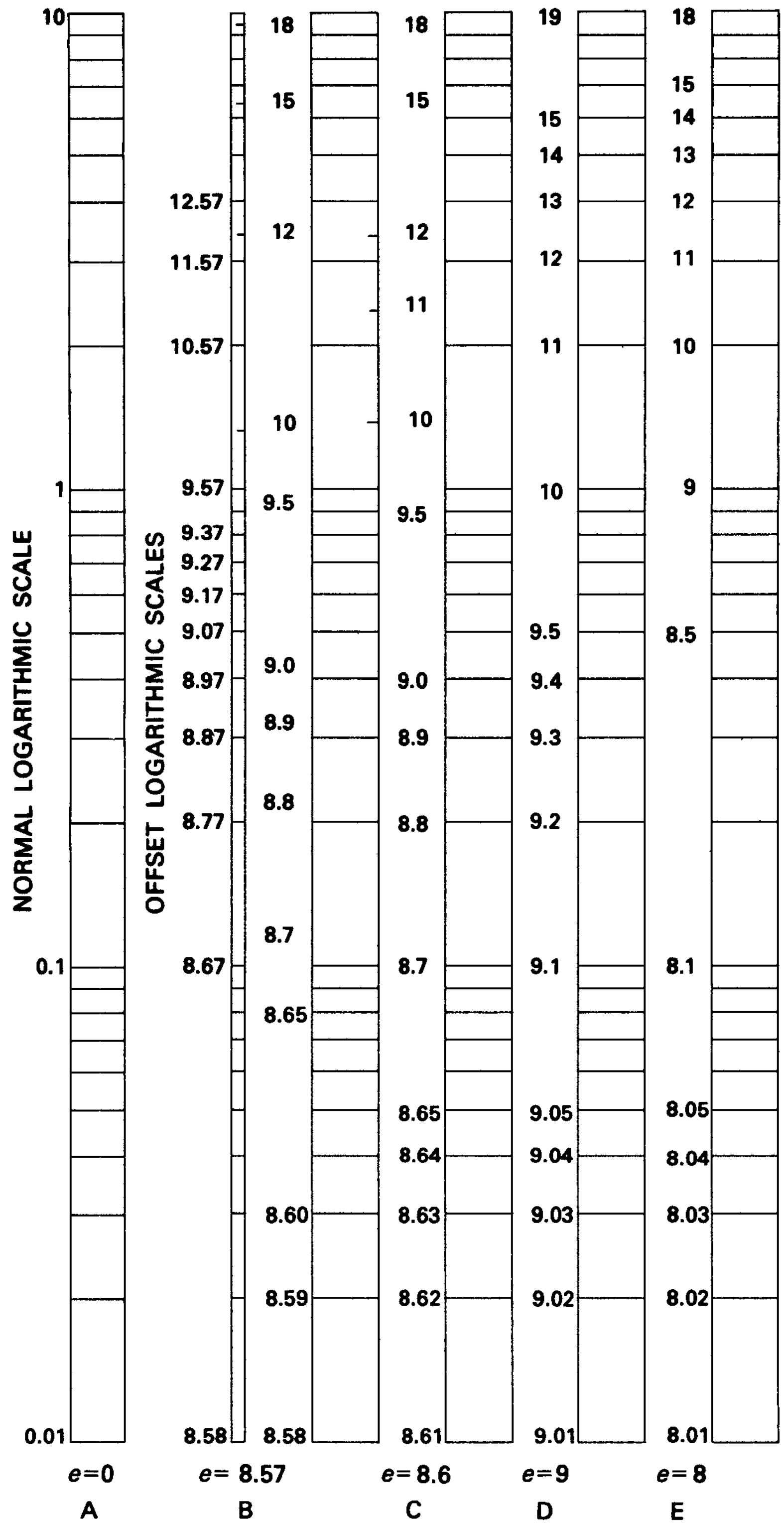


FIGURE 3.—Relation of various offset gage-height scales to the normal log scale.

flow is one type whose offset must sometimes be carried to a 0.01-ft refinement, and gage-height adjustment must be used rather than scale adjustment. A normal log ordinate scale label, "Gage height – (Scale offset value) feet," is used if the gage height is adjusted before plotting. Some ratings will allow rounding of the offset to 0.1 ft only, and either the scale or the gage-height adjustment option may be satisfactory. Master curve sheets, described in a subsequent section of this manual, are prepared after the discharge rating is developed, and then the curve alinement is secondary to a clear display of results. The master-sheet scale offset is usually rounded to the nearest foot for the best fit of the curve to the sheet having the best possible scale labels.

In developing most ratings, a single worksheet whose scales cover the entire range of stage and discharge is preferable, and the single offset used on that sheet is chosen to suit the scale labels. Supplementary worksheets for low-water, overbank, and, possibly, other rating segments, each with its own optimum value of e, may be needed. For instance, to straighten a low-water curve segment, an offset of 1.76 ft (*GZF*) may be needed, whereas an offset of 3 is required to straighten the middle range, and a value of 2 makes the best scale for the full-range sheet. The straight-line ratings developed on the supplementary worksheets are then transferred to the full-range sheet where, because of the rounded offset used on that sheet, they will become curved parts of the rating. Any logarithmic rating plot whose high-water part is concave downward will remain concave downward regardless of the value of any gage-height scale offset that is used.

A logarithmic scale can be adjusted further by multiplying the normal or offset scale values by a constant or by raising them to a power. Multiplying a gage-height scale by a constant raises or lowers the rating curve's position on the sheet but does not affect its size or shape. Raising the scale value to a power flattens or steepens the rating curve. Both procedures lead to scales that fail to fit the log paper's printed graduations and should ordinarily be avoided.

Finding the scale offset value needed to straighten a curved segment of a log rating requires either the trial-and-error solution illustrated in figure 4 or the direction solution illustrated in figure 5.

Figure 4 shows a typical stage-discharge relation between gage height (G) and discharge (Q) plotted as the top curve (e=0). The best value for e, when it is applied to G, will result in a straight-line relation between ($G-e$) and Q. In successive trials, the ordinate scale is varied by using e values of 1, 2, 3; each value results in a different curve, but each still represents the same rating as the top curve. The correct value of e is 2 because the rating plots as a straight line if the normal ordinate scale numbers shown on the logarithmic grid are increased by this value. If smaller values of e are used, the curve will be concave upward; if higher values of e are used, the curve will be concave downward. The value of e for a segment of a rating thus can be determined by adding or subtracting a variety of trial values to or from the numbered scales on the logarithmic grid until a straight-line rating is obtained.

A more direct method for finding e (Johnson, 1952) is illustrated in figure 5. The solid-line curve is straightened by subtracting the scale offset e from each value of G. The coordinates (G_1, Q_1) and (G_2, Q_2) for points near the extremeties of the curve to be straightened are picked by using the normal logarithmic scales. A value for Q_3 at the logarithmic midpoint is computed so that $Q_3=+\sqrt{Q_1Q_2}$. The corresponding gage height G_3 is picked from the solid curve. The scale offset computed from the equation for e (fig.5) will place the three points (Q_1, G_1-e), (Q_3, G_3-e), and (Q_2, G_2-e) in a straight line, and the solid-line curve plotted to the gage-height scale as offset by e will assume the dashed-curve position.

A straight-line rating rarely needs to be precisely straight. Although a curve slightly concave upward or downward throughout its range could be made straighter by further adjustment of e rounded to 0.01 or 0.1 ft, such a refinement might be unnecessary.

The value of e applicable to an already-labeled gage-height scale is the labeled value of any graduation minus its "normal" scale value. In figure 3*E*, e=8.1−0.1=8 or e=9−1=8. If the "normal" scale value is not obvious, the upper gage height G_U, and the

