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Techniques of Water-Resources Investigations of the United States Geological Survey

Chapter A10

DISCHARGE RATINGS AT GAGING STATIONS

By E. J. Kennedy

Book 3

APPLICATIONS OF HYDRAULICS

DEPARTMENT OF THE INTERIOR

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PREFACE

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TWI 5-A2. Determination of minor elements in water by emission spectroscopy; by P. R. Barnett and E. C. Mallory, Jr. 1971. 31 pages.

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TWI 7-C3. A model for simulation of flow in singular and interconnected channels, by R. W. Schaffranek, R. A. Baltzer, and D. E. Goldberg. 1981. 110 pages.

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TWI 8-A2. Installation and service manual for U.S. Geological Survey monometers, by J. D. Craig. 1983. 57 pages.

TWI 8-B2. Calibration and maintenance of vertical-axis type current meters, by G. F. Smoot and C. E. Novak. 1968. 15 pages.

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GLOSSARY

Term	***Definition***
ADP	Automatic data processing used to compute the discharge records for stations equipped with digital recorders
Bankfull Stage	Stage below which all discharge is confined to the main channel and above which part of the flow occurs in overbank areas of the flood plain
Complex rating	Discharge rating that relates discharge to stage plus some other independent variable such as rate of change in stage or fall in a reach between two gages
Control	Closest section or reach of a channel downstream from a gage, usually a natural constriction or artificial weir, where the channel is shallower, narrower, or rougher than it is elsewhere and where the water-surface slope is significantly steeper
Digital descriptors	Set of coordinates (usually gage heights and dependent variables), or the coefficients and exponents for some type of equation, that describes a curve of relation digitally for convenient use with a computer or calculator
Fall	Difference between the water-surface elevations of two locations on a stream, usually base and auxiliary gage sites for a slope station
Gage height	Water-surface elevation referred to some arbitrary gage datum; gage height is often used interchangeably with the more general term "stage," although gage height is more appropriately used for reading on a gage
Gage height of zero flow	Gage reading corresponding to infinitesimal discharge at a gaging station; the gage height of zero flow is often used interchangeably with the "point of zero flow," which is more appropriately used for a physical location in the streambed near the gage
Index-velocity rating	Complex rating in which a point velocity in a cross section is used as an indicator of mean velocity in the section
Permanent control	Natural or artificial control, the location and dimensions of which remain unchanged for very long periods
Point of zero flow	*See* Gage height of zero flow
Scalloping	Undesirable discharge rating characteristic in which the straight-line segments of a logarithmic rating curve, plotted by using rectangular coordinates, billow upward between nodes at the descriptor points; corresponding rating table shows erratic differences in discharge for each 0.10-ft difference in gage height
Shape curve	Curve similar in shape to that of a rating curve being developed, usually a previous rating or, for a new site, one derived form weir formulas or channel measurements
Shift adjustment	Adjustment, usually varying with time and stage, applied to gage heights to compensate for a change in the rating shape or position
Shifting-control method	Systematic use of shift adjustments as a substitute for revised ratings
Simple rating	Discharge rating that relates discharge to stage only
Slope rating	Complex rating that relates discharge to gage height at one gage (base gage) and to the fall in water-surface elevation between the base gage and an auxiliary gage at another site

Stable channel	Channel whose discharge rating remains unchanged for relatively long periods of time, generally between major floods
Stage	Gage height
Unstable channel	Channel whose discharge rating is changed frequently by minor rises or, in alluvial channels, continually during all flow conditions
V diagram	Graphic representation of the relation between shift adjustment and time or stage
WATSTORE User's Guide	Volumes 1 and 5 of a set of instruction manuals regarding the format of data input to the ADP system used for discharge-record computation (Hutchison and others, 1975, 1980)

SYMBOLS

Symbol	***Definition***
a	Constant
a_n	Coefficients for polynomial rating equations where the subscript n indicates the degree of the x term to which the coefficient applies ($y = a_0 + a_1x + a_2x^2 ...$)
b	Constant used to indicate the slope of a log rating curve (ratio of horizontal distance to vertical distance)
C_c	Value of a_0, in a one-curve equation for index-velocity rating, that makes $Q_m = Q_r$
C_r	Stage-related coefficient (Q_m/Q_b) in a two-curve analysis of index-velocity rating
C_u	Value of a_0 in a one-curve index-velocity rating, usable in a variation of the shifting-control method for one-curve index-velocity ratings, that applies at the time of a specific discharge measurement
dh/dt	Rate of change in stage; also J
e	Gage-height scale offset used for a log rating curve plot; where the log curve is a straight line, e is the effective gage height of zero flow
F	Fall, or difference in water-surface elevation, between two points on a stream
F_m	Fall measured by gage readings
F_r	Rating-fall value from a rating curve or table
G, GH, Ght	Gage height or stage
GZF	Gage height of the point of zero flow *(PZF)*
J	Rate of change in stage (also dh/dt); J is a more convenient symbol to use in an equation, especially in the numerator or denominator of a fraction
K	Channel conveyance
L	Length of a channel reach
log	Base 10 logarithm
Meas	Discharge measurement or its serial number
n	Manning roughness coefficient or constant-fall value other than 1.00
(n)	Column number in a computation sheet
P	A constant in a rating equation equal to Q when $(G-e)$ is 1.0 ft
PZF	Point of zero flow; the lowest point on the controlling section of a stream channel
Q	Discharge in general
Q_{adj}	Discharge adjusted by a rating factor
Q_b	Discharge from a base rating
Q_i	Discharge flowing into a channel reach
Q_m	Measured discharge
Q_o	Discharge flowing out of a channel reach
Q_r	Discharge from a rating
R	Hydraulic radius (area/wetted perimeter) of a cross section
S	Slope of energy gradient
S_c	Energy-gradient slope for steady-flow conditions measured at the same stage as Q_m
U	Velocity of a wave front at a specific stage
V	Mean velocity
V_g	Meter reading of index velocity or vane deflection
W	Average stream width in a reach of channel
X, x	Value measured as an abscissa on a plotted curve
Y, y	Value measured as an ordinate on a plotted curve
$1/US_c$	Term in the "Boyer equation" used for one type of rating (rate of change in stage) and also used as the name of the rating method
ΔQ	Change in discharge ($Q_i - Q_o$) at the ends of a channel reach caused by storage in the reach
$\Delta Q/J$	Storage effect ($Q_i - Q_o$) in a channel reach, per unit rate of change in stage, used as the name of a rating method

DISCHARGE RATINGS AT GAGING STATIONS

By E. J. Kennedy

Abstract

A discharge rating is the relation of the discharge at a gaging station to stage and sometimes also to other variables. This chapter of "Techniques of Water-Resources Investigations" describes the procedures commonly used to develop simple ratings where discharge is related only to stage and the most frequently encountered types of complex ratings where additional factors such as rate of change in stage, water-surface slope, or index velocity are used. Fundamental techniques of logarithmic plotting and the applications of simple storage routing to rating development are demonstrated. Computer applications, especially for handheld programmable calculators, and data handling are stressed.

Introduction

Most of the factors that affect the quality of a streamflow record are either determined by natural conditions or costly to improve. However, the hydrographer can greatly improve the quality of records through skillful use of proper procedures in the data analysis. The principal component of the data analysis—and the subject of this manual—is the discharge rating.

Discharge records for gaging stations are generally computed by applying a discharge rating for the site to a continuous or periodic record of stage. A discharge rating is the relation of discharge to stage and sometimes also to such variables as rate of change in stage, fall in a reach between gages, vane deflection, index velocity, gate opening, or turbine pressure differential.

A rating analysis is basically a process in which the data from a series of discharge measurements are plotted on graph paper, a curve defined by the measurements drawn, and a table prepared from the curve. Other data items might include bankfull stage, the dates of artificial changes to the channel or of floods that may have scoured or filled the channel, notes describing the presence or absence of backwater sources, and field surveys or other information defining the general shape of the rating curve.

Most ratings relate discharge to gage height only and are called simple ratings. A simple rating may be only one curve but is more often a compound curve consisting of three segments, one each for the low-, medium-, and high-water (or overbank) ranges. These segments of a compound curve may be connected by short transition curves. A complex rating is one that relates discharge to stage plus some other independent variable, usually either the rate of change in stage at one gage or the fall in a reach between two gages. Complex ratings usually have a stage-discharge relation curve plus one or more supplementary curves.

The purpose of this chapter of "Techniques of Water-Resources Investigations" is to enable the hydrographer to develop, with the least amount of effort, a discharge rating whose quality matches that of the available data. Logarithmic plotting, the channel-storage effect, the relation of various types of data to the rating, and the use of computers are stressed. Some complex rating situations that are rarely encountered are well described in published reports or manuals. Rating procedures for these special methods are mentioned only in general terms in this manual, and the detailed reports are given as references. Ratings for such universally used equipment as flumes and weirs, so well covered in handbooks by Brater and King (1976) and others, have been omitted.

Development of discharge ratings requires relatively minor applications of hydraulic theory, a greater amount of "handed-down" lore that applies mainly to streams in the general area, and a considerable amount of data manipulation. The data-handling aspect is emphasized in this manual. The procedures described are generally the simplest that apply to each type of rating but are not necessarily the only approaches. The methods used are mostly computer-based variations of standard, time-honored procedures. They apply to or can be modified to fit nearly all stream sites that are likely to be gaged. No attempt is made to credit the many hydrographers who first described the techniques that are demonstrated or those who subsequently added refinements.

Basic Concepts

A discharge rating is analyzed by applying some elementary arithmetic and algebraic processes and certain basic concepts of open-channel flow to the available field data. Familiarity with the basic concepts will allow a hydrographer to plan the rating analysis in logical steps and proceed with the least possible effort to develop a rating that makes the best use of all the data.

Controls

The relation of stage to discharge is usually controlled by a section or reach of channel below the gage, known as the station control, which eliminates the effect of all other downstream conditions on the velocity of flow at the gage. Section controls may be either natural or constructed and may consist of a ledge of rock across the channel, a boulder-covered riffle, an overflow dam, or any other physical feature capable of maintaining a fairly stable relation between stage and discharge. Section controls are often effective only at low discharges and are completely submerged by channel control at medium and high discharges. Relatively flat alluvial channels may have no section control at any discharge. Channel control consists of all the physical features of the channel that determine the stage of the river at a given point for a certain rate of flow. These features include the size, slope, roughness, alinement, constrictions and expansions, and shape of the channel. The reach of channel acting as the control may lengthen as the discharge increases and thus may introduce new features affecting the stage-discharge relation.

Knowledge of the channel features that control the stage-discharge relation is important in developing stage-discharge curves. If more than one control is effective and if the number of measurements is limited, interpolation between measurements and extrapolation beyond the highest measurements will require much judgement, particularly if the controls are not permanent and if various discharge measurements represent different positions of the stage-discharge curve.

When a stream overflows its banks, the configuration of, and perhaps the vegetation on, the flood plain affects the discharge rating, and the control becomes a combination of these features and those of the main channel.

As an earlier discussion stated, a discharge rating is often a compound curve consisting of three segments—one for low flow (section control), one for medium flow (channel control),and one for overbank high-water flow (combined channel and flood-plain control).

Gage height of zero flow

The stage that would occur at a gaging station if the discharge were infinitesimal is the gage height of zero flow (*GZF*). It is also defined as the gage height of the point of zero flow (*PZF*), the highest point on the thalweg (the longitudinal thread of the stream that follows the deepest point in each cross section) downstream from the gage. *GZF* can be measured by levels where the section control is artificial or naturally well defined and permanent. Otherwise, *GZF* should be measured at the time of each low-water wading measurement at an unstable channel site and less frequently in a relatively stable channel. *GZF* is usually measured by taking soundings along the thalweg near the control and subtracting the minimum thalweg sounding from the gage height. An accuracy of *GZF* measurement within a tenth of the sounding depth is possible over a smooth control. Errors may be much greater over a rough, boulder-lined control or where flow is great enough to obscure the control's location. A rough estimate of *GZF* for an alluvial channel that has no evident controlling section can be made by subtracting the deepest sounding in a wading measurement from the gage height. All *GZF* determinations should include the hydrographer's estimate of probable error. The logarithmic scale to be used in a rating analysis and the shape of the low-water rating curve are closely related to *GZF*, and a rigorous analysis of an unstable channel rating cannot be made wtihout determining *GZF* for most of the low-water discharge measurements.

Channel-storage effect

The measuring section or control for a gaging station may be a considerable distance from the gage. If that distance is great enough, the effect of channel storage on the discharge measurements or on the rating must be accounted for. A long stream reach having no tributaries has almost the same discharge at all locations only during periods when the water-surface elevations in that reach remain constant. If the water surface is rising or falling, the discharges at various locations may differ significantly because some of the flow is going into or coming out of storage in the channel.

If the inflow to the stream reach shown in figure 1 is greater than the outflow, the difference between inflow and outflow must be stored in the channel. The water surface must rise sufficiently during the period of imbalance to provide for the storage. Conversely, when the water surface is falling, the outflow must include the water coming out of storage and must be greater than the inflow. For a given reach, the relation between inflow and outflow depends on the rate of change in water-surface elevation and the average water-surface area in the reach. The general storage equation is

$$Q_i - Q_o = \frac{L \times W \times J}{3{,}600}$$

where Q_i is inflow in cubic feet per second, Q_o is outflow in cubic feet per second, L is the length of reach in feet, W is the average width of reach in feet, J is the average rate of change in water-surface elevation (positive for rising stage and negative for falling stage) in feet per hour, and $Q_i - Q_o$ is sometimes called ΔQ.

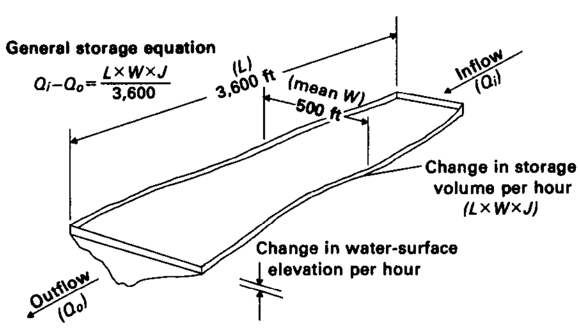


FIGURE 1.—Hypothetical stream-channel reach illustrating changing channel storage effect.

For the reach shown in figure 1, an inflow of 1,500 ft³/s, and an outflow of 1,000 ft³/s, the rate of change in water-surface elevation is computed as

$$1{,}500 - 1{,}000 = \frac{3{,}600 \times 500 \times J}{3{,}600}$$

Therefore, $J = +1.0$ ft/hr.

For the same reach, an inflow of 1,000 ft³/s, and a water surface falling at 0.5 ft/hr, the outflow is computed as

$$1{,}000 - Q_o = \frac{3{,}600 \times 500 \times (-0.5)}{3{,}600}$$

Therefore, $Q_o = 1{,}250$ ft³/s.

If the discharge Q_o of a stream whose average width W is 180 ft is measured as 1,000 ft³/s at a site 10,000 ft downstream from the gage while the water surface is falling 0.33 ft/hr at the gage and 0.27 ft/hr at the measuring site (average 0.30=J), the discharge Q_i at the gage is computed as

$$Q_i - 1{,}000 = \frac{10{,}000 \times 180 \times (-0.30)}{3{,}600}$$

Therefore, $Q_i = 850$ ft³/s.

The storage equation is used most frequently to adjust high-water discharge measurements made at a remote location while the stage is changing. The effect of storage on low-flow discharge measurements may be significant even where the rate of change in stage and the distance involved are surprisingly small, particularly for any large gage pool whose inflow was measured while the gage height was still falling after the control had been cleaned.

Channel storage is the dominant factor in defining rate of change in stage ratings covered in a subsequent section of this manual. For that type of rating, the storage reach falls between the gage and a control whose location may vary with stage. Graphic trial-and-error methods are used instead of the equation to evaluate storage, but the basic concept is the same.