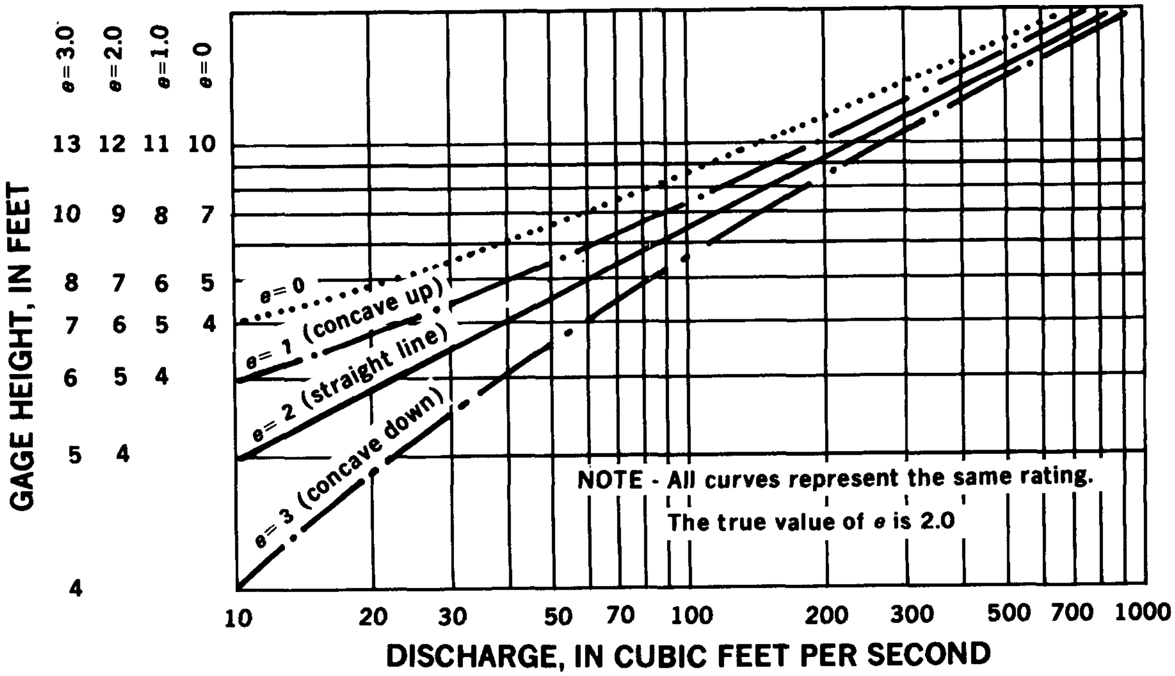
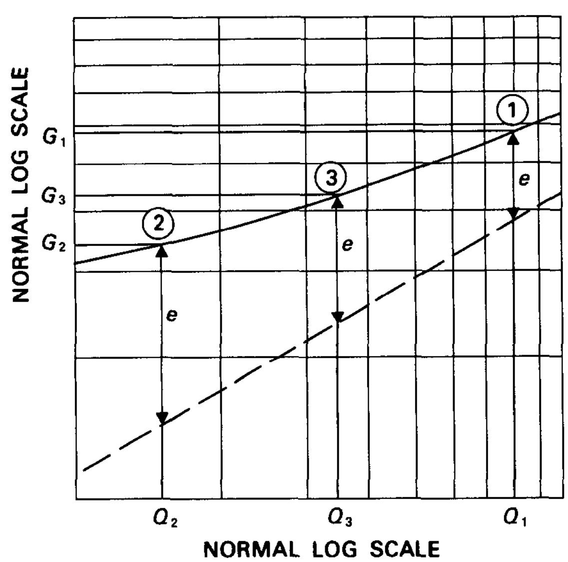


FIGURE 4.—Rating curve shapes resulting from different gage-height scale offsets.



$Q_3^2 = Q_1\,Q_2$ and when the dashed line is straight $(G_3-e)^2=(G_1-e)\,(G_2-e)$, solving for e

$$e=\frac{G_1\,G_2-G_3^2}{G_1+G_2-2G_3}$$

FIGURE 5.—Determination of scale offset by Johnson's (1952) method.

lower gage height G_L of any complete log cycle can be substituted in the equation

$$e=(10G_L-G_U)/9$$

In figure 3*B*,

$$e=[(10\times 8.58)-8.67]/9=8.57\text{ft}$$

or

$$e=[(10\times 8.67)-9.57]/9=8.57\text{ft}$$

Characteristics of low-water logarithmic rating curves

An especially useful characteristic of curves plotted on log-log paper applies mainly to the lower part of any discharge rating, particularly one that must extend to or near zero flow. Any straight line drawn on log paper, with a slope between 0° and 90° from the horizontal will pass through the "normal" scale origin (0,0) if it is extended downward on rectangular paper. All of the lines plotted in figure 6*A* intersect at the point (2.3,0.18). The same curves plotted on a rectangular grid in figure 6*B* also intersect at (0,0). For discharge ratings, it follows that any straight-line rating curve on a log-log grid must pass through zero discharge at gage height *e*. Actually, zero values cannot be plotted on a logarithmic grid. However, the knowledge that any straight line extended downward will pass through zero discharge at gage height *e* makes such a plot unnecessary. A rating curve on a logarithmic grid whose *e* is inaccurate will bend if it is extended downward through several log cycles and become either horizontal or vertical depending on the sign of the error. For this reason, it is usually unwise to develop on the same logarithmic worksheet the extreme low-water parts of two or more curves having different values of *GZF*.

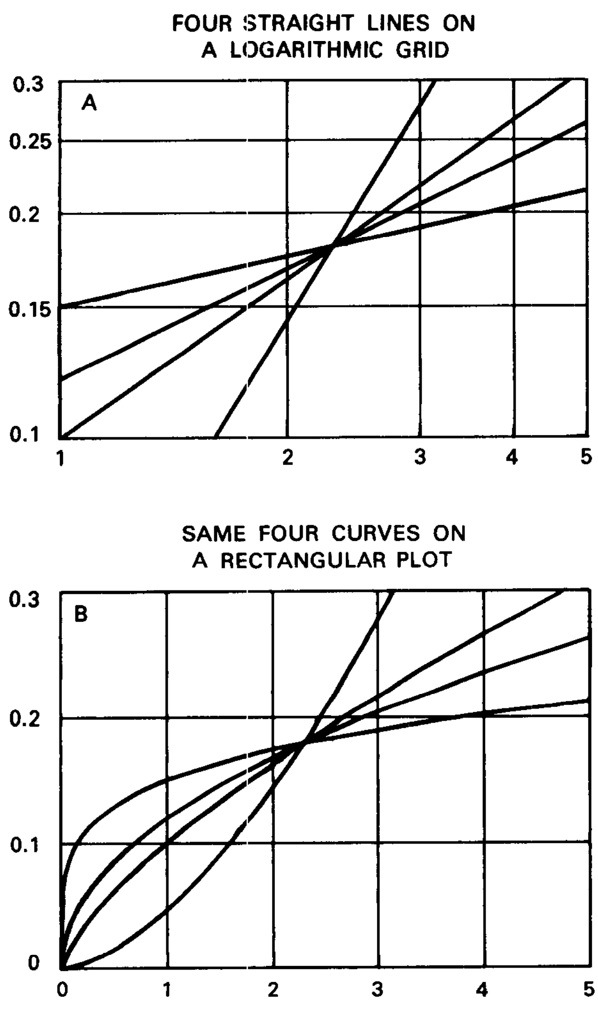


FIGURE 6.—Comparative plots of identical curves on logarithmic and rectangular grids.

Logarithmic curve coordinates

A rating starts as a plotted curve that must be converted into different formats for various uses. The first step in the conversion process is to approximate the logarithmic rating curve by using a series of straight-line segments, as figure 7 shows. A very close approximation of most logarithmic rating curves can be made with fewer than 10 straight lines. The maximum difference between the original curve and the approximation line is usually held to about 1 percent for high and medium rating parts and more for extreme low or sharply curved parts. If the logarithmic curve is concave upward, as it is in the curve near the 4-ft gage height (fig. 7), the use of long segments may lead to an undesirable but normally harmless condition called "scalloping." When the segments are plotted on a large-scale rectangular grid, especially by an automatic plotter, as a subsequent section describes, they may form a slightly scalloped curve rather than a smooth one. The scalloping is usually noticed when a table of discharges corresponding to the gage heights at 0.10-ft intervals (rating table) is prepared, and the discharge differences per 0.10 ft of gage height change abruptly at the gage heights of approximation line intersections. Scalloping can be minimized by using short, straight lines to approximate the logarithmic rating curve's concave upward parts, or it can be completely eliminated by increasing the gage-height scale offset until the logarithmic rating curve is straight or concave downward.

The scale offset *e* used for the consolidated worksheet and the coordinates of the straight-line segment intersections define the curve. These numbers are the digital descriptors of the rating and should be the only rating input for automatic processing of the daily discharge records. This set of descriptors should also be the only basis for compiling any rating tables that are prepared.