Data limitations

Some rating curves would seem to fit the data more closely if one or two of the discharge measurements were eliminated from the analysis. Sometimes an outlier measurement is obviously and seriously in error, but, often, the measurement is satisfactory and important to the rating analysis. Low- and medium-water measurements are normally made by using standard procedures, and their errors rarely exceed 5 percent. A flood measurement, on the other hand, may be made at night and may involve road overflow or concealed culvert flow, rapidly changing stage, improvised equipment, drift, or other conditions that reduce accuracy. Even these measurements are rarely in error by more than 10 percent. A stream may be so flashy that the best possible measurement may consist of only a few surface velocities, and the error may be as much as 15 percent. Indirect measurements are not usually made unless the conditions promise accuracy within about 20 percent. Such relatively large variations from the ratings are acceptable for measurements of these types. Summer flood measurements, made while inundated trees and brush are covered with leaves, tend to plot to the left of winter flood measurements, but this seasonal effect can be corrected by shifting-control adjustments and should not be confused with measuring error.

Unsynchronized base and auxiliary gage timers at slope stations may make an outlier out of an otherwise good discharge measurement made while the stage was changing rapidly. Such discharge measurements can be corrected if the timing errors can be determined closely, but it is desirable to make measurements at a slope station only while the stage is relatively constant.

Base and auxiliary datum errors are present at most slope stations and may cause erratic plotting of the extreme low-water measurements. Even first-order levels that are run to establish datum differences between gages 10 mi apart may have errors as great as 0.06 ft. Datum agreement can be checked at a site where ponded conditions occur throughout a slope reach during periods of negligible flow, when base and auxiliary gages set to the same datum should read the same. A sluggish intake at one gage or the other can affect the measured fall and cause a discharge measurement to plot erratically.

Discharge measurements at gaging stations where rate of change in stage is a rating factor pose a special problem if the gage-height record is subject to surging or bubble-gage stepping. The plotting position of this type of measurement depends on a substantial changing-stage adjustment as well as on the measured discharge, and a measurement can plot as an outlier if its adjustment is based on gage readings distorted by surging or stepping. This factor can be eliminated by manually smoothing a graphic gage-height record or by using a smoothing option, described in the WATSTORE User's Guide (Hutchison and others, 1975, 1980) for the primary computation of records for stations with rate of change in stage rat-ings. If a smoothed gage-height record is used for daily discharge computation, the same smoothed record must be used to plot and adjust the discharge measurements.

No discharge measurement, made either by current meter or indirectly, should be disregarded (left with an unaccountably high percentage difference from the rating) without a good reason. Disagreement with other measurements is generally not reason enough. If an outlier measurement is truly in error, the reason for the disparity often can be discovered. The hydrographer should check the arithmetic of an outlier measurement, compare the mean gage height with recorded and outside gage heights, compare the plotted cross sections and velocity distributions for several measurements made at the same site, consider the possibility of backwater, and check the equipment used. If these checks and others indicate that the outlier is a valid discharge measurement, it should be given appropriate weight in the analysis.

Use of computers and calculators

The procedures demonstrated in this manual include logarithmic interpolation between curve coordinate points, fitting equations to curves, and some repetitive chains of arithmetic for trial-and-error solutions. The long manual computations required by these operations can be performed rapidly with a handheld card-programmable calculator and even faster with more elaborate computers.

Mathematical and statistical packages are available as accessories for most programmable calculators. These packages contain instructions and program cards or modules for standard operations, including linear and polynomial regression for curve fitting. Nearly all rating-related computations can be performed by using the U.S. Geological Survey's central computer facility through an appropriate terminal and the National Water Data Storage and Retrieval System (WATSTORE). Instructions for preparing input for logarithmic interpolation between rating coordinates (RATLIST) and the analysis of some complex ratings (for example, rate of change in stage and slope) are included in the WATSTORE User's Guide (Hutchison and others, 1975, 1980).

A relatively inexpensive, handheld, card-programmable calculator with a large capacity for storage is particularly helpful for rating analysis, used either alone or as a supplement to more elaborate computers. The most useful program, available from U.S. Geological Survey personnel, for several makes and models, stores coordinates for a logarithmic rating and displays the discharge for any gage height that is entered and, conversely, the gage height for any discharge that is entered.

Plotting

A major part of a discharge rating analysis consists of plotting the gage heights and measured flows from relevant discharge measurements, drawing average or weighted curves based on the measurements and related data, and preparing the discharge rating in a format suitable for use in processing streamflow records. The actual analysis for ratings extending to near-zero flow is usually done on worksheets, which may be discarded after the final results are plotted neatly on one master curve sheet for reproduction and permanent filing.

Gage height, the independent variable, is almost always plotted as the ordinate (Y axis) in hydraulic usage, contrary to standard convention. Because of this practice, the slope of the rating curve is the cotangent of the vertical angle (the ratio of the discharge increment to the gage-height increment, or X/Y) rather than the customary tangent (Y/X).

Rectangular grids

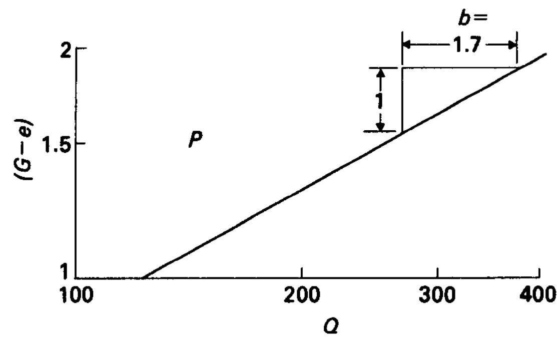
A curve of rectangular coordinates is the simplest and clearest graphic illustration of the relation between two variables. Rectangular plotting is usually appropriate for the low-water part of a master rating sheet and for some or all of the master curves in a complex slope rating. A curve that is developed on a logarithmic grid should always be replotted on rectangular paper and checked for reasonable shape before use. In general, however, a stage-discharge relation curve should not be plotted exclusively on a rectangular-grid work curve sheet.

Logarithmic grids

Logarithmic (log) plotting paper has some advantages over other types of grids that make it the principal tool for graphic rating analysis. A log rating curve has curvature, slope, and an intercept that are related to channel characteristics. Parts of a log-log curve or, in some instances, the entire curve can be straightened by adjusting the gage-height scale. A straight line on log paper represents a curve that can be described exactly by very few numbers or as a simple equation, convenient for computeror calculator use, and that is extended more readily than a curved line. Techniques for the use of log plotting are relatively simple and are essential for developing all types of discharge rating curves.

Straight-line rating curves

Figure 2 illustrates a rating curve plotted on a logarithmic grid. The gage-height values have been reduced before plotting by a constant value e, and, thus, the curve plots as a straight line. The magnitude of e, called the gage-height scale offset, is determined by one of several methods explained in the next section but usually approximates the gage height of zero flow. The slope of the straight-line rating can be determined graphically, as figure 2 shows, by drawing a vertical line one unit long starting at the rating curve and scaling the horizontal distance from the end of that line to the rating curve. The horizontal distance is the



Where:

b is the slope of the straight line.

e is the constant which when subtracted from *G* will result in a straight line on logarithmic paper for the plot *Q* vs ($G-e$). The value of "*e*" is usually the gage height of zero flow or the effective *GZF*. The value of "*e*" used, or any other constant that is subtracted from *G* is called the "Scale Offset."

G is the gage height.

P is the intercept equal to *Q* when ($G-e$) is equal to 1.0.

Q is the discharge.

The general equation for curves of this type is

$$Q=P\ (G-e)^b$$

The equation of the curve shown is $Q=125\ (G-e)^{1.7}$

FIGURE 2.—Hypothetical discharge rating plotted as a straight line on a logarithmic grid.

slope of the line, the exponent of the rating equation, and an indicator of the type of control. A slope greater than 2.0 generally indicates a section control, and a slope less than 2.0 indicates that a channel control is likely. A straight-line log rating usually can be extended upward but not beyond the stage where the channel changes shape (for example, at a terrace or at bankfull stage).

Adjustment of logarithmic scales

Figure 3*A* illustrates a "normal" logarithmic scale. Gage heights plotted to this scale first must be adjusted by subtracting the scale offset *e* to make the logarithmic plot of the rating curve a straight line. Conversely, this scale offset must be added to any value picked off the "normal" scale to convert that figure to a gage height.

Adjustment of the scale so that gage heights can be plotted directly is the general practice, universally followed where the scale offset can be rounded to the nearest foot or half foot without causing undue curvature of the plotted rating. Where the scale offset must be carried to a 0.01-ft refinement, gage-height adjustment is likely to be more convenient than scale adjustment. Figure 3*B* illustrates the difficulty of adjusting the scale by adding an offset carried to 0.01 ft. Only the lower cycle graduations can have useful labels. Figure 3*C* illustrates the effect of rounding the offset to 0.1 ft. Two cycles of graduations then can be labeled. Figures 3*D* and 3*E* illustrate how three cycles of useful labels result from rounding the scale offset to the nearest foot. Such rounding will yield a useful gage-height scale but may cause unsatisfactory curvature of the low-water rating.

If the stream depth at the control is more than about 3 ft when gage height is at the lower end of the rating, curvature of the log rating plot is relatively insensitive to changes of less than a foot in offset. The gage-height scale for a rating at substantial minimum depth can be adjusted by an offset rounded to the nearest foot, and the adjusted labels will fit the grid without significantly affecting the curvature of the rating. An artificially controlled stream whose rating must extend down to zero

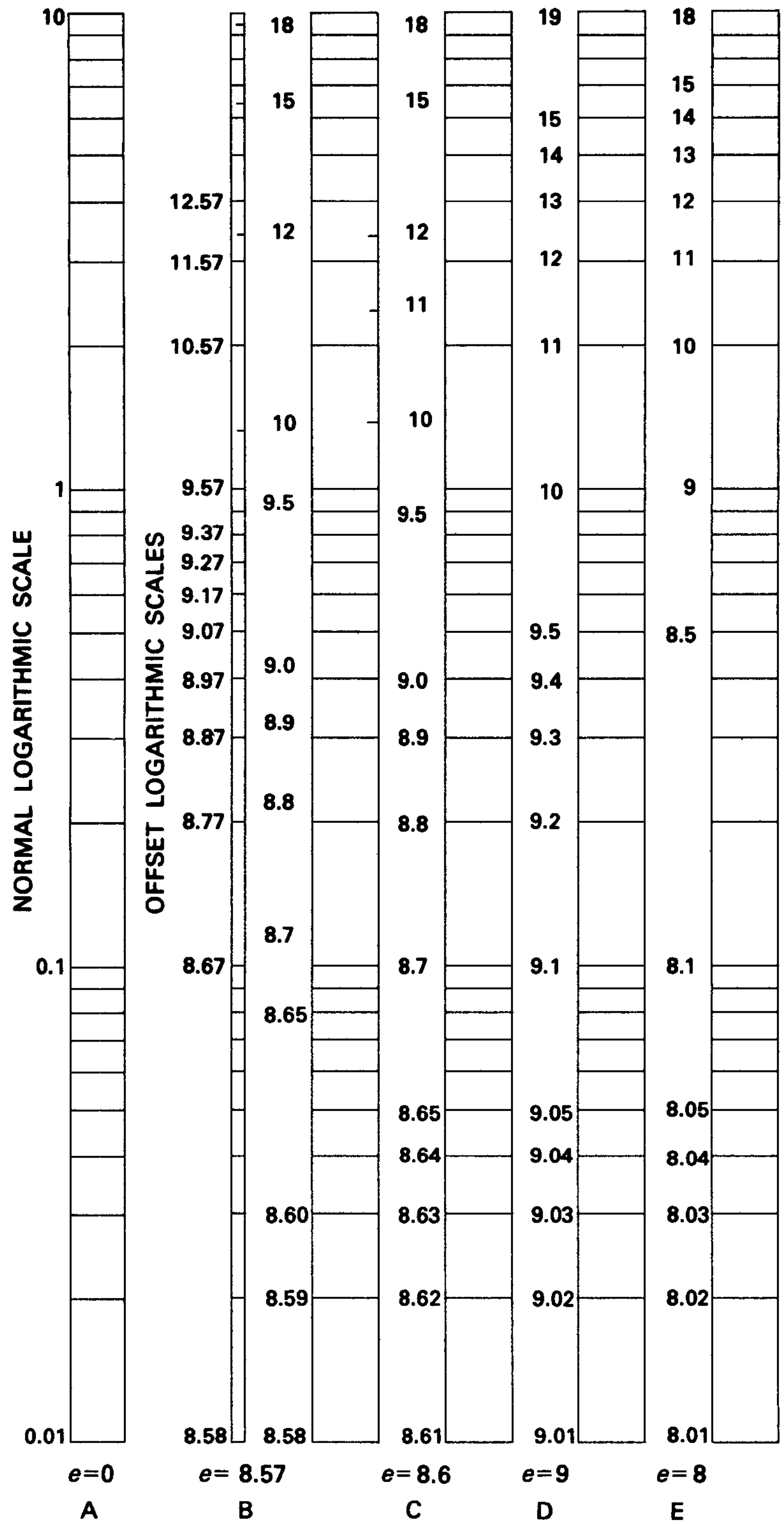


FIGURE 3.—Relation of various offset gage-height scales to the normal log scale.

flow is one type whose offset must sometimes be carried to a 0.01-ft refinement, and gage-height adjustment must be used rather than scale adjustment. A normal log ordinate scale label, "Gage height – (Scale offset value) feet," is used if the gage height is adjusted before plotting. Some ratings will allow rounding of the offset to 0.1 ft only, and either the scale or the gage-height adjustment option may be satisfactory. Master curve sheets, described in a subsequent section of this manual, are prepared after the discharge rating is developed, and then the curve alinement is secondary to a clear display of results. The master-sheet scale offset is usually rounded to the nearest foot for the best fit of the curve to the sheet having the best possible scale labels.

In developing most ratings, a single worksheet whose scales cover the entire range of stage and discharge is preferable, and the single offset used on that sheet is chosen to suit the scale labels. Supplementary worksheets for low-water, overbank, and, possibly, other rating segments, each with its own optimum value of e, may be needed. For instance, to straighten a low-water curve segment, an offset of 1.76 ft (*GZF*) may be needed, whereas an offset of 3 is required to straighten the middle range, and a value of 2 makes the best scale for the full-range sheet. The straight-line ratings developed on the supplementary worksheets are then transferred to the full-range sheet where, because of the rounded offset used on that sheet, they will become curved parts of the rating. Any logarithmic rating plot whose high-water part is concave downward will remain concave downward regardless of the value of any gage-height scale offset that is used.

A logarithmic scale can be adjusted further by multiplying the normal or offset scale values by a constant or by raising them to a power. Multiplying a gage-height scale by a constant raises or lowers the rating curve's position on the sheet but does not affect its size or shape. Raising the scale value to a power flattens or steepens the rating curve. Both procedures lead to scales that fail to fit the log paper's printed graduations and should ordinarily be avoided.

Finding the scale offset value needed to straighten a curved segment of a log rating requires either the trial-and-error solution illustrated in figure 4 or the direction solution illustrated in figure 5.

Figure 4 shows a typical stage-discharge relation between gage height (G) and discharge (Q) plotted as the top curve (e=0). The best value for e, when it is applied to G, will result in a straight-line relation between ($G-e$) and Q. In successive trials, the ordinate scale is varied by using e values of 1, 2, 3; each value results in a different curve, but each still represents the same rating as the top curve. The correct value of e is 2 because the rating plots as a straight line if the normal ordinate scale numbers shown on the logarithmic grid are increased by this value. If smaller values of e are used, the curve will be concave upward; if higher values of e are used, the curve will be concave downward. The value of e for a segment of a rating thus can be determined by adding or subtracting a variety of trial values to or from the numbered scales on the logarithmic grid until a straight-line rating is obtained.

A more direct method for finding e (Johnson, 1952) is illustrated in figure 5. The solid-line curve is straightened by subtracting the scale offset e from each value of G. The coordinates (G_1, Q_1) and (G_2, Q_2) for points near the extremeties of the curve to be straightened are picked by using the normal logarithmic scales. A value for Q_3 at the logarithmic midpoint is computed so that $Q_3=+\sqrt{Q_1Q_2}$. The corresponding gage height G_3 is picked from the solid curve. The scale offset computed from the equation for e (fig.5) will place the three points (Q_1, G_1-e), (Q_3, G_3-e), and (Q_2, G_2-e) in a straight line, and the solid-line curve plotted to the gage-height scale as offset by e will assume the dashed-curve position.

A straight-line rating rarely needs to be precisely straight. Although a curve slightly concave upward or downward throughout its range could be made straighter by further adjustment of e rounded to 0.01 or 0.1 ft, such a refinement might be unnecessary.