Interpolation between logarithmic curve coordinates

When the digital descriptors are used as a rating, each computation of the discharge corresponding to a specific gage height requires

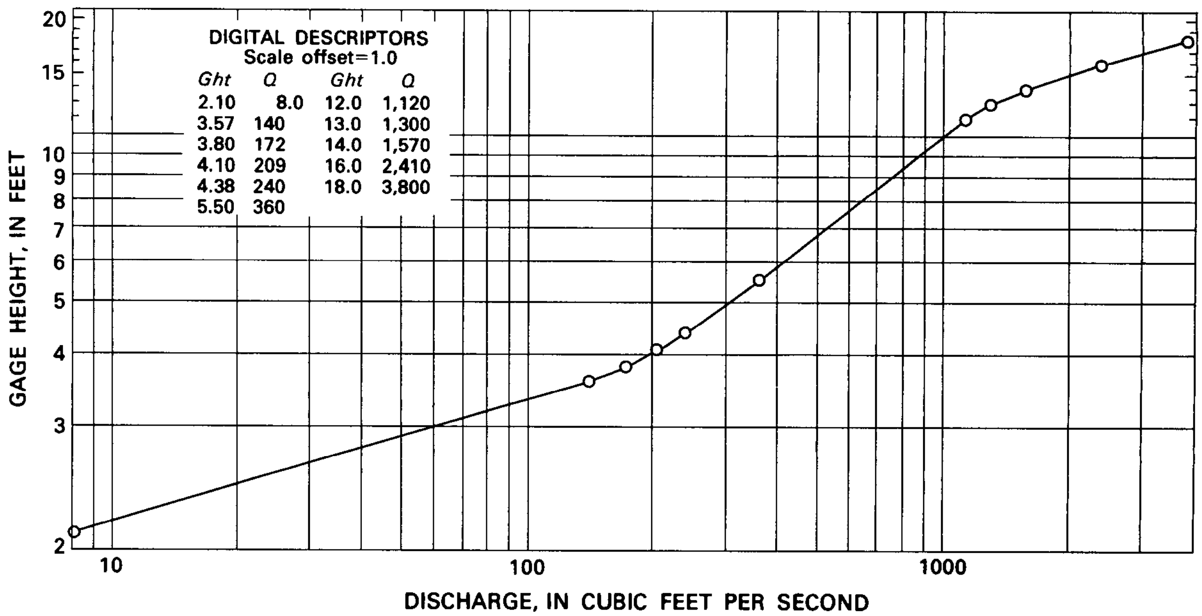


FIGURE 7.—Typical logarithmic rating curve with corresponding digital descriptors.

a solution of the formula illustrated and derived in figure 8. A calculator or computer is highly desirable because manual computations would be impractically slow. The formula is used in the WATSTORE program RATLIST which converts a set of descriptors into a rating table, and in rating-point interpolation programs for calculators. A calculator program stores the rating descriptors in the calculator's memory and displays the discharge value corresponding to any gage height that is entered.

Formats for ratings

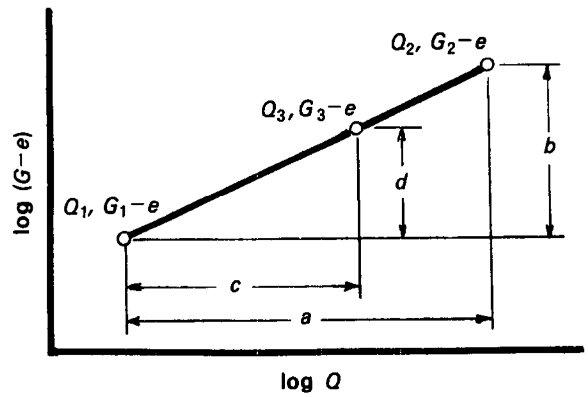
The same discharge rating can be a plotted curve, a table, an equation, or a list of descriptors. Each version has advantages over the others for specific purposes.

Rating curve sheets

The graphic format of a rating portrays the stage-discharge relation visually and simply and is the form used for the initial rating analysis. Work curve sheets are used to make graphic analyses of logarithmic ratings that have critical scale offsets, to determine auxiliary relations in complex ratings that need rectangular plotting and trial-and-error solutions, and to make rectangular plots of logarithmic curves to check them for reasonable shape. The work curve sheets can be discarded after the rating is developed and simpler scales are drafted in ink on a master curve sheet for the permanent record. Simple ratings for streams in relatively stable channels, where the minimum flow is greater than about 5 ft^3/s, usually can be developed on the same logarithmic sheet that will later be inked and used for the master sheet.

Work curve sheets

The actual development of most ratings is done most conveniently on log paper worksheets with enough cycles to cover the entire range of stage and discharge. For unstable channels, one worksheet is needed for each rating whose gage height of zero flow (*GZF*) is significantly different from that of its preceding rating, and its gage-height scale offset will correspond to the *GZF*. A rectangluar sheet is also needed to plot the curves developed on log paper and to check that their shapes are reasonable. Some auxiliary relations for complex ratings, explained in subsequent sections (storage, fall, and so forth), require rectangular work curve sheets for the development of the curve. Other auxiliary curves must be developed originally on log paper.



Where
e =Scale offset
G_1=*Ght* of lower coordinate
G_2=*Ght* of upper coordinate
G_3=*Ght* of intermediate coordinate
Q_1 =Discharge of lower coordinate
Q_2 =Discharge of upper coordinate
Q_3=Discharge corresponding to G
a,b,c,d= Lengths in log units
(1 log cycle length=1 log unit)

From similar triangles on diagram:

$$\frac{a}{c}=\frac{b}{d} \text{ or } c=\frac{ad}{b} \qquad (1)$$

Substituting equivalent values in equation 1:

$$c=\frac{(\log Q_2-\log Q_1)(\log(G_3-e)-\log(G_1-e))}{\log(G_2-e)-\log(G_1-e)} \qquad (2)$$

From diagram:

$$\log Q_3= Q_1+c \qquad (3)$$

Substituting equation 2 for "c" in equation 3, and taking antilogs, formula for Q_3 in algebraic format is:

$$Q_3= \text{antilog}\left[\log Q_1+\frac{(\log Q_2-\log Q)(\log(G_3-e)-\log(G_1-e))}{\log (G_2-e)-\log(G_1-e)}\right]$$

or in BASIC format:

```
Q3=10+(LGTQ1+(LGTQ2-LGTQ1)*(LGT(G3-E)
-LGT(G1-E)))/(LGT(G2-E)-LGT(G1-E)))
```

FIGURE 8.—Derivation of the logarithmic coordinate interpolation equation. Marginal notes, curve labels, effective dates, and such must be added manually.

Most wide-range simple ratings can be analyzed best on a diagonally ruled log sheet (Form 9–279M), which has 3×5 usable cycles. Excess cycles can be trimmed off or additional cycles taped in place. All discharge measurements relevant to the rating being analyzed and a shape curve (prior full-range rating for other-than-new gaging stations) are plotted on the worksheet. Colors can be used to distinguish the measurements that apply to a particular rating or those affected by ice or temporary backwater because the sheet will not be reproduced.

A print of the previous year's master curve sheet for a large stream sometimes can be used as a worksheet. Smaller streams, especially those whose minimum flow is near zero, have critical gage-height scale requirements and normally need separate worksheets for each new rating.

Master curve sheets

When the rating analysis is completed, the plotted curves are on penciled worksheets, often in a variety of sizes and formats. A curve sheet is needed for the permanent files and for reproducing copies for cooperating agencies, field folders, and planning purposes. A master sheet, inked and on good-quality paper, generally is prepared for reproduction and for the permanent record. Some compromises of curve characteristics that are vital for worksheets but less important for display of results may be tolerated in order to plot the curves on standard sheets of reasonable size. For instance, although the hybrid or rectangular scales used in this manual for some of the master curve sheets illustrating various kinds of ratings would be unsatisfactory on work curve sheets, they do present the ratings more clearly on the master curve sheets by minimizing clutter.

Standard (9–279 series) 11×17 in sheets have 2-×3-log cycles, and the larger sheets (17×22 in) have 3×4-log cycles. Two, or sometimes three, gage-height cycles may not cover the entire range, so some versions have a rectangular grid in the upper left-hand corner on which a rectangular plot can be substituted for the lower part of the log curve. Diagonally ruled log sheets (9–279M), the best worksheet forms, can also be used for master sheets.

Use of a standard log-rectangular combination sheet may require some gage-height scale adjustment. For instance, the rating shown on the master curve sheet used in a subsequent section of this manual (see fig. 15), plotted entirely on a log grid and using the scale offset (2 ft) that was used on a worksheet to straighten the curve's lower end and to locate its digital descriptors, would require three log cycles for gage height (2.8 to 35 ft) and seven discharge cycles (0.01 to 100,000 ft^3/s). By using a scale offset of 4 ft and rectangular plotting for the part of the curve below 100 ft^3/s, the curve fits easily on a 2×3-cycle combination sheet, and the curvature caused by changing the scale offset is not apparent in the part of the log curve plotted.

Several trials may be needed to find a satisfactory combination of scales for use on a combination log-rectangular master curve sheet. One procedure is to select a horizontal log scale so that the maximum discharge is as close as possible to the right margin. The best gage-height scale and offset make the log curve cross the left margin above but close to the bottom margin. If the point where the curve crosses the margin is too high or too low, the gage-height scale offset can be adjusted to correct the location. Most satisfactory rectangular discharge scales have as their highest value 2, 10, or 20 times the discharge at the left margin of the log scale. The rectangular gage-height scale should permit, if possible, plotting of the zero-flow gage height. All the rectangular scales should be chosen so that major divisions are 1, 2, or 5 (4 as a poor last resort) times some power of 10.