The value of e applicable to an already-labeled gage-height scale is the labeled value of any graduation minus its "normal" scale value. In figure 3*E*, e=8.1−0.1=8 or e=9−1=8. If the "normal" scale value is not obvious, the upper gage height G_U, and the

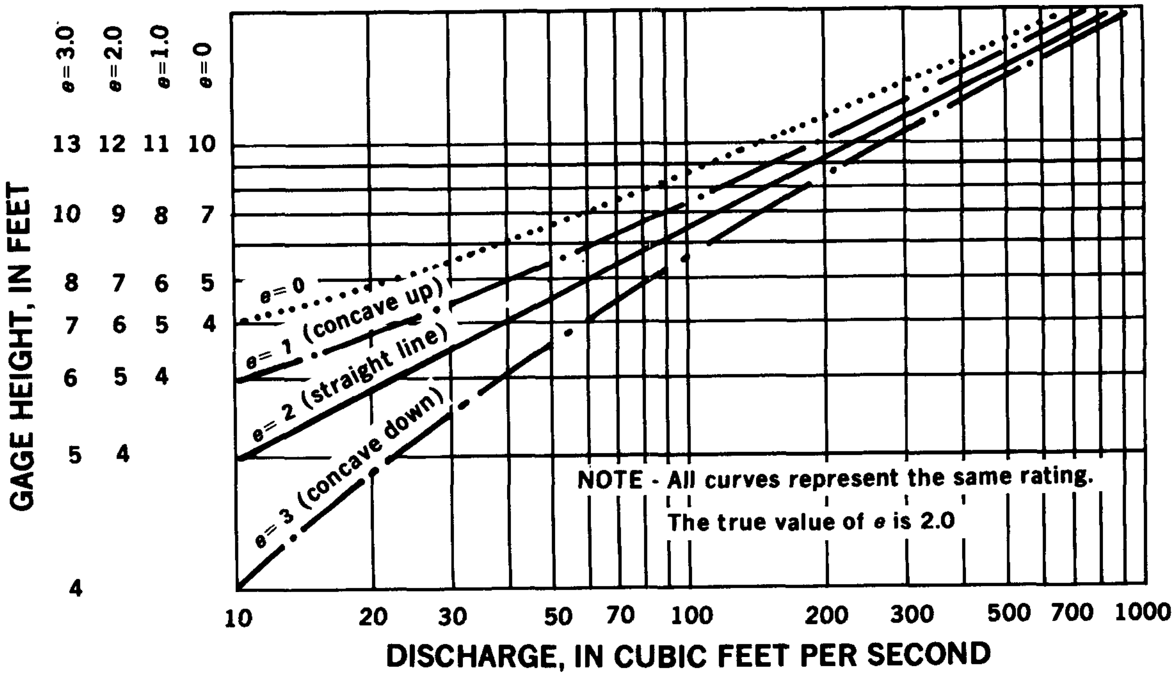
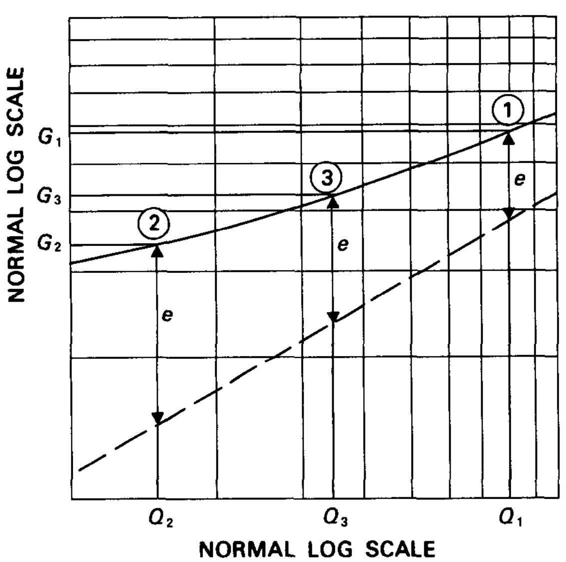


FIGURE 4.—Rating curve shapes resulting from different gage-height scale offsets.



$Q_3^2 = Q_1\ Q_2$ and when the dashed line is straight

$(G_3-e)^2=(G_1-e)\ (G_2-e)$, solving for e

$$e=\frac{G_1\ G_2-G_3^2}{G_1+G_2-2G_3}$$

FIGURE 5.—Determination of scale offset by Johnson's (1952) method.

lower gage height G_L of any complete log cycle can be substituted in the equation

$$e=(10G_L-G_U)/9$$

In figure 3*B*,

$$e=[(10\times8.58)-8.67]/9=8.57\text{ft}$$

or

$$e=[(10\times8.67)-9.57]/9=8.57\text{ft}$$

Characteristics of low-water logarithmic rating curves

An especially useful characteristic of curves plotted on log-log paper applies mainly to the lower part of any discharge rating, particularly one that must extend to or near zero flow. Any straight line drawn on log paper, with a slope between 0° and 90° from the horizontal will pass through the "normal" scale origin (0,0) if it is extended downward on rectangular paper. All of the lines plotted in figure 6*A* intersect at the point (2.3,0.18). The same curves plotted on a rectangular grid in figure 6*B* also intersect at (0,0). For discharge ratings, it follows that any straight-line rating curve on a log-log grid must pass through zero discharge at gage height *e*. Actually, zero values cannot be plotted on a logarithmic grid. However, the knowledge that any straight line extended downward will pass through zero discharge at gage height *e* makes such a plot unnecessary. A rating curve on a logarithmic grid whose *e* is inaccurate will bend if it is extended downward through several log cycles and become either horizontal or vertical depending on the sign of the error. For this reason, it is usually unwise to develop on the same logarithmic worksheet the extreme low-water parts of two or more curves having different values of *GZF*.

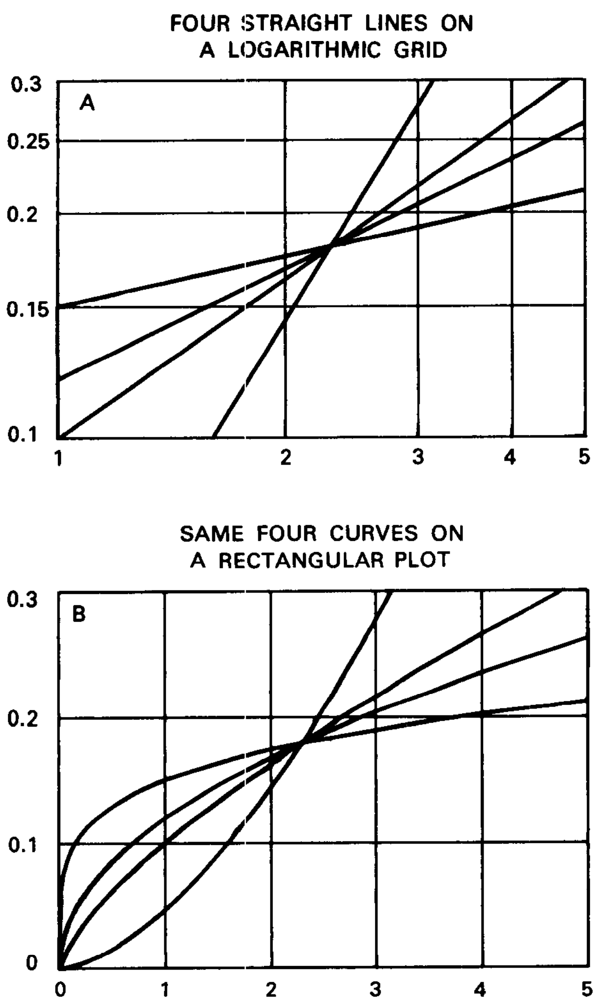


FIGURE 6.—Comparative plots of identical curves on logarithmic and rectangular grids.

Logarithmic curve coordinates

A rating starts as a plotted curve that must be converted into different formats for various uses. The first step in the conversion process is to approximate the logarithmic rating curve by using a series of straight-line segments, as figure 7 shows. A very close approximation of most logarithmic rating curves can be made with fewer than 10 straight lines. The maximum difference between the original curve and the approximation line is usually held to about 1 percent for high and medium rating parts and more for extreme low or sharply curved parts. If the logarithmic curve is concave upward, as it is in the curve near the 4-ft gage height (fig. 7), the use of long segments may lead to an undesirable but normally harmless condition called "scalloping." When the segments are plotted on a large-scale rectangular grid, especially by an automatic plotter, as a subsequent section describes, they may form a slightly scalloped curve rather than a smooth one. The scalloping is usually noticed when a table of discharges corresponding to the gage heights at 0.10-ft intervals (rating table) is prepared, and the discharge differences per 0.10 ft of gage height change abruptly at the gage heights of approximation line intersections. Scalloping can be minimized by using short, straight lines to approximate the logarithmic rating curve's concave upward parts, or it can be completely eliminated by increasing the gage-height scale offset until the logarithmic rating curve is straight or concave downward.

The scale offset *e* used for the consolidated worksheet and the coordinates of the straight-line segment intersections define the curve. These numbers are the digital descriptors of the rating and should be the only rating input for automatic processing of the daily discharge records. This set of descriptors should also be the only basis for compiling any rating tables that are prepared.

Interpolation between logarithmic curve coordinates

When the digital descriptors are used as a rating, each computation of the discharge corresponding to a specific gage height requires

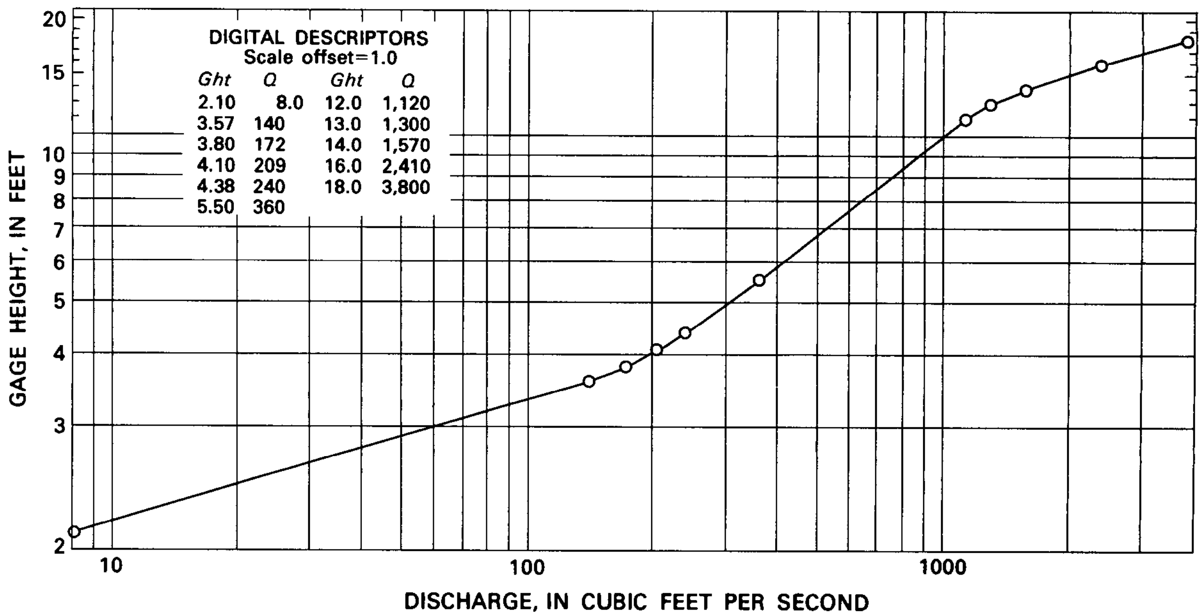


FIGURE 7.—Typical logarithmic rating curve with corresponding digital descriptors.

a solution of the formula illustrated and derived in figure 8. A calculator or computer is highly desirable because manual computations would be impractically slow. The formula is used in the WATSTORE program RATLIST which converts a set of descriptors into a rating table, and in rating-point interpolation programs for calculators. A calculator program stores the rating descriptors in the calculator's memory and displays the discharge value corresponding to any gage height that is entered.

Formats for ratings

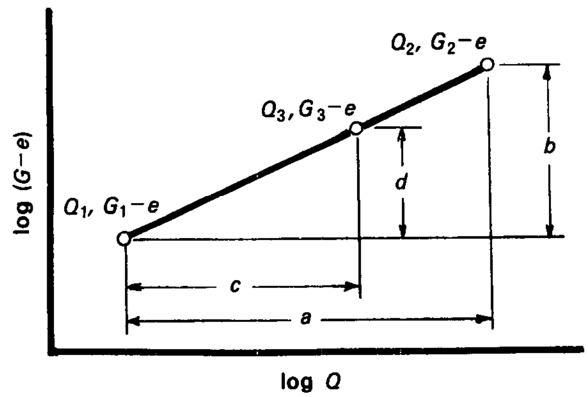
The same discharge rating can be a plotted curve, a table, an equation, or a list of descriptors. Each version has advantages over the others for specific purposes.

Rating curve sheets

The graphic format of a rating portrays the stage-discharge relation visually and simply and is the form used for the initial rating analysis. Work curve sheets are used to make graphic analyses of logarithmic ratings that have critical scale offsets, to determine auxiliary relations in complex ratings that need rectangular plotting and trial-and-error solutions, and to make rectangular plots of logarithmic curves to check them for reasonable shape. The work curve sheets can be discarded after the rating is developed and simpler scales are drafted in ink on a master curve sheet for the permanent record. Simple ratings for streams in relatively stable channels, where the minimum flow is greater than about 5 ft^3/s, usually can be developed on the same logarithmic sheet that will later be inked and used for the master sheet.

Work curve sheets

The actual development of most ratings is done most conveniently on log paper worksheets with enough cycles to cover the entire range of stage and discharge. For unstable channels, one worksheet is needed for each rating whose gage height of zero flow (*GZF*) is significantly different from that of its preceding rating, and its gage-height scale offset will correspond to the *GZF*. A rectangluar sheet is also needed to plot the curves developed on log paper and to check that their shapes are reasonable. Some auxiliary relations for complex ratings, explained in subsequent sections (storage, fall, and so forth), require rectangular work curve sheets for the development of the curve. Other auxiliary curves must be developed originally on log paper.



Where
e =Scale offset
G_1=*Ght* of lower coordinate
G_2=*Ght* of upper coordinate
G_3=*Ght* of intermediate coordinate
Q_1 =Discharge of lower coordinate
Q_2 =Discharge of upper coordinate
Q_3=Discharge corresponding to G
a,b,c,d= Lengths in log units
(1 log cycle length=1 log unit)

From similar triangles on diagram:

$$\frac{a}{c}=\frac{b}{d} \quad \text{or} \quad c=\frac{ad}{b} \tag{1}$$

Substituting equivalent values in equation 1:

$$c=\frac{(\log Q_2-\log Q_1)(\log(G_3-e)-\log(G_1-e))}{\log(G_2-e)-\log(G_1-e)} \tag{2}$$

From diagram:

$$\log Q_3= Q_1+c \tag{3}$$

Substituting equation 2 for "c" in equation 3, and taking antilogs, formula for Q_3 in algebraic format is:

$$Q_3= \text{antilog}\left[\log Q_1+\frac{(\log Q_2-\log Q)(\log(G_3-e)-\log(G_1-e))}{\log (G_2-e)-\log(G_1-e)}\right]$$

or in BASIC format:

```
Q3=10+(LGTQ1+(LGTQ2-LGTQ1)*(LGT(G3-E)
-LGT(G1-E)))/(LGT(G2-E)-LGT(G1-E)))
```

FIGURE 8.—Derivation of the logarithmic coordinate interpolation equation. Marginal notes, curve labels, effective dates, and such must be added manually.

Most wide-range simple ratings can be analyzed best on a diagonally ruled log sheet (Form 9–279M), which has 3×5 usable cycles. Excess cycles can be trimmed off or additional cycles taped in place. All discharge measurements relevant to the rating being analyzed and a shape curve (prior full-range rating for other-than-new gaging stations) are plotted on the worksheet. Colors can be used to distinguish the measurements that apply to a particular rating or those affected by ice or temporary backwater because the sheet will not be reproduced.

A print of the previous year's master curve sheet for a large stream sometimes can be used as a worksheet. Smaller streams, especially those whose minimum flow is near zero, have critical gage-height scale requirements and normally need separate worksheets for each new rating.

Master curve sheets

When the rating analysis is completed, the plotted curves are on penciled worksheets, often in a variety of sizes and formats. A curve sheet is needed for the permanent files and for reproducing copies for cooperating agencies, field folders, and planning purposes. A master sheet, inked and on good-quality paper, generally is prepared for reproduction and for the permanent record. Some compromises of curve characteristics that are vital for worksheets but less important for display of results may be tolerated in order to plot the curves on standard sheets of reasonable size. For instance, although the hybrid or rectangular scales used in this manual for some of the master curve sheets illustrating various kinds of ratings would be unsatisfactory on work curve sheets, they do present the ratings more clearly on the master curve sheets by minimizing clutter.

Standard (9–279 series) 11×17 in sheets have 2-×3-log cycles, and the larger sheets (17×22 in) have 3×4-log cycles. Two, or sometimes three, gage-height cycles may not cover the entire range, so some versions have a rectangular grid in the upper left-hand corner on which a rectangular plot can be substituted for the lower part of the log curve. Diagonally ruled log sheets (9–279M), the best worksheet forms, can also be used for master sheets.

Use of a standard log-rectangular combination sheet may require some gage-height scale adjustment. For instance, the rating shown on the master curve sheet used in a subsequent section of this manual (see fig. 15), plotted entirely on a log grid and using the scale offset (2 ft) that was used on a worksheet to straighten the curve's lower end and to locate its digital descriptors, would require three log cycles for gage height (2.8 to 35 ft) and seven discharge cycles (0.01 to 100,000 ft^3/s). By using a scale offset of 4 ft and rectangular plotting for the part of the curve below 100 ft^3/s, the curve fits easily on a 2×3-cycle combination sheet, and the curvature caused by changing the scale offset is not apparent in the part of the log curve plotted.

Several trials may be needed to find a satisfactory combination of scales for use on a combination log-rectangular master curve sheet. One procedure is to select a horizontal log scale so that the maximum discharge is as close as possible to the right margin. The best gage-height scale and offset make the log curve cross the left margin above but close to the bottom margin. If the point where the curve crosses the margin is too high or too low, the gage-height scale offset can be adjusted to correct the location. Most satisfactory rectangular discharge scales have as their highest value 2, 10, or 20 times the discharge at the left margin of the log scale. The rectangular gage-height scale should permit, if possible, plotting of the zero-flow gage height. All the rectangular scales should be chosen so that major divisions are 1, 2, or 5 (4 as a poor last resort) times some power of 10.

Master curve sheets for streams that are limited in range are best with only a log plot; those for some streams that have complex ratings may be better illustrated by rectangular plotting alone. Complex slope stations (see fig. 21) often need a sheet that has been prepared specially by a pasteup and photographic transfer process.

Automatically plotted curve sheets

Some hydrographers use an onsite minicomputer or a programmable desk calculator equipped with a plotter to analyze ratings. A program to plot a rating curve and the relevant discharge measurements on the master curve sheet is written to fit the available equipment. If the plot is made on Form 9–279P, it may resemble the one shown in figure 9.

Digital descriptors of ratings

A rating that is to be a part of a computer program should be as concise as possible. The average discharge rating table describes that rating with several thousand digits. The same rating can be described with about 50 digits by

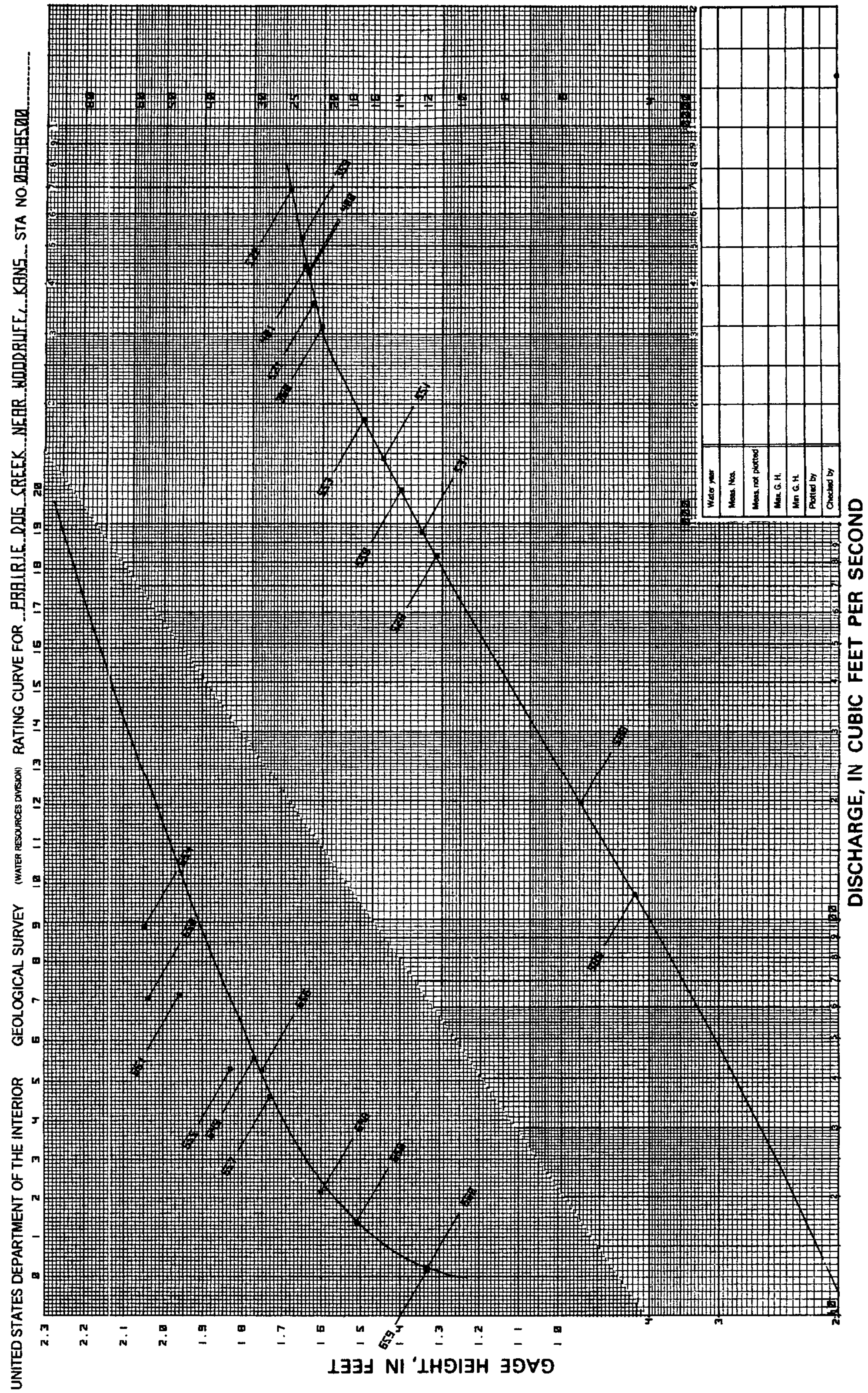


FIGURE 9.—Automatically plotted rating curve-sheet.

using logarithmic coordinates or with about 15 digits by using an equation.

Logarithmic interpolation

Single-offset logarithmic interpolation uses one gage-height scale offset and the coordinates (gage height and discharge) of the ends of straight-line segments that approximate the logarithmic rating curve. This type of descriptor system, described in a previous section and illustrated in figure 7, is the system used most commonly for stage-discharge ratings.

Multiple-offset logarithmic interpolation, an elegant development of the single-offset method, uses offsets that vary with stage for both the gage height and the discharge scales. The method can be used in conjunction with a computer terminal and the WATSTORE system to calculate the fewest possible special descriptor sets that can define a curve through selected coordinates or with a manually prepared rating table, within a user-defined degree of variation. Multiple-offset descriptor use is described in the WATSTORE User's Guide (Hutchison and others, 1975, 1980) and can be adapted to some special rating problems. However, in its present (1980) state of development, the procedure has little application to the processes covered in this manual.

Linear interpolation

The descriptors for some auxiliary relations to complex ratings (fall, storage, and so forth) are used with linear interpolations rather than with logarithmic interpolations. This system requires more descriptors than the logarithmic method but can be used if negative values are involved.

Rating equations

A simple equation is the most convenient rating format for certain applications, especially the auxiliary relations used with some complex ratings. Equations for ordinary discharge ratings are too difficult to fit and too unwieldy for practical use.

Most complex ratings (slope, index velocity, and so forth) involve auxiliary relations (for example, fall versus factor or stage versus coefficient) that are straight lines on logarithmic grids or even on arithmetic grids. These curves (indeed, most parabolic curves) can be described by simple equations. The curve-fitting process can be expedited by using programmable calculators, some of which have linear regression programs built into the hardware. Most programmable calculators and computers have standard program packages (such as STATPAC or MATHPAC) and easy-to-follow user's guides for fitting equations to parabolic, logarithmic, power function, and other common types of curves.

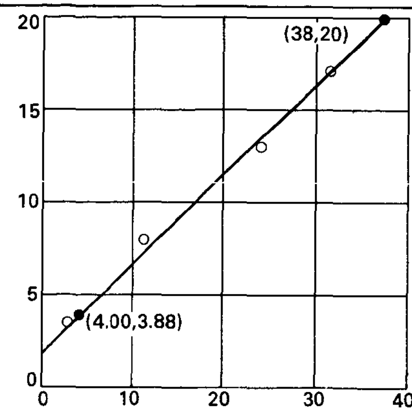
Curve-fitting programs involve inputting data points that are fitted to the appropriate curve by least-squares regression. The fitted equation, particularly for a parabola, may not be applicable outside the range of the data points. To counter this objectionable feature, a trial curve is drawn manually well beyond the range of the data points, and points from that curve are used for the input. Figure 10 illustrates fitting equations to various types of curves.

Rating tables

A table is the preferred rating format for manual computation of discharge records, shift adjustments, percentage differences, and periods of special conditions (fragmentary gage-height record, ice, backwater, and so forth) and for the file copies of the ratings.

Manual preparation of a rating table is a very slow process. Discharge figures for each foot or so of gage height are picked off the curvilinear plot and entered on an appropriate (standard 9–210 series) form. Starting at the low end, trial differences per 0.10 ft of gage height are added to each discharge value and adjusted as necessary until the values vary smoothly, yield properly rounded figures, and match the figures picked off the curve. Some compromises are usually necessary to accommodate all three objectives. Some hydrographers adjust second differences to assure smoothness in the rate of change of the differences per 0.10 ft of gage height. The lower ends of many ratings require an expanded table that lists discharges for gage heights at increments of 0.01 ft. The difference in dis-

A. LINEAR



General equation: $y = a_0 + a_1 x$

a_0 = y axis intercept
a_1 = slope of line

Options:
1. Compute from two points on a known line (x_1,y_1) and (x_2,y_2)
   $a_1 = \frac{y_1 - y_2}{x_1 - x_2}$ $\quad a_0 = y_n - a_1 x_n$
2. Regression of actual data points *(computer)

Examples using the above curve:
1. Two points are (4.00,3.88), (38.0,20.0):
   $a_1 = \frac{3.88-20.0}{4.00-38.0} = 0.474$
   $a_0 = 3.88-(0.474 \times 4.00) = 1.984$
   The equation is: $y = 1.984+0.474$
2. Regression of data points by computer

Computer input

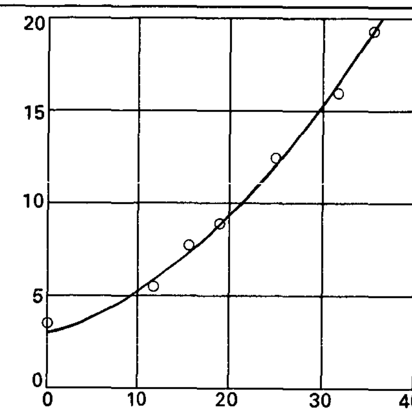
x	y
3.6	3.5
12.0	8.0
24.0	13.0
32.0	17.0

Computer output

$a_0 = 2.024$
$a_1 = 0.467$
The equation is:
$y = 2.024+0.467\,x$

*Central computer, minicomputer, or programmable calculator with appropriate program from manufacturer's library (STATPAC, MATHPAC, etc.)

B. PARABOLIC



General equation: $y = a_0 + a_1 x + a_2 x^2$

a_0 = y axis intercept

Options:
1. *Computer input of points from manually-drawn curve
2. *Computer input of actual data. (See text for drawbacks of this option)

Examples using the above curve:
1. Points from manually drawn curve input:

Computer input

x	y
0	3
10	5
20	9
30	15
40	23

Computer output

$a_0 = 3.00$
$a_1 = .10$
$a_2 = 0.10$

The equation is:
$y = 3.00+0.10\,x+0.01x^2$

2. Actual data points input:

Computer input

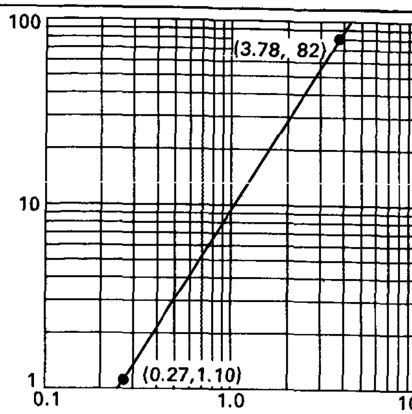
x	y
0.	3.2
12.4	5.3
15.2	7.3
19.0	7.9
22.5	12.1
32.5	16.0
35.4	19.1

Computer output

$a_0 = 2.998$
$a_1 = 0.133$
$a_2 = 0.0089$

The equation is:
$y = 2.998+0.133\,x+0.0089\,x^2$

C. LOG LINEAR



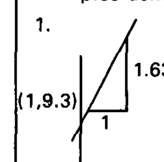
General equation: $y = a_0 x^{a_1}$

$a_0 = y$ intercept at $x = 1$
a_1 = slope of line

Options:
1. Measure slope graphically, read intercept
2. From two points on curve (x_1,y_1) (x_2,y_2)
   $a_1 = \frac{\log y_1 - \log y_2}{\log x_1 - \log x_2}$ $\quad a_0 = \frac{y_n}{x_n^{a_1}}$

Examples using the above curve:

1.



$a_0 = 9.3$ (scaled)
$a_1 = 1.63$ (scaled)
The equation is:
$y = 9.3\ x^{1.63}$

2. $a_1 = \frac{\log 1.10 - \log 82}{\log 0.27 - \log 3.78} = 1.634$

$a_0 = \frac{82}{3.78^{1.634}}$ or $\frac{1.10}{0.27^{1.634}} = 9.34$

The equation is:
$y = 9.34\ x^{1.634}$

FIGURE 10.—Examples of equations fitted to various types of curves.

charge per 0.01-ft stage change in an expanded table may require smoothing.

The rating table is usually prepared by a computer that interpolates between the logarithmic curve descriptors and prints out a table similar to table 1*A*. Some minicomputers can be programmed to prepare tables in the RATLIST format. Others with 80-line printers can prepare tables in a format similar to that of table 1*B*. An appropriate handheld calculator, programmed to display the same discharge figures, can substitute for a rating table during the rating analysis process and can be used later to compute and enter the figures manually on a rating table form.

The WATSTORE system accepts tables as rating input but computes and uses digital descriptors from them that will not exactly match the descriptors used in the table's preparation. Exclusive use of digital descriptors rather than tables as rating input eliminates discrepancies and may prevent some problems.

Stage-discharge ratings

Simple ratings—the most common ones—involve only the relation of discharge to stage at one location and use discharge measurements as the primary data for analysis. Most ratings also need the gage height of zero flow to select the best logarithmic gage-height scale offset, the dates of floods or other channel-changing events to establish dates of shifts from one rating to another, and possibly some cross-section surveys to help determine the shape of the curve.

Curve shaping

Few discharge ratings for new sites are so well defined by discharge measurements that the shape of the curve is apparent throughout its entire range. A shape curve similar to the discharge rating being developed is always helpful in shaping the rating curve and, for some ratings, is absolutely necessary. The shape curve may be a well-defined rating previously used at the site or an approximate rating based on a conveyance curve, a step-backwater model, or defined by a weir formula. For a site where only a part of the previously used rating has shifted, the newly defined part may be merged with the previous curve. Some sites may require cross-section surveying to derive a shape curve from channel data.