Master curve sheets for streams that are limited in range are best with only a log plot; those for some streams that have complex ratings may be better illustrated by rectangular plotting alone. Complex slope stations (see fig. 21) often need a sheet that has been prepared specially by a pasteup and photographic transfer process.

Automatically plotted curve sheets

Some hydrographers use an onsite minicomputer or a programmable desk calculator equipped with a plotter to analyze ratings. A program to plot a rating curve and the relevant discharge measurements on the master curve sheet is written to fit the available equipment. If the plot is made on Form 9–279P, it may resemble the one shown in figure 9.

Digital descriptors of ratings

A rating that is to be a part of a computer program should be as concise as possible. The average discharge rating table describes that rating with several thousand digits. The same rating can be described with about 50 digits by

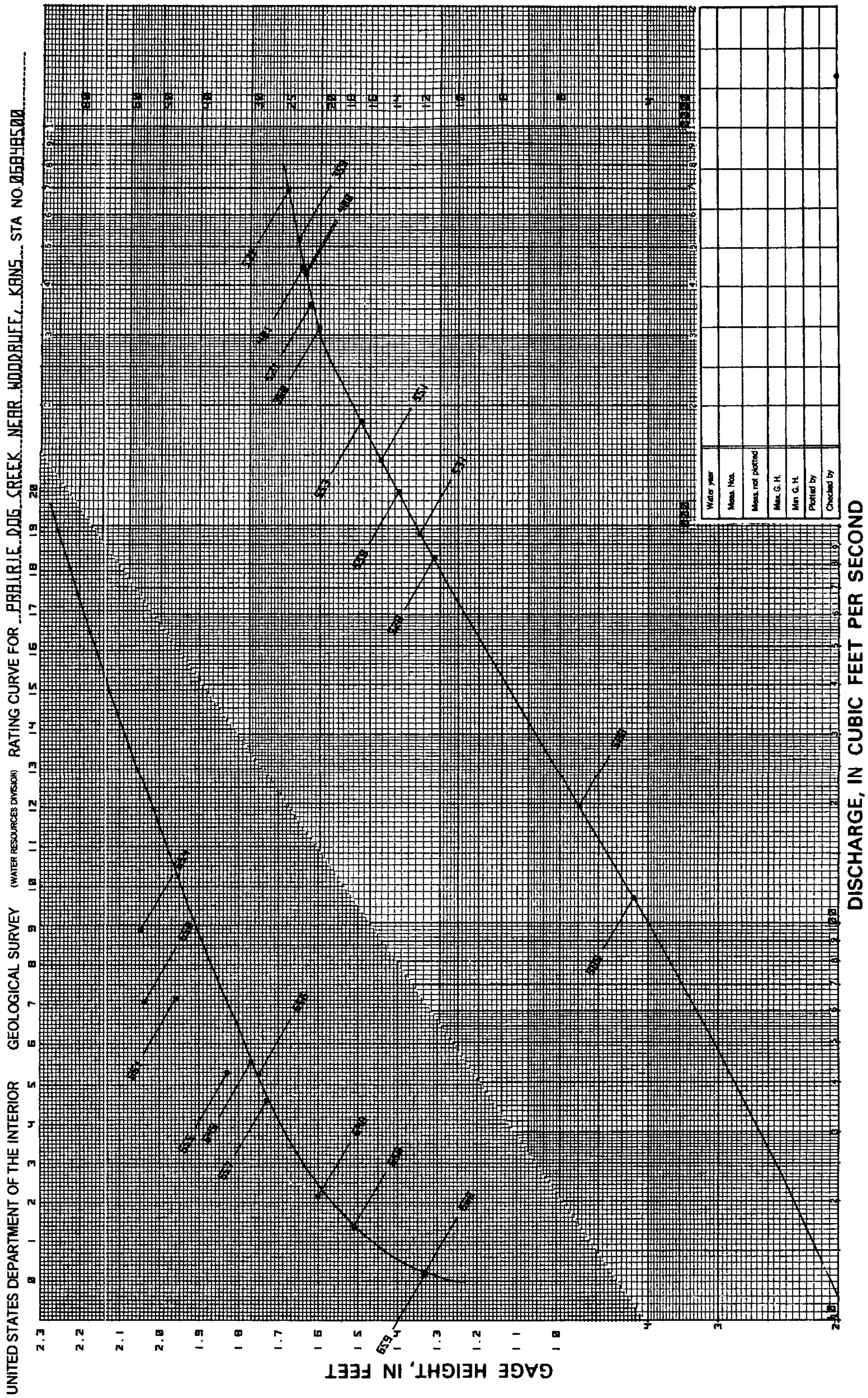


FIGURE 9.—Automatically plotted rating curve-sheet.

using logarithmic coordinates or with about 15 digits by using an equation.

Logarithmic interpolation

Single-offset logarithmic interpolation uses one gage-height scale offset and the coordinates (gage height and discharge) of the ends of straight-line segments that approximate the logarithmic rating curve. This type of descriptor system, described in a previous section and illustrated in figure 7, is the system used most commonly for stage-discharge ratings.

Multiple-offset logarithmic interpolation, an elegant development of the single-offset method, uses offsets that vary with stage for both the gage height and the discharge scales. The method can be used in conjunction with a computer terminal and the WATSTORE system to calculate the fewest possible special descriptor sets that can define a curve through selected coordinates or with a manually prepared rating table, within a user-defined degree of variation. Multiple-offset descriptor use is described in the WATSTORE User's Guide (Hutchison and others, 1975, 1980) and can be adapted to some special rating problems. However, in its present (1980) state of development, the procedure has little application to the processes covered in this manual.

Linear interpolation

The descriptors for some auxiliary relations to complex ratings (fall, storage, and so forth) are used with linear interpolations rather than with logarithmic interpolations. This system requires more descriptors than the logarithmic method but can be used if negative values are involved.

Rating equations

A simple equation is the most convenient rating format for certain applications, especially the auxiliary relations used with some complex ratings. Equations for ordinary discharge ratings are too difficult to fit and too unwieldy for practical use.

Most complex ratings (slope, index velocity, and so forth) involve auxiliary relations (for example, fall versus factor or stage versus coefficient) that are straight lines on logarithmic grids or even on arithmetic grids. These curves (indeed, most parabolic curves) can be described by simple equations. The curve-fitting process can be expedited by using programmable calculators, some of which have linear regression programs built into the hardware. Most programmable calculators and computers have standard program packages (such as STATPAC or MATHPAC) and easy-to-follow user's guides for fitting equations to parabolic, logarithmic, power function, and other common types of curves.

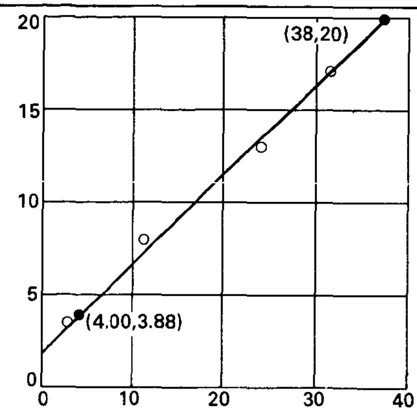
Curve-fitting programs involve inputting data points that are fitted to the appropriate curve by least-squares regression. The fitted equation, particularly for a parabola, may not be applicable outside the range of the data points. To counter this objectionable feature, a trial curve is drawn manually well beyond the range of the data points, and points from that curve are used for the input. Figure 10 illustrates fitting equations to various types of curves.

Rating tables

A table is the preferred rating format for manual computation of discharge records, shift adjustments, percentage differences, and periods of special conditions (fragmentary gage-height record, ice, backwater, and so forth) and for the file copies of the ratings.

Manual preparation of a rating table is a very slow process. Discharge figures for each foot or so of gage height are picked off the curvilinear plot and entered on an appropriate (standard 9–210 series) form. Starting at the low end, trial differences per 0.10 ft of gage height are added to each discharge value and adjusted as necessary until the values vary smoothly, yield properly rounded figures, and match the figures picked off the curve. Some compromises are usually necessary to accommodate all three objectives. Some hydrographers adjust second differences to assure smoothness in the rate of change of the differences per 0.10 ft of gage height. The lower ends of many ratings require an expanded table that lists discharges for gage heights at increments of 0.01 ft. The difference in dis-

A. LINEAR



General equation: $y = a_0 + a_1 x$

a_0 = y axis intercept
a_1 = slope of line

Options:

1. Compute from two points on a known line (x_1, y_1) and (x_2, y_2)
   $a_1 = \frac{y_1 - y_2}{x_1 - x_2}$ $a_0 = y_n - a_1 x_n$
2. Regression of actual data points *(computer)

Examples using the above curve:

1. Two points are (4.00,3.88), (38.0,20.0):
   $a_1 = \frac{3.88-20.0}{4.00-38.0} = 0.474$
   $a_0 = 3.88-(0.474 \times 4.00) = 1.984$
   The equation is: $y = 1.984+0.474$
2. Regression of data points by computer

Computer input

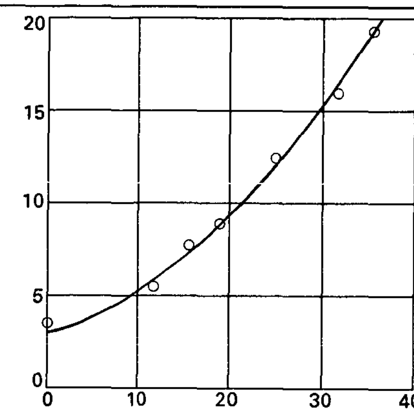
x	y
3.6	3.5
12.0	8.0
24.0	13.0
32.0	17.0

Computer output

a_0 = 2.024
a_1 = 0.467
The equation is:
$y = 2.024+0.467\,x$

*Central computer, minicomputer, or programmable calculator with appropriate program from manufacturer's library (STATPAC, MATHPAC, etc.)

B. PARABOLIC



General equation: $y = a_0 + a_1 x + a_2 x^2$

a_0 = y axis intercept

Options:

1. *Computer input of points from manually-drawn curve
2. *Computer input of actual data. (See text for drawbacks of this option)

Examples using the above curve:

1. Points from manually drawn curve input:

Computer input

x	y
0	3
10	5
20	9
30	15
40	23

Computer output

a_0 = 3.00
a_1 = .10
a_2 = 0.10
The equation is:
$y = 3.00+0.10\,x+0.01x^2$

2. Actual data points input:

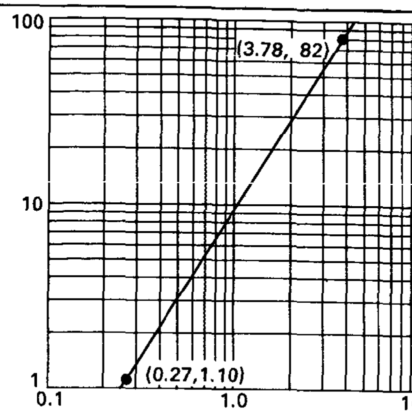
Computer input

x	y
0.	3.2
12.4	5.3
15.2	7.3
19.0	7.9
22.5	12.1
32.5	16.0
35.4	19.1

Computer output

a_0 = 2.998
a_1 = 0.133
a_2 = 0.0089
The equation is:
$y = 2.998+0.133\,x+0.0089\,x^2$

C. LOG LINEAR



General equation: $y = a_0 x^{a_1}$

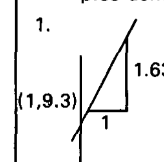
a_0 = y intercept at $x = 1$
a_1 = slope of line

Options:

1. Measure slope graphically, read intercept
2. From two points on curve (x_1, y_1) (x_2, y_2)
   $a_1 = \frac{\log y_1 - \log y_2}{\log x_1 - \log x_2}$ $a_0 = \frac{y_n}{x_n^{a_1}}$

Examples using the above curve:

1.



a_0 = 9.3 (scaled)
a_1 = 1.63 (scaled)
The equation is:
$y = 9.3\ x^{1.63}$

2. $a_1 = \frac{\log 1.10 - \log 82}{\log 0.27 - \log 3.78} = 1.634$
   $a_0 = \frac{82}{3.78^{1.634}}$ or $\frac{1.10}{0.27^{1.634}} = 9.34$
   The equation is:
   $y = 9.34\ x^{1.634}$

FIGURE 10.—Examples of equations fitted to various types of curves.

charge per 0.01-ft stage change in an expanded table may require smoothing.

The rating table is usually prepared by a computer that interpolates between the logarithmic curve descriptors and prints out a table similar to table 1*A*. Some minicomputers can be programmed to prepare tables in the RATLIST format. Others with 80-line printers can prepare tables in a format similar to that of table 1*B*. An appropriate handheld calculator, programmed to display the same discharge figures, can substitute for a rating table during the rating analysis process and can be used later to compute and enter the figures manually on a rating table form.

The WATSTORE system accepts tables as rating input but computes and uses digital descriptors from them that will not exactly match the descriptors used in the table's preparation. Exclusive use of digital descriptors rather than tables as rating input eliminates discrepancies and may prevent some problems.

Stage-discharge ratings

Simple ratings—the most common ones—involve only the relation of discharge to stage at one location and use discharge measurements as the primary data for analysis. Most ratings also need the gage height of zero flow to select the best logarithmic gage-height scale offset, the dates of floods or other channel-changing events to establish dates of shifts from one rating to another, and possibly some cross-section surveys to help determine the shape of the curve.

Curve shaping

Few discharge ratings for new sites are so well defined by discharge measurements that the shape of the curve is apparent throughout its entire range. A shape curve similar to the discharge rating being developed is always helpful in shaping the rating curve and, for some ratings, is absolutely necessary. The shape curve may be a well-defined rating previously used at the site or an approximate rating based on a conveyance curve, a step-backwater model, or defined by a weir formula. For a site where only a part of the previously used rating has shifted, the newly defined part may be merged with the previous curve. Some sites may require cross-section surveying to derive a shape curve from channel data.

Figures 11*A*, *B*, and *C* illustrate some general relations between the cross-section shape in the controlling reach of a stable channel and the logarithmic rating curve shape. Wide flood plains usually cause the rating to break sharply to the right at bankfull stage, and the transition from section to channel control usually causes the curve to break upward. Additional section controls or some channel constrictions may cause additional rating-curve breaks. Specific shape curves often can be defined from relatively simple channel geometry studies.

Slope-conveyance method

Figure 11*D* illustrates the slope-conveyance method, a versatile tool for converting channel-shape data into rating-curve shape. Discharge, conveyance, and energy slope are interrelated by the equations shown, which are based on the Manning formula. The method requires surveys of one or more typical cross sections in the channel-controlling reach. The fall in the water-surface elevation between the gage and the cross section may be substantial; therefore, the conveyance curve is normally adjusted by adding or subtracting this fall. A conveyance curve at a section can be used at a gage site by assuming that the elevation of the water surface at the section equals the gage height at the time of the survey. A better means of transferring the curve to the gage is to assume that the elevation of a high-water mark at the section is equal to the corresponding peak gage height recorded at the gage. Then the cross-section levels are started at the high-water mark, the recorded peak gage height being used as its adjusted elevation. When the formula shown in figure 11*D* is used and the surveyed cross sections are appropriately subdivided and assigned n values (Manning roughness coefficients), K (conveyance) can be computed at as many gage heights as needed to define the full range of the stage-conveyance curve. If more than one cross section was surveyed, a conveyance curve is computed for each, and an average K curve is drawn. A value of slope corresponding to each discharge measurement is computed and plotted by using

TABLE 1.

A CENTRAL COMPUTER FACILITY VERSION
(WATSTORE "RATLIST" Program)

UNITED STATES DEPARTMENT OF INTERIOR - GEOLOGICAL SURVEY - WATER RESOURCES DIVISION

EXPANDED RATING TABLE DATE PRINTED 02-09-78

03580990 JACK DANIEL SPRING AT LYNCHBURG, TENN. TYPE LOG(SCALE OFFSET = 1.04) RATING NO 01

BASED ON ______ DISCHARGE MEASUREMENTS, NO.__________, AND ______, AND IS _____ WELL DEFINED BETWEEN _____ AND _____

FROM ________ TO _________, FROM _________ TO _________ *COMP BY _____ DATE _____ *CKD. BY _____ DATE _____

GAGE HEIGHT IN FEET	DISCHARGE IN CUBIC FEET PER SECOND (STANDARD PRECISION) .00	.01	.02	.03	.04	.05	.06	.07	.08	.09	DIFF IN Q PER TENTH GH
1.00								.01	.01	.02	
1.10	.03	.04	.05	.07	.08	.10	.12	.14	.17	.20	.20
1.20	.23	.26	.29	.33	.37	.41	.45	.50	.55	.60	.42
1.30	.65	.71	.77	.84	.91	.98	1.1	1.1	1.1	1.3	.7
1.40	1.4	1.5	1.6	1.8	1.9	2.0	2.2	2.3	2.5	2.6	1.4
1.50	2.8	2.9	3.1	3.3	3.5	3.7	3.9	4.1	4.3	4.6	2.0
1.60	4.8	5.1	5.3	5.6	5.8	6.1	6.4	6.7	7.0	7.3	2.8
1.70	7.6	7.9	8.3	8.6	9.0	9.3	9.7	10	10	11	3
1.80	11	12	12	13	13	14	14	14	14	15	4
1.90	15	15	15	16	16	16	16	16	17	17	2
2.00	17	17	17	17	18	18	18	18	18	18	2
2.10	19	19	19	19	19	19	19	20	20	20	1
2.20	20	20	20	20	21	21	21	21	21	21	1
2.30	22	22	22	22	22	22	23	23	23	23	1
2.40	24	24	24	25	25	26	26	27	27	28	4
2.50	28	29	29	30	30	31	32	32	33	33	6
2.60	34	35	35	36	37	37	38	39	40	40	7
2.70	41	42	43	45	46	47	48	50	51	52	13
2.80	54	55	56	58	59	61	62	64	66	67	15
2.90	69	71	73	75	77	79	81	83	85	88	21
3.00	90										

TABLE 1.—Continued.