Figures 11*A*, *B*, and *C* illustrate some general relations between the cross-section shape in the controlling reach of a stable channel and the logarithmic rating curve shape. Wide flood plains usually cause the rating to break sharply to the right at bankfull stage, and the transition from section to channel control usually causes the curve to break upward. Additional section controls or some channel constrictions may cause additional rating-curve breaks. Specific shape curves often can be defined from relatively simple channel geometry studies.

Slope-conveyance method

Figure 11*D* illustrates the slope-conveyance method, a versatile tool for converting channel-shape data into rating-curve shape. Discharge, conveyance, and energy slope are interrelated by the equations shown, which are based on the Manning formula. The method requires surveys of one or more typical cross sections in the channel-controlling reach. The fall in the water-surface elevation between the gage and the cross section may be substantial; therefore, the conveyance curve is normally adjusted by adding or subtracting this fall. A conveyance curve at a section can be used at a gage site by assuming that the elevation of the water surface at the section equals the gage height at the time of the survey. A better means of transferring the curve to the gage is to assume that the elevation of a high-water mark at the section is equal to the corresponding peak gage height recorded at the gage. Then the cross-section levels are started at the high-water mark, the recorded peak gage height being used as its adjusted elevation. When the formula shown in figure 11*D* is used and the surveyed cross sections are appropriately subdivided and assigned n values (Manning roughness coefficients), K (conveyance) can be computed at as many gage heights as needed to define the full range of the stage-conveyance curve. If more than one cross section was surveyed, a conveyance curve is computed for each, and an average K curve is drawn. A value of slope corresponding to each discharge measurement is computed and plotted by using

TABLE 1.

A CENTRAL COMPUTER FACILITY VERSION
(WATSTORE "RATLIST" Program)

UNITED STATES DEPARTMENT OF INTERIOR - GEOLOGICAL SURVEY - WATER RESOURCES DIVISION

EXPANDED RATING TABLE DATE PRINTED 02-09-78

03580990 JACK DANIEL SPRING AT LYNCHBURG, TENN. TYPE LOG(SCALE OFFSET = 1.04) RATING NO 01

BASED ON ______ DISCHARGE MEASUREMENTS, NO.__________, AND ______, AND IS _____ WELL DEFINED BETWEEN _____ AND _____
FROM ________ TO ________, FROM ________ TO ________ *COMP BY _____ DATE _____
*CKD. BY _____ DATE _____

GAGE HEIGHT IN FEET	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09	DIFF IN Q PER TENTH GH
	DISCHARGE IN CUBIC FEET PER SECOND (STANDARD PRECISION)										
1.00								.01	.01	.02	
1.10	.03	.04	.05	.07	.08	.10	.12	.14	.17	.20	.20
1.20	.23	.26	.29	.33	.37	.41	.45	.50	.55	.60	.42
1.30	.65	.71	.77	.84	.91	.98	1.1	1.1	1.1	1.3	.7
1.40	1.4	1.5	1.6	1.8	1.9	2.0	2.2	2.3	2.5	2.6	1.4
1.50	2.8	2.9	3.1	3.3	3.5	3.7	3.9	4.1	4.3	4.6	2.0
1.60	4.8	5.1	5.3	5.6	5.8	6.1	6.4	6.7	7.0	7.3	2.8
1.70	7.6	7.9	8.3	8.6	9.0	9.3	9.7	10	10	11	3
1.80	11	12	12	13	13	14	14	14	14	15	4
1.90	15	15	15	16	16	16	16	16	17	17	2
2.00	17	17	17	17	18	18	18	18	18	18	2
2.10	19	19	19	19	19	19	19	20	20	20	1
2.20	20	20	20	20	21	21	21	21	21	21	1
2.30	22	22	22	22	22	22	23	23	23	23	1
2.40	24	24	24	25	25	26	26	27	27	28	4
2.50	28	29	29	30	30	31	32	32	33	33	6
2.60	34	35	35	36	37	37	38	39	40	40	7
2.70	41	42	43	45	46	47	48	50	51	52	13
2.80	54	55	56	58	59	61	62	64	66	67	15
2.90	69	71	73	75	77	79	81	83	85	88	21
3.00	90										

TABLE 1.—Continued.

B. TYPICAL MINICOMPUTER VERSION
(User Prepared Program)

U.S.DEPARTMENT OF INTERIOR-GEOLOGICAL SURVEY-WATER RESOURCES DIVISION

RATING TABLE FOR JACK DANIEL SPRING AT LYNCHBURG, TENN.

TABLE NO. 1 STATION NO. 03580990 SCALE OFFSET= 1.04

USED FROM TO

DISCHARGE IN CUBIC FEET PER SECOND

GHT	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.0								0.01	0.01	0.02
1.1	0.03	0.04	0.05	0.07	0.08	0.10	0.12	0.14	0.17	0.20
1.2	0.23	0.26	0.29	0.33	0.37	0.41	0.45	0.50	0.55	0.60
1.3	0.65	0.71	0.77	0.84	0.91	0.98	1.06	1.14	1.22	1.31
1.4	1.40	1.51	1.63	1.75	1.88	2.01	2.15	2.30	2.45	2.61
1.5	2.78	2.95	3.13	3.31	3.51	3.71	3.91	4.13	4.35	4.58
1.6	4.81	5.06	5.31	5.57	5.84	6.11	6.40	6.69	6.99	7.30
1.7	7.62	7.94	8.28	8.62	8.98	9.34	9.71	10.10	10.50	10.90
1.8	11.3	11.7	12.1	12.6	13.0	13.5	13.8	14.1	14.4	14.7
1.9	15.0	15.2	15.4	15.6	15.8	16.0	16.2	16.4	16.6	16.8

GHT	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
2	17.0	18.5	20.0	21.5	23.5	28.0	34.1	41.0	53.6	69.0
3	84.0									

SCALE OFFSET= 1.04

COORDINATES USED

1.07	0.006	2.30	21.500
1.30	0.650	2.40	23.500
1.40	1.400	2.50	28.000
1.85	13.500	2.70	41.000
1.90	15.000	2.90	69.000
2.00	17.000	3.00	84.000

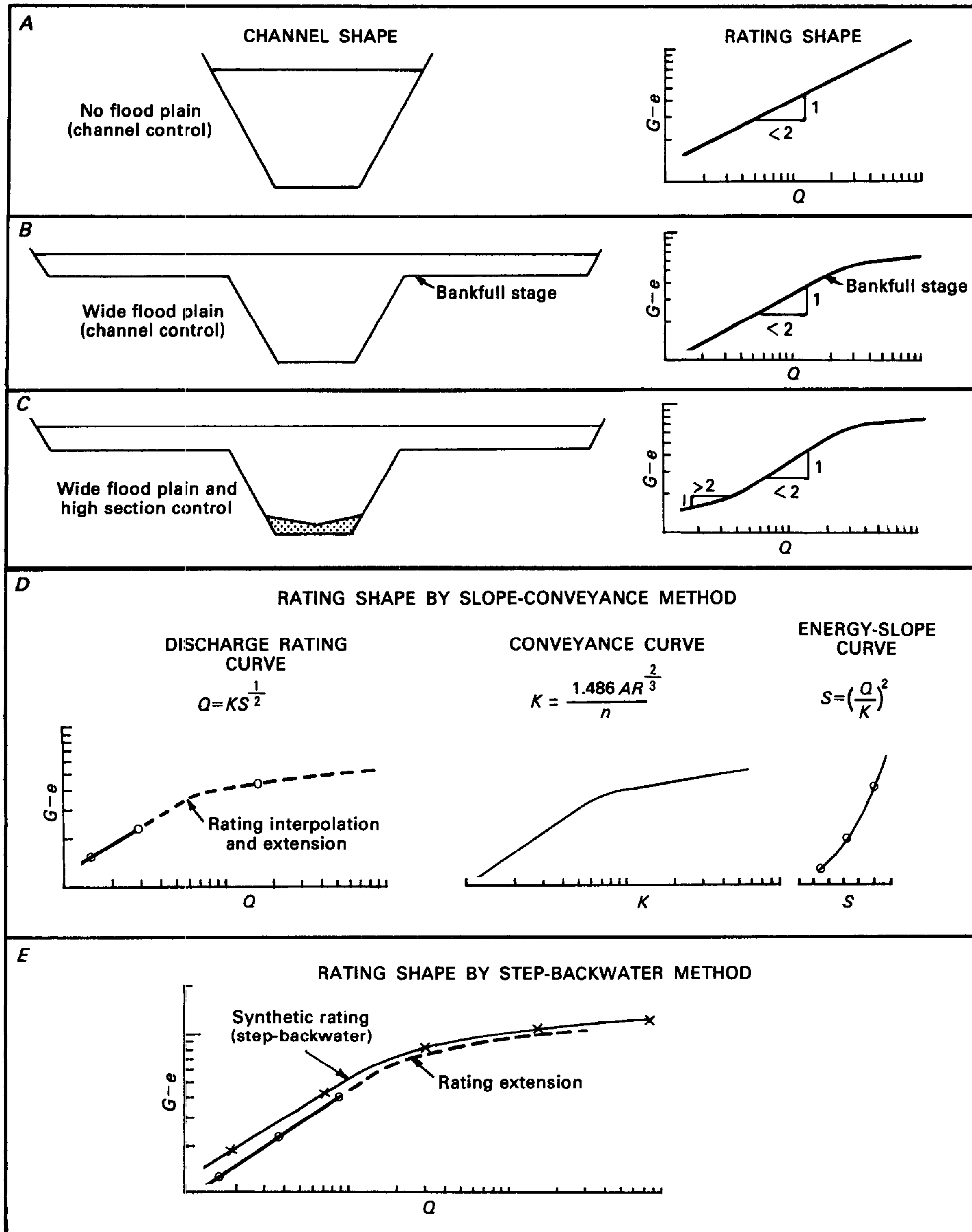


FIGURE 11.—Relation of rating-curve shape to cross-section properties.

the K curve. The S (slope) curve, drawn through the plotted points, may increase with stage, as figure 11D shows, or decrease, but it should be smooth, and it usually approaches valley slope at high stages. After the K and S curves have been defined, discharge values at all stages can be computed and the rating curve drawn, complete with all breaks. This procedure is especially useful for interpolating the undefined parts of rating curves known to contain breaks or for making moderate extensions of rating curves.

Step-backwater rating curves

Figure 11E illustrates one way in which the step-backwater method can be used to obtain an approximate shape curve to use as a guide in a rating extension. The step-backwater method consists of a survey of a reach downstream from the gage, including several cross sections (usually 10), an estimate of the stage-discharge relation at the downstream end of the reach, and a computation of water-surface profiles for several selected discharges. The end result is a curve drawn through the computed water-surface elevation at the gage corresponding to each selected discharge. The rating curve is extended generally parallel to the logarithmic shape curve. If conditions are favorable, the step-backwater model can be calibrated. This procedure requires some revision of the roughness coefficients and section subdivision until the shape curve coincides with the defined part of the rating. The upper part of the shape curve then becomes the rating extension. The step-backwater procedure, explained in detail by Bailey and Ray (1966) and Shearman (1976), is reliable for all natural channels, although the length of reach needed may be impractically long for a channel whose slope is less than about 0.0005 ft/ft. Although this shaping method is expensive, it is by far the best indirect procedure for defining a discharge rating shape.

Stable channel techniques

Channel stability is a relative term. Most channels in stable material remain unchanged between floods that may scour or fill the controls. If these periods are relatively long and if the discharge measurements made during them are adequate to define the applicable ratings, the channel is considered stable. High-water controlling reaches are usually changed only by major floods, but low-water controls may be modified by minor rises.

One procedure for making a rating analysis for a stable channel is as follows:

1. List the relevant discharge measurements (see table 2). The list should include all measurements made within the period for which daily records are to be computed and, preferably, some that were made later. All previous high-water measurements should also be considered.
2. Prepare a logarithmic work curve sheet. A print of the previous year's master curve sheet may be a satisfactory worksheet if no major rating change occurred and if the scales are satisfactory.
3. Plot the discharge measurements and a shape curve on the worksheet. The last rating used or some other superseded rating curve may be the best available shape curve. If more than one new rating is indicated, the measurements that apply to each rating may be plotted on the worksheet by using a distinctive symbol or a color.
4. Draw the curve or curves on the basis of the measurements. A great deal of judgment is needed to strike a balance between the closeness of fit of the curves to the data and the reasonableness of the curve shape. The upper end of the rating curve should be merged with the high-water rating used previously unless there is evidence of a high-water shift. The lower end should be carried down below the minimum recorded stage or to near-zero flow (0.006 ft^3/s, the lowest discharge that can be used as a descriptor) if the data warrant it.
5. Plot the curve, or curves, on at least one rectangular grid to verify the reasonableness of shape and the absence of significant scalloping.
6. Select the digital descriptors.
7. Enter the digital descriptors in the memory of an appropriately programmed calculator. In the absence of a calculator, a rating-table printout can be obtained through the RATLIST program (table 1A) or a desktop cal-

TABLE 2.

9—207
(Nov. 1957)

UNITED STATES DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY (WATER RESOURCES DIVISION)
DISCHARGE MEASUREMENT SUMMARY SHEET

Station No. 03580990

Discharge measurements of Jack Daniel Spring at Lynchburg, Tenn., during the year ending Sept. 30, 1970.

No.	Date 1970	Made by—	Width	Area	Mean velocity	Gage height	Discharge	Rating 1: Shift adj.	Rating 1: Percent diff.	Method	Number meas sections	Gage height change	Time	Meas rated	Outs Ght.	REMARKS
			Feet	Sqft	Fps	Feet	Cfs	Feet				Feet	Hr			
1	Apr.26	V.J. May C.S. Bennett	15	40.0	.99	2.70	39.5	-	-3.8	.6	11	+.04	.30	P	2.72	Wall downstream is section control
2	26	do	21	57.4	1.08	2.86	61.9	-	-1.0	.6 .2-.8	22	+.06	.58	F	2.90	do
3	26	do	21	58.1	1.16	2.92	67.2	-	-6.8	do	21	+.01 -.01	.52	F	2.95	do
4	Apr.26	do	21	57.3	1.05	2.82	60.3	-	+6.5	.2-.8	23	-.04	.67	G	2.84	do
5	May 10	do	1.00	-	-	1.12	.049	-	-2.9	Incr. vol.	4	+.07	.18	P	1.12	All flow in weir notch
6	10	do	1.52	-	-	1.18	.168	-	-1.2	do	6	+.04	.17	F	1.18	do
7	10	do	1.80	-	-	1.23	.331	-	+.6	do	7	+.02	.15	G	1.23	do
8	10	do	2.10	-	-	1.25	.403	-	-1.5	do	7	0	.15	G	1.25	do
9	10	do	2.65	-	-	1.30	.627	-	-3.7	do	8	+.04	.18	G	1.30	do
10	10	do	2.90	-	-	1.34	.902	-	-1.0	do	7	+.04	.15	G	1.34	do
11	Dec.23	J. Anderson Bennett	7.1	6.01	1.88	1.81	11.3	-	-3.5	.6	17	0	.42	G	-	Weir drowned out--channel controlling
12	23	do	7.1	5.95	1.92	1.79	11.4	-	+4.4	.6	17	0	.38	G	-	do
13	Dec.24	Anderson	7.2	2.17	1.96	1.58	4.26	-	-2.1	.6	18	0	.42	G	1.58	Full width weir control shallow depth C = 1.18
14	24	do	6.1	1.86	2.39	1.58	4.44	-	+2.0	.6	16	0	.42	G	1.58	do Pygmy meter

Copied by EJK Computed by EJK Checked by VJM culator (table 1*B*). The rating is still tentative at this stage and can be modified easily in the storage of a handheld calculator. If any descriptor in a comptuer-generated table is changed, the table must be rerun.

8. Compute the percentage differences between discharge measurements and rating values. If these differences are judged to be satisfactory, as they are in table 2, the rating may be final. If they are unsatisfactory, the process is repeated from step 6.

9. Plot the rating values between the descriptors on the rectangular grid to inspect the plot for scalloping or abrupt breaks in slope. If the curve is satisfactory, it is final. If the curve requires further smoothing, the process is repeated from step 6.

10. Plot the master curve sheet as it has been done in figure 12, and, if a rating table similar to table 1*A* or 1*C* has not been made previously, prepare one.

11. Prepare written notes on the thought processes and assumptions used in developing the rating for subsequent use in documenting the rating analysis.

The steps listed will ensure a well-analyzed rating for a site on a stable channel. Most ratings are unstable for very low flows, and their analyses will require additional steps.