B. TYPICAL MINICOMPUTER VERSION
(User Prepared Program)

U.S.DEPARTMENT OF INTERIOR-GEOLOGICAL SURVEY-WATER RESOURCES DIVISION

RATING TABLE FOR JACK DANIEL SPRING AT LYNCHBURG, TENN.
TABLE NO. 1 STATION NO. 03580990 SCALE OFFSET= 1.04

USED FROM TO

DISCHARGE IN CUBIC FEET PER SECOND

GHT	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.0								0.01	0.01	0.02
1.1	0.03	0.04	0.05	0.07	0.08	0.10	0.12	0.14	0.17	0.20
1.2	0.23	0.26	0.29	0.33	0.37	0.41	0.45	0.50	0.55	0.60
1.3	0.65	0.71	0.77	0.84	0.91	0.98	1.06	1.14	1.22	1.31
1.4	1.40	1.51	1.63	1.75	1.88	2.01	2.15	2.30	2.45	2.61
1.5	2.78	2.95	3.13	3.31	3.51	3.71	3.91	4.13	4.35	4.58
1.6	4.81	5.06	5.31	5.57	5.84	6.11	6.40	6.69	6.99	7.30
1.7	7.62	7.94	8.28	6.62	8.98	9.34	9.71	10.10	10.50	10.90
1.8	11.3	11.7	12.1	12.6	13.0	13.5	13.8	14.1	14.4	14.7
1.9	15.0	15.2	15.4	15.6	15.8	16.0	16.2	16.4	16.6	16.8

GHT	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
2	17.0	18.5	20.0	21.5	23.5	28.0	34.1	41.0	53.6	69.0
3	84.0									

SCALE OFFSET= 1.04
COORDINATES USED

1.07	0.006	2.30	21.500
1.30	0.650	2.40	23.500
1.40	1.400	2.50	28.000
1.85	13.500	2.70	41.000
1.90	15.000	2.90	69.000
2.00	17.000	3.00	84.000

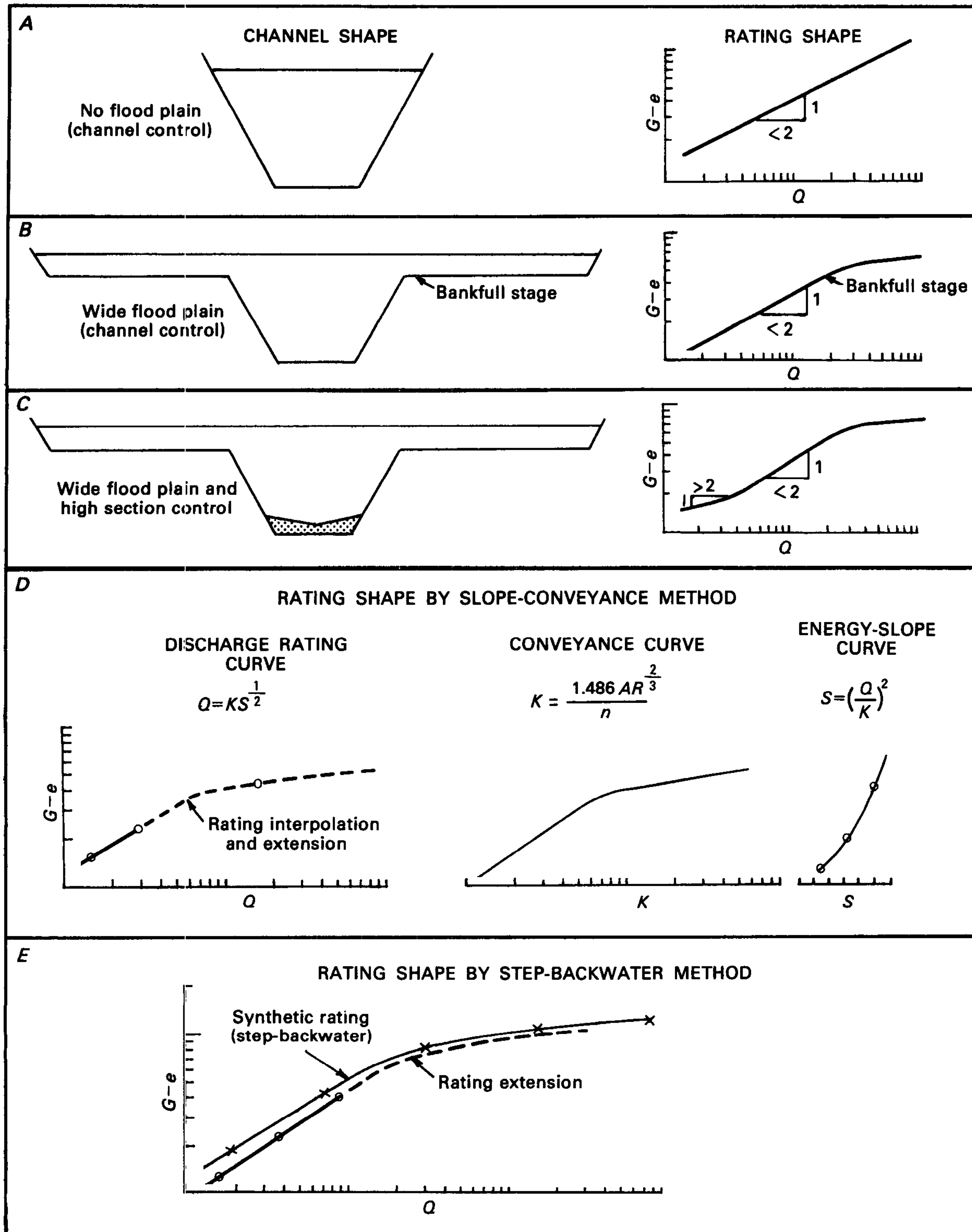


FIGURE 11.—Relation of rating-curve shape to cross-section properties.

the K curve. The S (slope) curve, drawn through the plotted points, may increase with stage, as figure 11D shows, or decrease, but it should be smooth, and it usually approaches valley slope at high stages. After the K and S curves have been defined, discharge values at all stages can be computed and the rating curve drawn, complete with all breaks. This procedure is especially useful for interpolating the undefined parts of rating curves known to contain breaks or for making moderate extensions of rating curves.

Step-backwater rating curves

Figure 11E illustrates one way in which the step-backwater method can be used to obtain an approximate shape curve to use as a guide in a rating extension. The step-backwater method consists of a survey of a reach downstream from the gage, including several cross sections (usually 10), an estimate of the stage-discharge relation at the downstream end of the reach, and a computation of water-surface profiles for several selected discharges. The end result is a curve drawn through the computed water-surface elevation at the gage corresponding to each selected discharge. The rating curve is extended generally parallel to the logarithmic shape curve. If conditions are favorable, the step-backwater model can be calibrated. This procedure requires some revision of the roughness coefficients and section subdivision until the shape curve coincides with the defined part of the rating. The upper part of the shape curve then becomes the rating extension. The step-backwater procedure, explained in detail by Bailey and Ray (1966) and Shearman (1976), is reliable for all natural channels, although the length of reach needed may be impractically long for a channel whose slope is less than about 0.0005 ft/ft. Although this shaping method is expensive, it is by far the best indirect procedure for defining a discharge rating shape.

Stable channel techniques

Channel stability is a relative term. Most channels in stable material remain unchanged between floods that may scour or fill the controls. If these periods are relatively long and if the discharge measurements made during them are adequate to define the applicable ratings, the channel is considered stable. High-water controlling reaches are usually changed only by major floods, but low-water controls may be modified by minor rises.

One procedure for making a rating analysis for a stable channel is as follows:

1. List the relevant discharge measurements (see table 2). The list should include all measurements made within the period for which daily records are to be computed and, preferably, some that were made later. All previous high-water measurements should also be considered.
2. Prepare a logarithmic work curve sheet. A print of the previous year's master curve sheet may be a satisfactory worksheet if no major rating change occurred and if the scales are satisfactory.
3. Plot the discharge measurements and a shape curve on the worksheet. The last rating used or some other superseded rating curve may be the best available shape curve. If more than one new rating is indicated, the measurements that apply to each rating may be plotted on the worksheet by using a distinctive symbol or a color.
4. Draw the curve or curves on the basis of the measurements. A great deal of judgment is needed to strike a balance between the closeness of fit of the curves to the data and the reasonableness of the curve shape. The upper end of the rating curve should be merged with the high-water rating used previously unless there is evidence of a high-water shift. The lower end should be carried down below the minimum recorded stage or to near-zero flow (0.006 ft^3/s, the lowest discharge that can be used as a descriptor) if the data warrant it.
5. Plot the curve, or curves, on at least one rectangular grid to verify the reasonableness of shape and the absence of significant scalloping.
6. Select the digital descriptors.
7. Enter the digital descriptors in the memory of an appropriately programmed calculator. In the absence of a calculator, a rating-table printout can be obtained through the RATLIST program (table 1A) or a desktop cal-

TABLE 2.

9—207
(Nov. 1957)

UNITED STATES DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY (WATER RESOURCES DIVISION)
DISCHARGE MEASUREMENT SUMMARY SHEET

Station No. 03580990

Discharge measurements of Jack Daniel Spring at Lynchburg, Tenn., during the year ending Sept. 30, 1970.

No.	Date 1970	Made by—	Width	Area	Mean velocity	Gage height	Discharge	Rating 1 Shift adj.	Percent diff.	Method	Number meas sections	Gage height change	Time	Meas rated	Outs Ght.	REMARKS
			Feet	Sqft	Fps	Feet	Cfs	Feet				Feet	Hr			
1	Apr.26	V.J. May C.S. Bennett	15	40.0	.99	2.70	39.5	-	-3.8	.6	11	+.04	.30	P	2.72	Wall downstream is section control
2	26	do	21	57.4	1.08	2.86	61.9	-	-1.0	.6 .2-.8	22	+.06	.58	F	2.90	do
3	26	do	21	58.1	1.16	2.92	67.2	-	-6.8	do	21	+.01 -.01	.52	F	2.95	do
4	Apr.26	do	21	57.3	1.05	2.82	60.3	-	+6.5	.2-.8	23	-.04	.67	G	2.84	do
5	May 10	do	1.00	-	-	1.12	.049	-	-2.9	Incr. vol.	4	+.07	.18	P	1.12	All flow in weir notch
6	10	do	1.52	-	-	1.18	.168	-	-1.2	do	6	+.04	.17	F	1.18	do
7	10	do	1.80	-	-	1.23	.331	-	+ .6	do	7	+.02	.15	G	1.23	do
8	10	do	2.10	-	-	1.25	.403	-	-1.5	do	7	0	.15	G	1.25	do
9	10	do	2.65	-	-	1.30	.627	-	-3.7	do	8	+.04	.18	G	1.30	do
10	10	do	2.90	-	-	1.34	.902	-	-1.0	do	7	+.04	.15	G	1.34	do
11	Dec.23	J. Anderson Bennett	7.1	6.01	1.88	1.81	11.3	-	-3.5	.6	17	0	.42	G	-	Weir drowned out--channel controlling
12	23	do	7.1	5.95	1.92	1.79	11.4	-	+4.4	.6	17	0	.38	G	-	do
13	Dec.24	Anderson	7.2	2.17	1.96	1.58	4.26	-	-2.1	.6	18	0	.42	G	1.58	Full width weir control shallow depth C = 1.18
14	24	do	6.1	1.86	2.39	1.58	4.44	-	+2.0	.6	16	0	.42	G	1.58	do Pygmy meter

Copied by EJK Computed by EJK Checked by VJM culator (table 1*B*). The rating is still tentative at this stage and can be modified easily in the storage of a handheld calculator. If any descriptor in a comptuer-generated table is changed, the table must be rerun.

8. Compute the percentage differences between discharge measurements and rating values. If these differences are judged to be satisfactory, as they are in table 2, the rating may be final. If they are unsatisfactory, the process is repeated from step 6.

9. Plot the rating values between the descriptors on the rectangular grid to inspect the plot for scalloping or abrupt breaks in slope. If the curve is satisfactory, it is final. If the curve requires further smoothing, the process is repeated from step 6.

10. Plot the master curve sheet as it has been done in figure 12, and, if a rating table similar to table 1*A* or 1*C* has not been made previously, prepare one.

11. Prepare written notes on the thought processes and assumptions used in developing the rating for subsequent use in documenting the rating analysis.

The steps listed will ensure a well-analyzed rating for a site on a stable channel. Most ratings are unstable for very low flows, and their analyses will require additional steps.

Unstable channel techniques

Nearly all stream channels are unstable at some stages at some times. A very few channels, usually steep ones in sandy streams, are unstable at all stages at all times and can be gaged only with extraordinary methods and equipment, if they can be gaged at all. Horizontal movement of stream reaches in alluvial fans may be too extensive to permit gaging by any means. Other streams ordinarily not gaged include some in sand channels having certain combinations of slope and fine sediment, whose ratings are affected by frequent changes in bed configuration (plane, dune, antidune, and so forth). Bed changes often cause a complete absence of any relation between stage and discharge at some stages. Discharge at some sites can be related only to average depth or to hydraulic radius, neither of which can be automatically recorded, and that relation is discontinuous in that the top of the low-water rating curve is above and to the left of the bottom of the high-water curve. If a bed remains reasonably stable throughout the low-flow regime and changes to another reasonably stable configuration at high flow, it may be practical to develop a discontinuous stage-discharge rating. The bedform effect on ratings is also a low-water phenomenon in some otherwise relatively stable channels, especially those that fluctuate diurnally owing to dam operations. This very complex effect is described in detail by Dawdy (1961) and by Simons and Richardson (1962).

The low water ratings of most streams, especially shallow ones having riffles or other section controls, change or shift gradually as algae or grass grows in in the channel and causes backwater. This effect often starts in the spring, peaks in late summer, and disappears during winter. Backwater from leaves, an autumn occurrence, builds up rapidly and continues erratically until rises flush the leaves from the control. Debris buildup that causes backwater on the control often occurs just after a substantial rise and continues until another rise flushes the channel or until the debris is removed manually. Alluvial stream bottoms and their corresponding ratings become lower when the streams' sediment-transport mechanism picks up more sediment that it deposits. Conversely, the streambed rises when the quantity of sediment deposited exceeds the quantity picked up. The transport process depends on a continuously changing balance of discharge, water temperature, and sediment concentration, among other factors. The shift or change in rating position is smooth and gradual when it is caused by aqueous growth, sometimes sudden and erratic when it is caused by leaves, unchanging for long periods when the backwater is caused by debris, and smooth but extremely variable in some clean, sandy channels. Rating analyses for unstable channels often can be made by using the stable-stream procedures for the medium- and high-water parts. Low-water rating analysis for streams whose channels are unstable often requires a special approach and involves the use of the shifting-control method, an important tool in nearly all rating analyses.