Unstable channel techniques

Nearly all stream channels are unstable at some stages at some times. A very few channels, usually steep ones in sandy streams, are unstable at all stages at all times and can be gaged only with extraordinary methods and equipment, if they can be gaged at all. Horizontal movement of stream reaches in alluvial fans may be too extensive to permit gaging by any means. Other streams ordinarily not gaged include some in sand channels having certain combinations of slope and fine sediment, whose ratings are affected by frequent changes in bed configuration (plane, dune, antidune, and so forth). Bed changes often cause a complete absence of any relation between stage and discharge at some stages. Discharge at some sites can be related only to average depth or to hydraulic radius, neither of which can be automatically recorded, and that relation is discontinuous in that the top of the low-water rating curve is above and to the left of the bottom of the high-water curve. If a bed remains reasonably stable throughout the low-flow regime and changes to another reasonably stable configuration at high flow, it may be practical to develop a discontinuous stage-discharge rating. The bedform effect on ratings is also a low-water phenomenon in some otherwise relatively stable channels, especially those that fluctuate diurnally owing to dam operations. This very complex effect is described in detail by Dawdy (1961) and by Simons and Richardson (1962).

The low water ratings of most streams, especially shallow ones having riffles or other section controls, change or shift gradually as algae or grass grows in in the channel and causes backwater. This effect often starts in the spring, peaks in late summer, and disappears during winter. Backwater from leaves, an autumn occurrence, builds up rapidly and continues erratically until rises flush the leaves from the control. Debris buildup that causes backwater on the control often occurs just after a substantial rise and continues until another rise flushes the channel or until the debris is removed manually. Alluvial stream bottoms and their corresponding ratings become lower when the streams' sediment-transport mechanism picks up more sediment that it deposits. Conversely, the streambed rises when the quantity of sediment deposited exceeds the quantity picked up. The transport process depends on a continuously changing balance of discharge, water temperature, and sediment concentration, among other factors. The shift or change in rating position is smooth and gradual when it is caused by aqueous growth, sometimes sudden and erratic when it is caused by leaves, unchanging for long periods when the backwater is caused by debris, and smooth but extremely variable in some clean, sandy channels. Rating analyses for unstable channels often can be made by using the stable-stream procedures for the medium- and high-water parts. Low-water rating analysis for streams whose channels are unstable often requires a special approach and involves the use of the shifting-control method, an important tool in nearly all rating analyses.

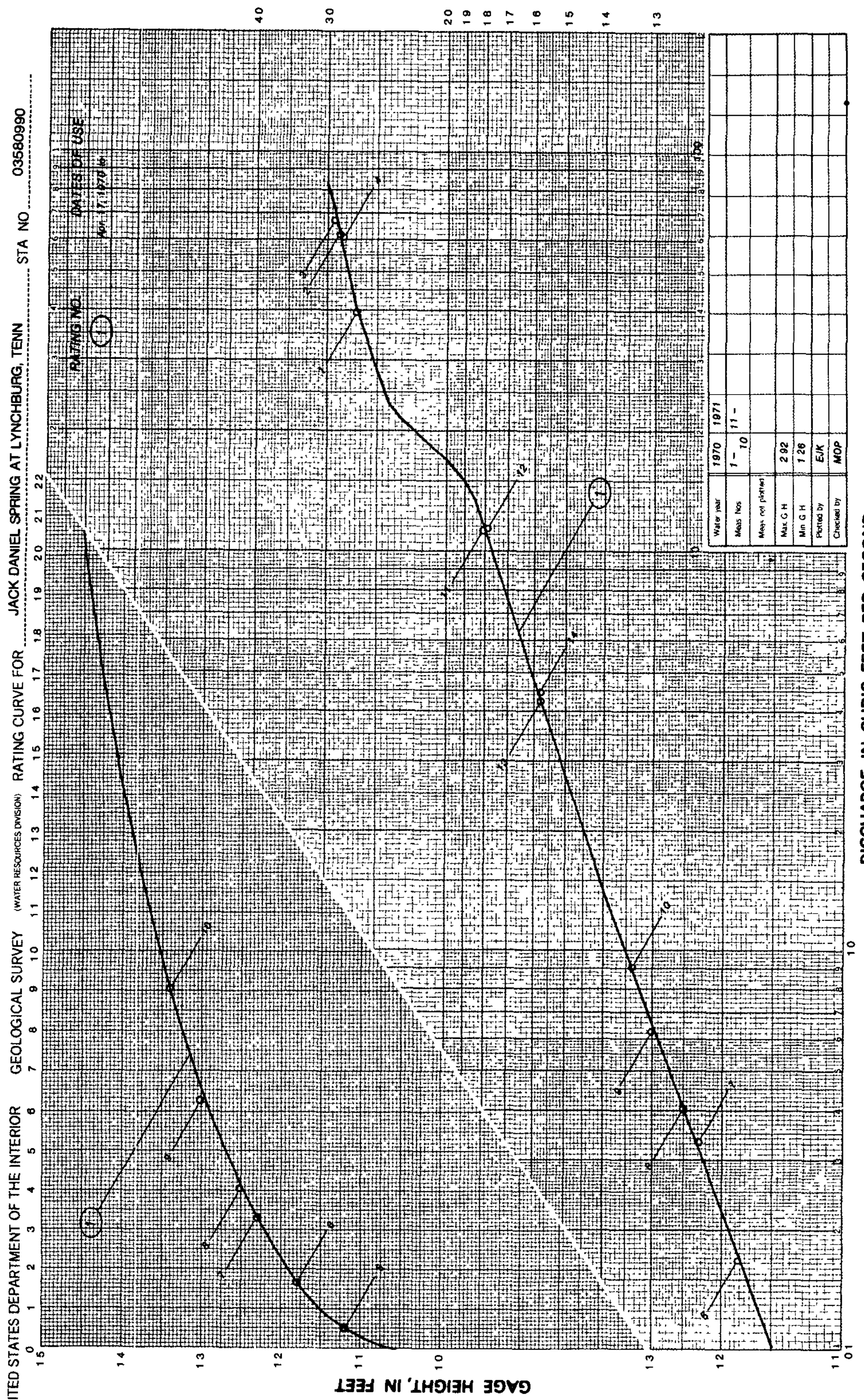


FIGURE 12.—Master rating-curve sheet.

Shifting-control method

A shift adjustment is a correction made to a recorded gage height that compensates for the vertical movement or shifting of that rating. The shift adjustment of a discharge measurement to a base rating curve is computed by subtracting the gage height of the discharge measurement from the gage height of the rating curve that corresponds to the measured discharge. This "observed" shift adjustment is used when a discharge measurement is given full weight. A shift adjustment of −0.27 ft for a gage height of 3.19 ft, for instance, means that the effective rating curve at 3.19 ft is 0.27 ft above the base rating curve. Daily discharges for periods when the shifting-control method is used are computed by adding the applicable shift adjustment to the daily gage height before entering the base rating to obtain the discharge. In some complex uses of shifting-control method and in cases where several measurements not in exact agreement define a shifting condition, an average shift adjustment may be used.

Base rating curves for sites affected by backwater caused by temporary phenomena such as aqueous growth or leaves are best drawn by using the discharge measurements made while the control was observed to be clean; these measurements usually plot farthest to the right. The resulting curve represents clean channel conditions, and shift adjustments are required only when the control is obstructed. All shift adjustments are negative for this type of analysis. Sand bed streams usually have much larger shifts, and the sign of an adjustment has no special significance. The base curve for an alluvial streambed site is best drawn in an average location; both plus and minus shifts should be used to keep shift adjustments small enough for visual interpolation of daily shift values between measurements. Small shifts also simplify the smooth transition to zero shift during higher water periods if the upper part of the rating is stable.

Some channels and their ratings shift upward or downward by more than 5 ft in a single high-water day. Stable channels obstructed by grass or algae have much smaller shifts. Values of daily shift adjustments must be interpolated between discharge measurements in order to compute the daily discharges, and hourly adjustments are needed for some flood records. Shift adjustments may be varied, manually or by computer, with time, stage, or both, or the adjustment can be kept constant during the low-water periods between rises. Where shifting is particularly erratic, hydrographic comparison of daily discharges with those of other streams may help distribution. Shift distribution is simpler and more accurate where the rating curve is properly shaped. Otherwise, the shift distribution must compensate simultaneously for channel changes and rating shortcomings—a difficult assignment.

The shifting-control method can be used, most practically with the ADP initial processing of daily records, to simulate a rating that changes its shape and position gradually because of grass or aqueous growth or the accumulation of debris in the channel. This simulation is accomplished by varying the shift adjustment with both time and stage. Details of applying this method using ADP are described in the WATSTORE User's Guide (Hutchison and others, 1975, 1980).

Low-water rating analysis for unstable channels

Most low-water discharge measurements of an unstable channel stream have different *GZF*'s and, consequently, do not define the same rating or adapt to logarithmic plotting. A special approach must be used, and frequent *GZF* determinations, preferably one for each low-water wading measurement, are essential.

The fundamental assumptions in low-water rating analysis for an unstable channel are as follows:

1. A basic curve shape prevails for the low-water rating and is substantially changed only by floods that change the channel location, shape, or meander pattern downstream.
2. The basic rating-curve shape can be defined by the relation of maximum depth at the control ($Gh-GZF$) to measured discharge.
3. Any straight line plotted to "normal" scale on a logarithmic grid whose slope is between 0° and 90° is a parabola that goes through the coordinates (0,0).
4. The basic low-water rating curve defined by using the previous assumptions, in ef-

fect, can be moved vertically on a rectangular grid without error by use of the shifting-control method.

Assumptions 1 and 4 are approximations, and there is no way to verify how closely they may apply at a particular site. However, any related errors affect only the discharge for days on which interpolated shift adjustments were made and are minor in comparison with errors caused by grossly misshaped rating curves.

Figure 13 illustrates a low-water rating analysis for a stream whose section control of alluvial material over the remnants of a beaver dam is unstable but whose medium- and high-water ratings are relatively stable. The basic data are tabulated in figure 13*A* and include frequent *GZF* determinations. The depth column is *Ght*−*GZF*. The measured discharges, plotted against corresponding gage heights in figure 13*B* on a rectangular grid, give no reliable information as to the shape of the rating. The same discharges, plotted against depth at the control on a logarithmic grid in figure 13*C*, give a well-defined curve because no measurement plots farther above or below the curve than the expected error in the *GZF* determination. The heavy curve on the rectangular grid in figure 13*D* is the depth-discharge curve raised by 3.04 ft (any other value within the range of shifts would do about as well) to match the rating position on October 4. The light curves illustrate the effective rating location on other days. Shift adjustments listed in figure 13*A* are distances between the curve positions at the times of discharge measurement and the heavy base curve. If *GZF*'s had not been measured and if the October and May discharge measurements had not been made, the other measurements would have led to a differently shaped base curve, the shift adjustment variation between measurements would have been erratic, and the computed record would have been less reliable.

Complex ratings

A complex rating is used for a site where the water-surface slope is variable and where no simple relation exists between stage and discharge. Discharge must be related to stage and some other variable. Rate of change in stage is the additional variable for rating streams where storage causes the stage-discharge relation to loop (figure 14*A*). A slope rating is used, along with an auxiliary gage to measure fall in a reach, where tributaries, dams, or the return of overbank flow to the channel causes variable backwater. Index-velocity ratings, which involve special mechanical or electronic devices to measure velocity, are used where special rating problems exist.

A complex rating requires more discharge measurements for adequate definition than a simple stage-discharge rating, and the type of complex rating that will apply usually cannot be predicted before the measurements are made. A prudent procedure to follow at a newly established site where a complex rating is anticipated is to assume that a slope rating will be needed, establish temporary gages at potential auxiliary sites so that readings can be made during all discharge measurements, and measure a few rises over the entire flood hydrograph. Then the loop ratings can be plotted as one indicator of the appropriate rating type. The simplest analysis can be tried first. If it is not satisfactory, various slope ratings can be tried until an adequate rating is developed or until the need for an index-velocity rating is apparent.

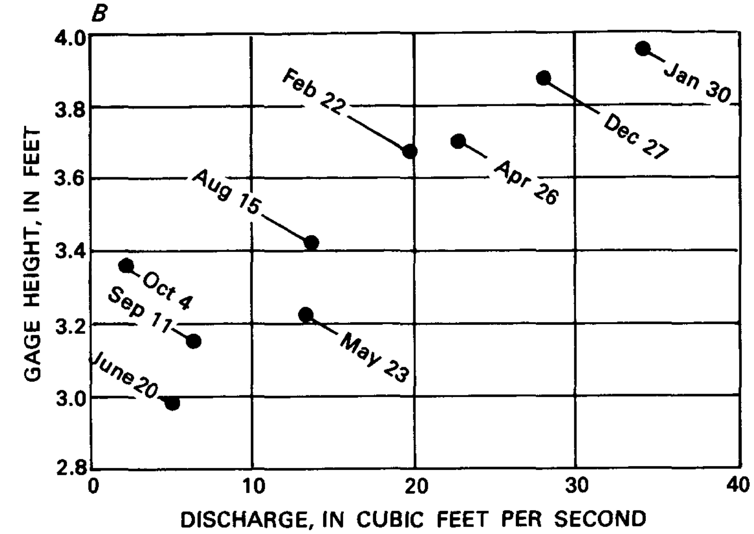
A loop rating can be drawn by connecting plotted consecutive discharge measurements made during a single rise. If a rating has been developed, the loop for each major rise can be plotted without discharge measurements by connecting the successive plots of recorded instantaneous gage heights and the corresponding adjusted discharges. Typical single-storm storage loops are shown in figure 14*A*. This type of loop is distinctive in that one occurs on every rise and is roughly symmetrical about the stage-discharge curve for constant-stage conditions. Such loops are related to channel storage between the gage and the control and indicate the applicability of a rate of change in stage rating. Figure 14*D* shows typical backwater loops of the type caused by the return of overbank flow to the main channel. A backwater loop occurs only after an overbank rise—the greater the overbank depth, the wider the loop. An overbank return loop is always to the left of the free-fall rating (the rating defined

Soldier Creek near Circleville, Kansas

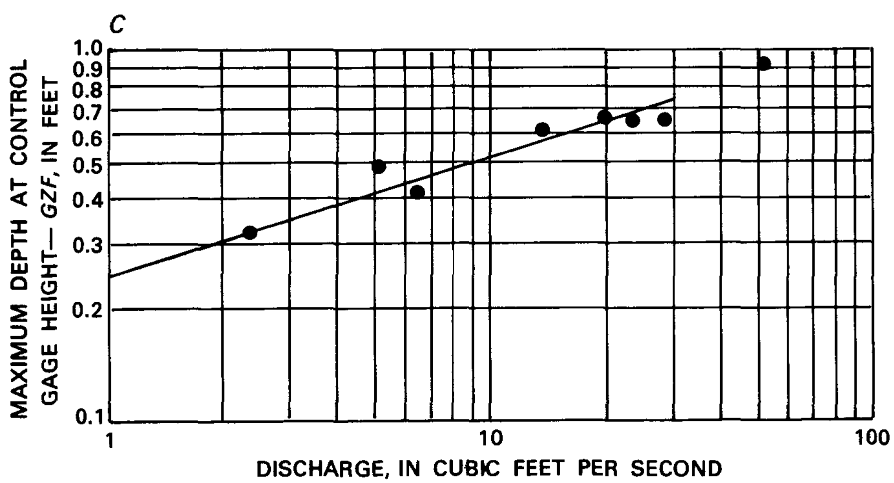
A BASIC DATA

Meas. No.	Date	*Ght* (ft)	*Disc* (ft^3/s)	*GZF* (ft)	Depth at control (ft)	Shift Adjust. (ft)
139	10-04-72	3.36	2.35	3.04	0.32	0
140	11-15	4.22	51.4	3.31	.91	-.31
141	12-27	3.87	28.0	3.22	.65	-.12
142	1-30-73	3.95	34.2	--	--	-.15
143	2-22	3.67	19.6	3.02	.65	0
144	3-26	4.43	110	--	--	0
145	4-26	3.70	22.7	3.06	.64	0
146	5-23	3.22	13.3	--	--	+.38
147	6-20	2.98	5.11	2.50	.48	+.47
148	7-20	5.66	298	--	--	0
149	8-15	3.42	13.6	2.82	.60	+.18
150	9-11	3.15	6.43	2.74	.41	+.33

B



C



D

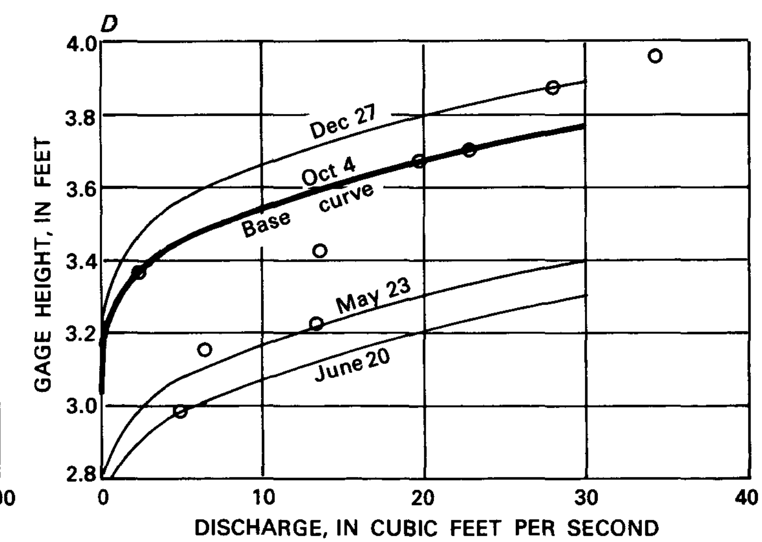


FIGURE 13.—Example of a low-water rating analysis with periodic *GZF* observations used to define unstable low-water ratings.