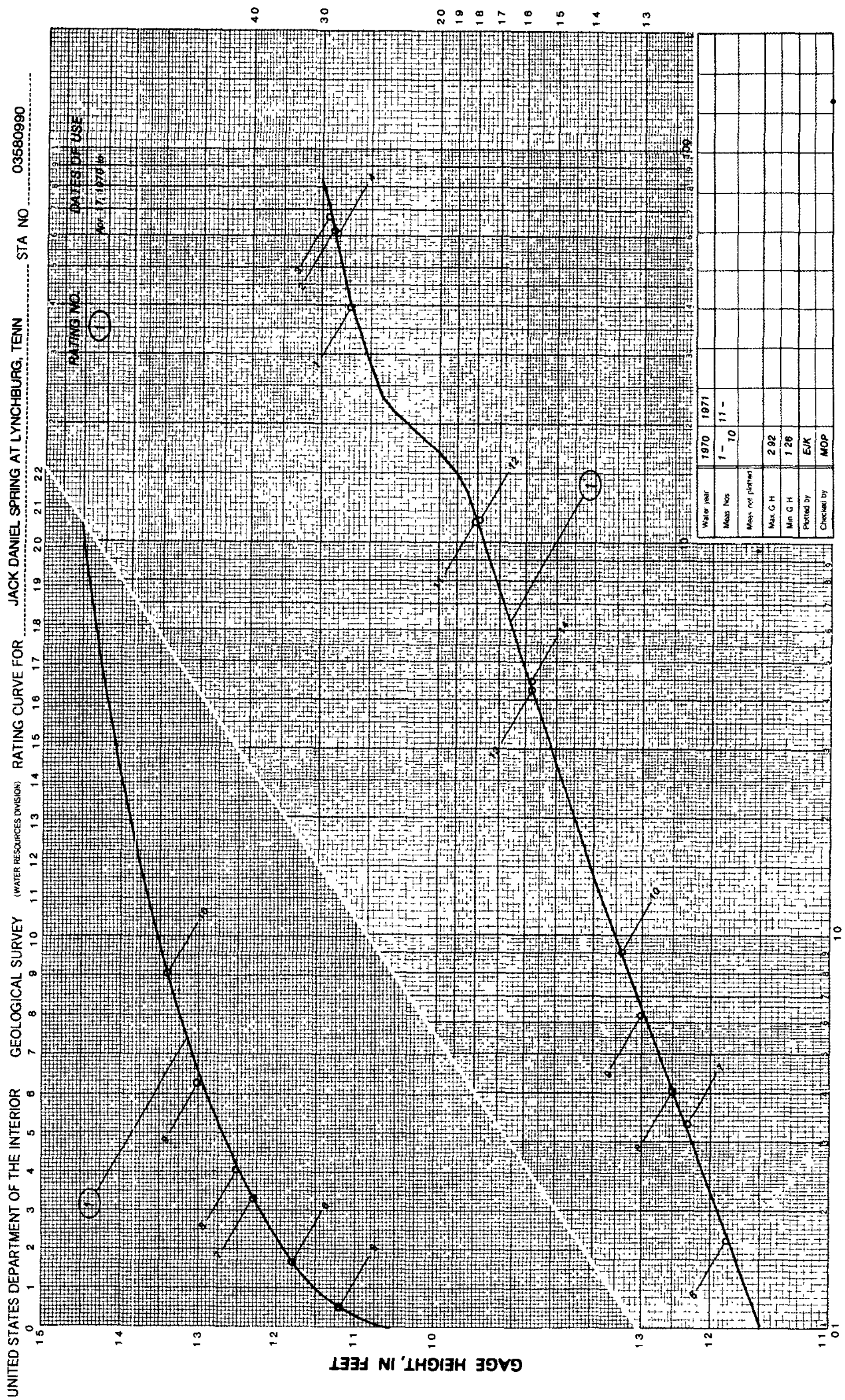


FIGURE 12.—Master rating-curve sheet.

Shifting-control method

A shift adjustment is a correction made to a recorded gage height that compensates for the vertical movement or shifting of that rating. The shift adjustment of a discharge measurement to a base rating curve is computed by subtracting the gage height of the discharge measurement from the gage height of the rating curve that corresponds to the measured discharge. This "observed" shift adjustment is used when a discharge measurement is given full weight. A shift adjustment of −0.27 ft for a gage height of 3.19 ft, for instance, means that the effective rating curve at 3.19 ft is 0.27 ft above the base rating curve. Daily discharges for periods when the shifting-control method is used are computed by adding the applicable shift adjustment to the daily gage height before entering the base rating to obtain the discharge. In some complex uses of shifting-control method and in cases where several measurements not in exact agreement define a shifting condition, an average shift adjustment may be used.

Base rating curves for sites affected by backwater caused by temporary phenomena such as aqueous growth or leaves are best drawn by using the discharge measurements made while the control was observed to be clean; these measurements usually plot farthest to the right. The resulting curve represents clean channel conditions, and shift adjustments are required only when the control is obstructed. All shift adjustments are negative for this type of analysis. Sand bed streams usually have much larger shifts, and the sign of an adjustment has no special significance. The base curve for an alluvial streambed site is best drawn in an average location; both plus and minus shifts should be used to keep shift adjustments small enough for visual interpolation of daily shift values between measurements. Small shifts also simplify the smooth transition to zero shift during higher water periods if the upper part of the rating is stable.

Some channels and their ratings shift upward or downward by more than 5 ft in a single high-water day. Stable channels obstructed by grass or algae have much smaller shifts. Values of daily shift adjustments must be interpolated between discharge measurements in order to compute the daily discharges, and hourly adjustments are needed for some flood records. Shift adjustments may be varied, manually or by computer, with time, stage, or both, or the adjustment can be kept constant during the low-water periods between rises. Where shifting is particularly erratic, hydrographic comparison of daily discharges with those of other streams may help distribution. Shift distribution is simpler and more accurate where the rating curve is properly shaped. Otherwise, the shift distribution must compensate simultaneously for channel changes and rating shortcomings—a difficult assignment.

The shifting-control method can be used, most practically with the ADP initial processing of daily records, to simulate a rating that changes its shape and position gradually because of grass or aqueous growth or the accumulation of debris in the channel. This simulation is accomplished by varying the shift adjustment with both time and stage. Details of applying this method using ADP are described in the WATSTORE User's Guide (Hutchison and others, 1975, 1980).

Low-water rating analysis for unstable channels

Most low-water discharge measurements of an unstable channel stream have different *GZF*'s and, consequently, do not define the same rating or adapt to logarithmic plotting. A special approach must be used, and frequent *GZF* determinations, preferably one for each low-water wading measurement, are essential.

The fundamental assumptions in low-water rating analysis for an unstable channel are as follows:

1. A basic curve shape prevails for the low-water rating and is substantially changed only by floods that change the channel location, shape, or meander pattern downstream.
2. The basic rating-curve shape can be defined by the relation of maximum depth at the control ($Gh-GZF$) to measured discharge.
3. Any straight line plotted to "normal" scale on a logarithmic grid whose slope is between 0° and 90° is a parabola that goes through the coordinates (0,0).
4. The basic low-water rating curve defined by using the previous assumptions, in ef-

fect, can be moved vertically on a rectangular grid without error by use of the shifting-control method.

Assumptions 1 and 4 are approximations, and there is no way to verify how closely they may apply at a particular site. However, any related errors affect only the discharge for days on which interpolated shift adjustments were made and are minor in comparison with errors caused by grossly misshaped rating curves.

Figure 13 illustrates a low-water rating analysis for a stream whose section control of alluvial material over the remnants of a beaver dam is unstable but whose medium- and high-water ratings are relatively stable. The basic data are tabulated in figure 13*A* and include frequent *GZF* determinations. The depth column is *Ght*−*GZF*. The measured discharges, plotted against corresponding gage heights in figure 13*B* on a rectangular grid, give no reliable information as to the shape of the rating. The same discharges, plotted against depth at the control on a logarithmic grid in figure 13*C*, give a well-defined curve because no measurement plots farther above or below the curve than the expected error in the *GZF* determination. The heavy curve on the rectangular grid in figure 13*D* is the depth-discharge curve raised by 3.04 ft (any other value within the range of shifts would do about as well) to match the rating position on October 4. The light curves illustrate the effective rating location on other days. Shift adjustments listed in figure 13*A* are distances between the curve positions at the times of discharge measurement and the heavy base curve. If *GZF*'s had not been measured and if the October and May discharge measurements had not been made, the other measurements would have led to a differently shaped base curve, the shift adjustment variation between measurements would have been erratic, and the computed record would have been less reliable.

Complex ratings

A complex rating is used for a site where the water-surface slope is variable and where no simple relation exists between stage and discharge. Discharge must be related to stage and some other variable. Rate of change in stage is the additional variable for rating streams where storage causes the stage-discharge relation to loop (figure 14*A*). A slope rating is used, along with an auxiliary gage to measure fall in a reach, where tributaries, dams, or the return of overbank flow to the channel causes variable backwater. Index-velocity ratings, which involve special mechanical or electronic devices to measure velocity, are used where special rating problems exist.

A complex rating requires more discharge measurements for adequate definition than a simple stage-discharge rating, and the type of complex rating that will apply usually cannot be predicted before the measurements are made. A prudent procedure to follow at a newly established site where a complex rating is anticipated is to assume that a slope rating will be needed, establish temporary gages at potential auxiliary sites so that readings can be made during all discharge measurements, and measure a few rises over the entire flood hydrograph. Then the loop ratings can be plotted as one indicator of the appropriate rating type. The simplest analysis can be tried first. If it is not satisfactory, various slope ratings can be tried until an adequate rating is developed or until the need for an index-velocity rating is apparent.

A loop rating can be drawn by connecting plotted consecutive discharge measurements made during a single rise. If a rating has been developed, the loop for each major rise can be plotted without discharge measurements by connecting the successive plots of recorded instantaneous gage heights and the corresponding adjusted discharges. Typical single-storm storage loops are shown in figure 14*A*. This type of loop is distinctive in that one occurs on every rise and is roughly symmetrical about the stage-discharge curve for constant-stage conditions. Such loops are related to channel storage between the gage and the control and indicate the applicability of a rate of change in stage rating. Figure 14*D* shows typical backwater loops of the type caused by the return of overbank flow to the main channel. A backwater loop occurs only after an overbank rise—the greater the overbank depth, the wider the loop. An overbank return loop is always to the left of the free-fall rating (the rating defined