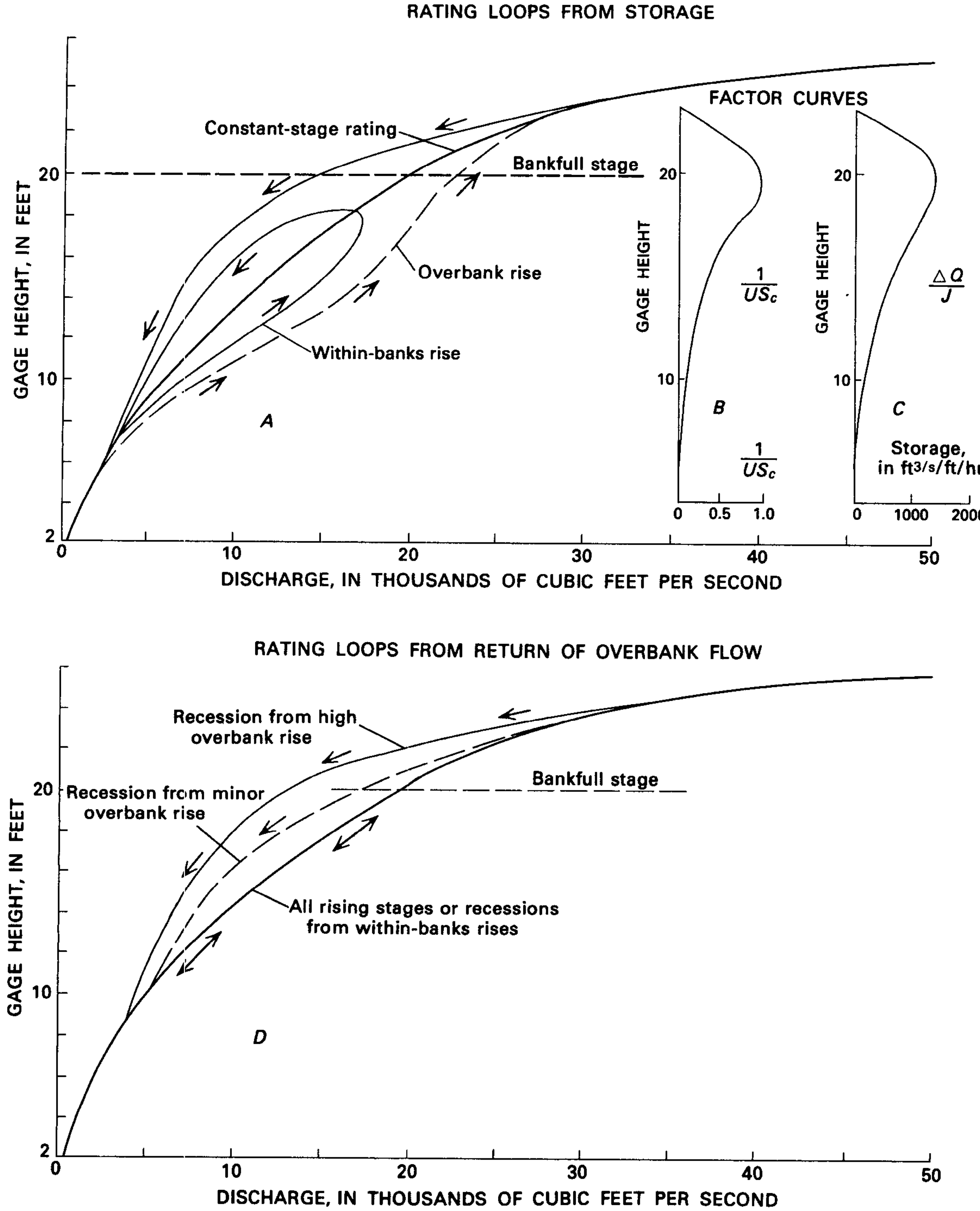


FIGURE 14.—Typical shapes of single-storm loop ratings and factor curves.

by rising-stage measurements and those falling-stage measurements that follow a within-banks rise). Loops of this type are rarely as clearcut as the illustration. They are often superimposed on storage loops and may be impossible to identify. The presence of backwater loops, alone or in combination with storage loops, rules out the use of a rate of change in stage rating and requires a slope or index-velocity rating.

The ordinary types of complex ratings (rate of change in stage, slope, or index velocity) are explained and illustrated by actual examples in this manual. The WATSTORE User's Guide (Hutchison and others, 1975, 1980) contains the instructions necessary for preparing data so that trial-and-error solutions for most of the complex rating types can be made by using a computer.

Rate of change in stage ratings

Two types of rate of change in stage ratings are in general use: (1) $\Delta Q/J$ (storage effect per unit of rate of change in stage), which treats a rating loop as a simple storage phenomenon, and (2) $1/US_c$, which relates the magnitude of the rating loop to the velocity of flood waves (U) and to the water-surface slope at constant discharge (S_c). Either method can be used at most sites that have rating loops similar to those in figure 14*A*, but one method may be clearly superior to the other at a site where the rating loop is wide. The best practice is to try both methods and select the one that best fits the discharge measurements.

A rate of change in stage rating is subject to subtle errors that are not apparent until the rating is tested by using actual data. Serious irregularities occur most often when an auxiliary curve (figs. 14*B*, *C*) is bent too sharply in the stage range where rates of change are most rapid. The sharp bends can cause false peaks and troughs in the hydrograph. Other causes of erratic record include stilling-well surge, manometer stepping, and sluggish intakes that suddenly plug or clear. Much of the gage-height surge present in some wells or bubble gages can be removed during ADP processing by using a smoothing option covered in the WATSTORE User's Guide (Hutchison and others 1975, 1980). Some errors can be prevented by checking a rating through a major rise (see figs. 16, 19) before it is used and by drawing the hydrograph and loop ratings for all subsequent major rises from ADP-generated gage heights and discharges. If the hydrographs and loop ratings are always reasonable, the rating probably is accurate and is the correct type for the site. If the hydrographs and loops are unreasonable and if adjustments to the rating do not correct them, the rating type is probably inapplicable, and a slope rating should be tried.

Daily discharges computed by using both the constant-stage discharge and the factor curves of a rate of change in stage rating are called adjusted discharges. Those computed by using only the constant-stage discharge curve as a simple rating are called unadjusted discharges. The choice of methods depends on the use of the records and the definition of the rating. The adjusted discharges from a rate of change in stage rating represent flow at the gage. Unadjusted discharges from the constant-stage curve can be considered to represent flow at the control, wherever the control happens to be at the time. If unadjusted discharges are used, the peak discharge usually will be slightly below the adjusted peak discharge, and the unadjusted discharge hydrograph will be similar in shape to and a few hours later than the adjusted hydrograph. Adjusted daily discharges for the rising and falling high-water days will differ substantially from unadjusted discharges, but the total flow for each rise will be about the same. If the rating tests are favorable, adjusted daily discharges are always preferable. If water samples collected at the gage are involved and if the constituents analyzed are related to the quantity of flow, adjusted daily discharges should always be used. If no water samples are involved and if the rating's auxiliary curve is poorly defined, unadjusted daily discharges computed from the constant-stage rating may be the best choice.

$\Delta Q/J$ ratings

The $\Delta Q/J$ type of rating is a logical first-trial choice if the rate of change of stage is the likely cause of loop ratings. The principal components are a constant-stage discharge curve (central curve in fig. 14*A*) and a storage curve (fig. 14*C*). Actual discharge is computed by adding a storage correction to the discharge obtained from the constant-stage rating. The storage correction is the value from the storage curve multiplied by the rate of change in stage. If the symbols as defined in figure 15*B* are used, the relation can be written:

$$Q_m = Q_r + \left[\left(\frac{\Delta Q}{J}\right)\right] \times J$$

The rating is developed by trial and error, starting with a trial constant, a stage rating curve drawn close to the measurements made during near-steady stages. The difference between each measured discharge and the constant-stage discharge is divided by the rate of change in stage and plotted against stage on a separate graph. The storage curve, which represents the storage correction per foot-per-hour change in stage, is based on these plotted points. Each discharge measurement is adjusted to constant-stage conditions (corrected for storage) by using the storage curve. The constant-stage rating curve is refined by using the adjusted measurement values. The process is repeated, usually about three times, until further refinement of rating or storage curve is unlikely. The sequence of steps used is listed in figure 15*B*. The WATSTORE User's Guide (Hutchison and others, 1975, 1980) outlines a similar procedure to be used with an appropriate computer facility.

The $\Delta Q/J$ curve must be drawn with due regard to the unequal weights of the plotted points $[(Q_m - Q_r)/J]$. Those based on discharge measurements whose rates of change in stage were high have considerable weight, and the storage curve should be drawn close to them. Measuring error is a large part of the variance between the discharge curve and the measured flow when the rate of change in stage is less than about 0.10 ft/hr. Storage values are not usually computed for those measurements, and only a little weight is given to values based on discharge measurements whose rates of change in stage are less than 0.20 ft/hr. A large departure of the storage curve from a point based on a slowly changing stage measurement has little effect on that measurement's percentage difference.

The general shape of the $\Delta Q/J$ curve is predictable. Most storage curves go through zero at the stage where the low-water control becomes submerged and again where the overbank contains more than about half the flow. The maximum storage value usually occurs at about bankfull stage. The curve should bend as gently as the data will allow.

Figure 15*A* illustrates a typical $\Delta Q/J$ method application. A $1/US_c$ analysis (not shown) also was made, and the resulting rating was not significantly different from the $\Delta Q/J$ rating. The gage is just downstream from a long highway embankment with a relatively short bridge that spans all flow. A riffle near the gage is the section control for low water and is drowned out above about the 7-ft stage. The location of the channel-controlling reach for medium stages is not apparent in the field or on a map, but its centroid is probably several miles downstream. Above bankfull stage (27 ft), the flow fans out into the relatively shallow flood plain just downstream from the gage. The flood plain that acts as control and the gage are so close together at very high stage that storage is negligible, and there is no changing-stage effect. The discharge and storage curves are typically shaped, the computations in figure 15*C* indicate only one outlier, and the testing by manual computation shown in figure 16 is favorable. The rating is sufficiently well defined to justify the use of adjusted daily discharges for the published record.

Most $\Delta Q/J$ ratings are used where the medium- and high-water ratings loop owing to storage change between the gage and a high-water control whose location depends on the stage. The process also can be used where changing-stage effect is caused by a section control far downstream and is present only at low water. Figure 17*A* illustrates this type of rating. At high stages, backwater from a downstream dam makes a slope rating necessary. All discharge measurements are made from a cableway at the gage. A rock riffle section control just downstream is submerged at a very low stage, and a series of shoals about 2 mi downstream becomes the low-water control. A storage curve was developed by the trial-and-error procedure used in the previous example, and the results of the final trial are shown in figure 17*B*.

$1/US_c$ ratings

The $1/US_c$ rating type, also called the Boyer method, is generally used if changing-stage effect cannot be related to simple storage. The method is based on the Boyer equation:

$$\frac{Q_m}{Q_r} = \sqrt{1+\left(\frac{1}{US_c}\right) \times J}$$

This equation, whose symbols are defined in figure 18, evolved from two earlier, similar equations that were used to adjust individual discharge measurements for changing-stage effect. In the early equations, the variables U and S_c were evaluated separately. The Boyer method treats the entire term $1/US_c$ as one empirical variable, and its relation to stage is defined by discharge measurements made during periods of rapidly changing stage. The rating components are a constant-stage rating (central curve, fig. 14*A*) and a stage versus $1/US_c$ curve (fig. 14*B*).

The rating is developed by trial and error, starting with a trial constant-stage rating drawn close to the measurements that were made during near-steady stages. Then the ratio of each changing-stage measured discharge to the constant-stage discharge and the rate of change in stage are entered in the Boyer equation. The equation is solved for $1/US_c$, and the result is plotted against the stage of the discharge measurement. A $1/US_c$ curve (factor curve) is drawn next on the basis of the plotted points. Each discharge measurement is then adjusted to constant-stage conditions by using the factor curve and the Boyer equation. The constant-stage curve is refined by using the adjusted measurements. The process is repeated, usually about three times, until further refinement of either curve is unlikely. The sequence of steps for manual computation is listed in figure 18*B*. The WATSTORE User's Guide (Hutchison and others, 1975, 1980) outlines the procedure to be used with appropriate computer equipment for the rating analysis.

The factor curve must be drawn so that the $1/US_c$ values computed from discharge measurements whose rate of change in stage is high are given more weight than those calculated from measurements made while stage changed slowly. Values of $1/US_c$ for measurements whose rate of change in stage is less than about 0.10 ft/hr are not usually computed because their variation from the constant-stage curve is greatly affected by normal measuring error. A large departure of the factor curve from a $1/US_c$ value based on a nearly constant stage discharge measurement has little effect on that measurement's percentage difference.

The shape of the factor curve is similar to that of a $\Delta Q/J$ curve. A typical $1/US_c$ curve goes through zero at the stage where the section control is submerged, reaches its maximum value at about bankfull stage, and approaches zero at the stage where the overbank area of the channel contains about half the total flow. The factor curve should bend as gently as the data will allow. If the value of $1/US_c$ at any stage is too great because of an erroneously drawn curve, the value under the radical in the Boyer equation may become negative for periods of rapidly falling stage in that range. The computed factor then would be the square root of a negative number, and a meaningful value could not be determined. Correcting this condition may require revision of both the constant-stage curve and the factor curve.

Figure 18*A* illustrates a typical $1/US_c$ rating. A $\Delta Q/J$ analysis (not shown) was tried for this site, and the resulting rating was essentially the same as the $1/US_c$ rating. The stream has a flat, narrow, uniform main channel and a flood plain 1 mi wide. Rating loops occur only at stages between 3 and 16 ft and rarely vary from the constant-stage rating by more than 15 percent. The rating was analyzed by using the procedure outlined in figure 18*B*, which is designed for either manual or minicomputer computation. A similar outline to be used for computing the trial curves on an appropriate terminal is contained in the WATSTORE User's Guide (Hutchison and others, 1975, 1980).

The final trial computations, tabulated in figure 18*C*, indicate a close fit of data to the rating, and the testing by manual computations shown in figure 19 is favorable. Daily discharge

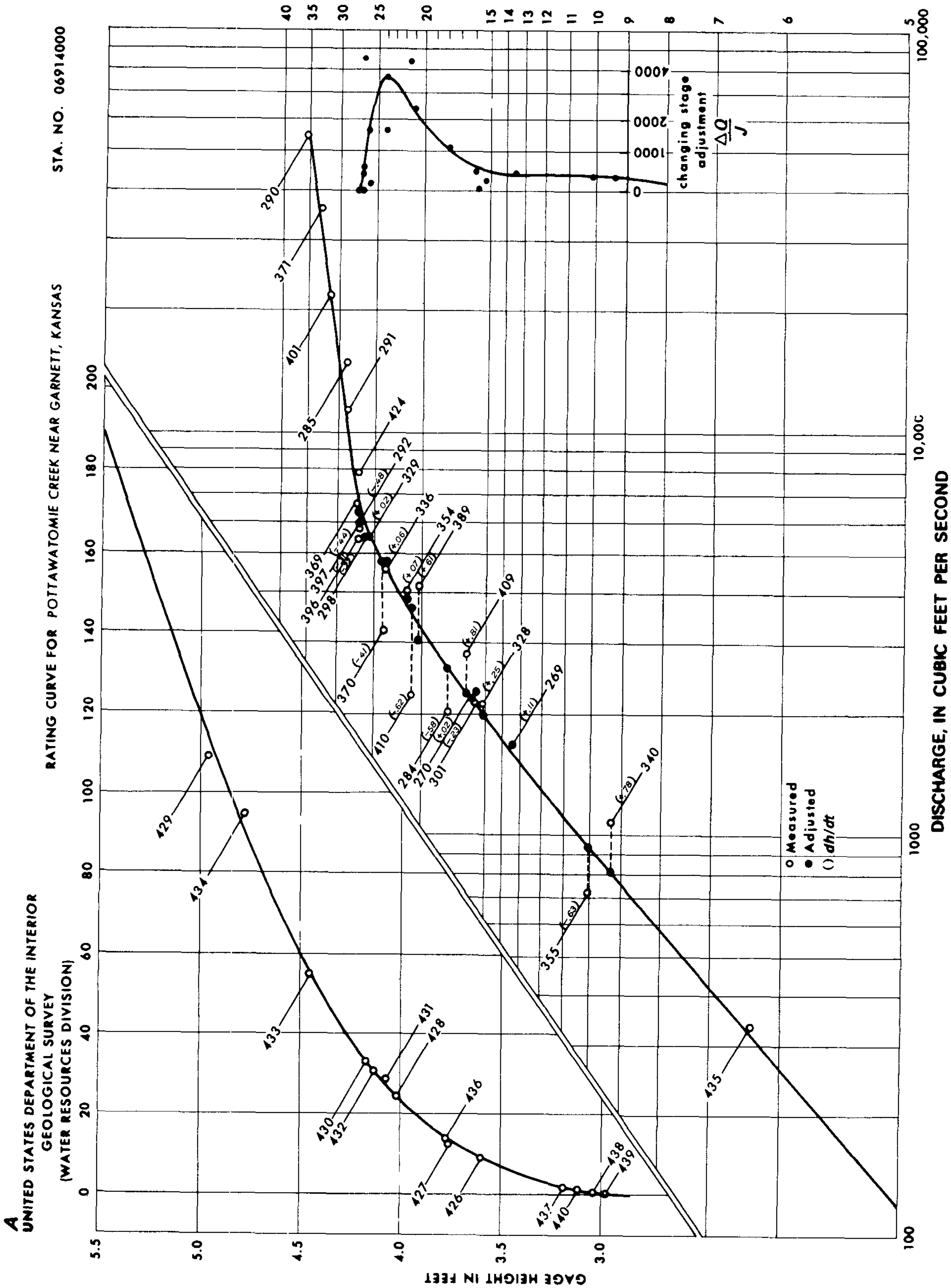


FIGURE 15.—Typical $\Delta Q/J$ discharge rating.

B

ANALYSIS PROCEDURE

STEP	OPERATION	INSTRUCTIONS
1	Prepare work sheets and table	Needed: Sheet 1, a log-log rating grid for the constant-stage discharge curve; sheet 2, a rectangular grid for the $\Delta Q/J$ curve; and a computation sheet with columns titled and numbered 1 to 11 as in the example below.
2	Enter data	Fill in 1 to 4 with data from the discharge measurements.
3	Plot Q curve data	Using sheet 1, with the appropriate *Ght* scale offset, plot *Ght* (2) vs. Q_m (3). Flag each print with J (4).
4	Draw first trial	*This trial curve should be close to constant-stage measurements, left of rising-stage measurements and to the right of falling-stage measurements. Skip to step 7 for the first trial computation.*
5	Plot Q curve data	On sheet *1, plot *Ght* (2) vs. Q_{adj} (10).
6	Draw refined Q curve	This discharge curve should average the step 5 points as well as possible.
7	List Q_r values	Fill in * 5 from step 4 curve (first trial), step 6 curve (subsequent trials), or from the curve's descriptors for the final trial.
8	Compute and list ΔQ	In 6 , if J (4) is between +0.1 and −0.1 enter a dash. Otherwise, * 6 = 3 − 5
9	Compute and list $\Delta Q/J$	If a dash is entered in 6 , enter a dash in 7 . Otherwise, * 7 = 6 ÷ 4
10	Plot storage curve data	Plot $\Delta Q/J$ (7) vs. *Ght* (2) on sheet *2. Use a distinctive symbol for rapid-change points.
11	Draw storage curve	The storage curve should resemble figure 14*C* and be closest to those step 10 points defined by rapid-change measurements. Maximum $\Delta Q/J$ is usually just above bankfull stage. $\Delta Q/J$ is zero when section control is effective and again when the flood plain contains most of the total discharge.
12	List of curve values of $\Delta Q/J$	Fill in * 8 from the sheet 2 curve values for the early trials. For the final trial, use values interpolated from the curve's descriptors.
13	List computed ΔQ	Fill in * 9 for all measurements, regardless of magnitude of J. 9 =− 8 x 4
14	List Q_{adj}	Fill in * 10 = 3 + 9
15	Next trial (step 5)	If both the Q and $\Delta Q/J$ curves are unlikely to improve with further trials, proceed to step 16. For an additional trial, return to step 5.
16	Finalize	Prepare descriptors or tables for both curves.
17	Finalize	*Recompute* * 5 *to* 10 *and compute* 11 *using the step 16 materials;* 11 =100x(10 − 5)− 5 . If 11 values are satisfactory, proceed to step 18. Otherwise, return to step 5.
18	Test	See text and figure 16. If test is satisfactory, proceed to step 19. Otherwise, return to step 5 and adjust curve shapes as necessary.
19	Finalize	Prepare master curve sheet.

*Erase any entries or plotting from previous trials.

C

COMPUTATIONS

Pottawatomie Creek near Garnett, Kansas

Meas. No.	Ght	Q_m	J	Q_r	$Q_m - Q_r = \Delta Q$	Comp. $\frac{\Delta Q}{J}$	Curve $\frac{\Delta Q}{J}$	$-\frac{\Delta Q}{J} \times J = \Delta Q$	$Q_m + \Delta Q = Q_{adj}$	%Diff.
1	2	3	4	5	6	7	8	9	10	11
269	13.49	1690	0.11	1620	70	636	475	-52	1638	1.1
284	17.86	2030	-0.50	2680	-650	1121	1087	630	2660	-0.7
285	26.80	14900	-0.21	12800	2100	0	0	0	14900	16.4
290	35.27	54200	-0.14	57600	-3400	17895	0	0	54200	-5.9
291	28.60	11300	-0.33	12200	-900	2727	0	0	11300	-7.4
340	9.34	1080	0.78	801	279	358	311	-243	837	4.5
355	10.07	720	-0.63	936	-216	343	336	211	931	-0.5
370	23.97	3240	-0.41	4880	-1640	4000	3941	1616	4856	-0.5
371	32.85	35900	0.30	33900	2000	0	0	0	35900	5.9
401	31.11	23000	-0.10	22600	200	0	0	0	23000	0.9
409	16.42	2820	0.81	2310	510	630	763	-618	2202	-4.7
410	20.93	2210	-0.62	3600	-1390	2242	2076	1287	3497	-2.9
424	26.98	7940	0.30	7880	60	200	2030	-609	7331	-7.0

SYMBOLS

Comp.	Computed value	Q_m	Measured discharge (ft^3/s)
Curve	Value from curve	Q_r	Discharge from ratings (ft^3/s)
Ght	Gage height (ft)	ΔQ	Storage correction (ft^3/s)
J=**dh/dt*	Rate of change in stage (ft/hr)	%Diff.	Variation of Q_{adj} from Q_r
Q_{adj}	Adjusted discharge (ft^3/s)		

**J* and *dh/dt* are both conventional symbols for rate of change in stage. *J* is more convenient to use in an equation, especially as part of the numerator or denominator of a fraction.

FIGURE 15.—Continued.

A. DAILY DISCHARGE COMPUTATION

Flood subdivision with factor
(Experimental)
(Mar. 1960)

UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
WATER RESOURCES DIVISION

Stream Pottawatomie Creek near Garnett, Kans

Gage height, in feet, and discharge, in cubic feet per second, at indicated time, 19 59

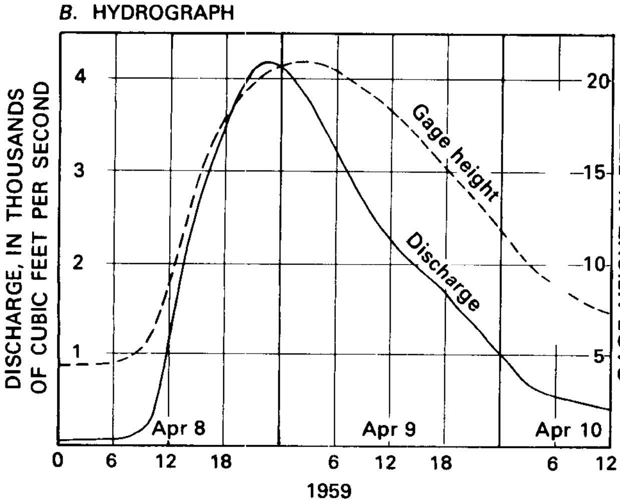
April 8, 1959

HOUR	GAGE HEIGHT	SHIFT ADJ.	j	$\frac{\Delta Q}{J}$		ΔQ	Q_r	DISCH.
0	4.33							42
1								
2	4.34							43
3								
4	4.38							45
5								
6	4.44							50
7								
8	4.83							92
9								
10	5.75							228
11								
12	8.85	+	1.75	290		507	674	1180
13								
14	12.55	+	1.87	400		748	1430	2180
15								
16	15.88	+	1.25	610		762	2160	2920
17								
18	17.80	+	.75	1100		825	2660	3480
19								
20	19.15	+	.58	1560		905	3060	3960
21								
22	20.12	+	.37	2050		758	3430	4190 ←
23	20.43	+	.28	2150		602	3560	4160
24	20.65	+	.20	2250		450	3650	4100
Mean	10.88							1700

April 9, 1959

HOUR	GAGE HEIGHT	SHIFT ADJ.	j	$\frac{\Delta Q}{J}$		ΔQ	Q_r	DISCH.
0	20.65							4100
1								
2								
3	20.94	←	0					3770
4								
5								
6	20.47		-.18	2160	-	388	3580	3190
7								
8								
9								
10								
11								
12	18.36		-.44	1260	-	554	2810	2260
13								
14								
15								
16								
17								
18	15.19		-.52	550	-	286	1990	1700
19								
20								
21								
22								
23								
24	11.74		-.60	360	-	216	1240	1020
Mean	17.60							2440

B. HYDROGRAPH



C. LOOP RATING FOR RISE OF APRIL 8–10, 1959

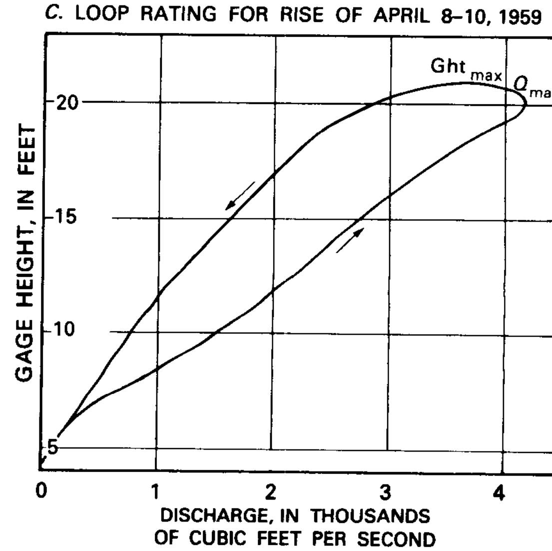
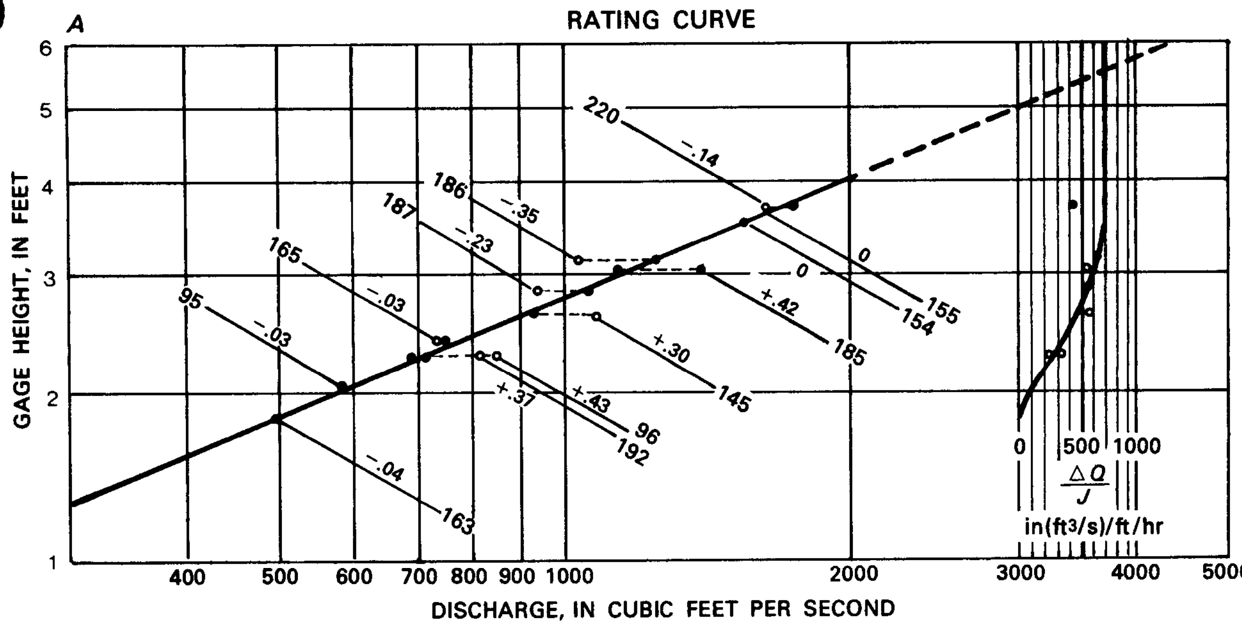


FIGURE 16.—Formats for testing a $\Delta Q/J$ rating.

Hiwassee River above Charleston, Tennessee



B

COMPUTATIONS

Meas. No.	*Ght*	Q_m	*J*	*Q_r	$\frac{\Delta Q}{J}$ (computed)	$\frac{\Delta Q}{J}$ (curve)	ΔQ	Q_{adj}	Percent Diff.
95	2.06	586	-.03	604	-	120	+ 5	591	-2.2
96	2.28	846	+.43	714	307	265	-114	732	+2.5
145	2.63	1080	+.30	912	560	470	-141	939	+3.0
154	3.55	1550	0	1570	-	700	0	1550	-1.3
155	3.63	1620	0	1640	-	700	0	1620	-1.2
163	1.84	492	-.04	505	-	0	0	492	-2.6
165	2.41	733	-.03	784	-	350	+ 10	743	-5.2
185	3.04	1400	+.42	1180	524	610	-256	1140	-3.3
186	3.14	1030	-.35	1250	628	635	+222	1250	0
187	2.86	934	-.23	1060	547	560	+129	1060	0
192	2.29	805	+.37	719	232	270	-100	705	-1.9
220	3.71	1650	-.14	1710	429	700	+ 98	1750	+2.3

*Scale offset −1.0, Coordinates 0.45, 100; 4.03, 2000.

SYMBOLS

Ght	Gage height (ft)	Q_r	Discharge from rating (ft3/s)
J	Rate of stage change (ft/hr)	ΔQ	Storage correction (ft3/s)
Q_{adj}	Adjusted discharge (ft3/s)	% Diff.	Variation of Q_{adj} from Q_r
Q_m	Measured discharge (ft3/s)		

FIGURE 17.—Typical storage-affected low-water discharge rating.

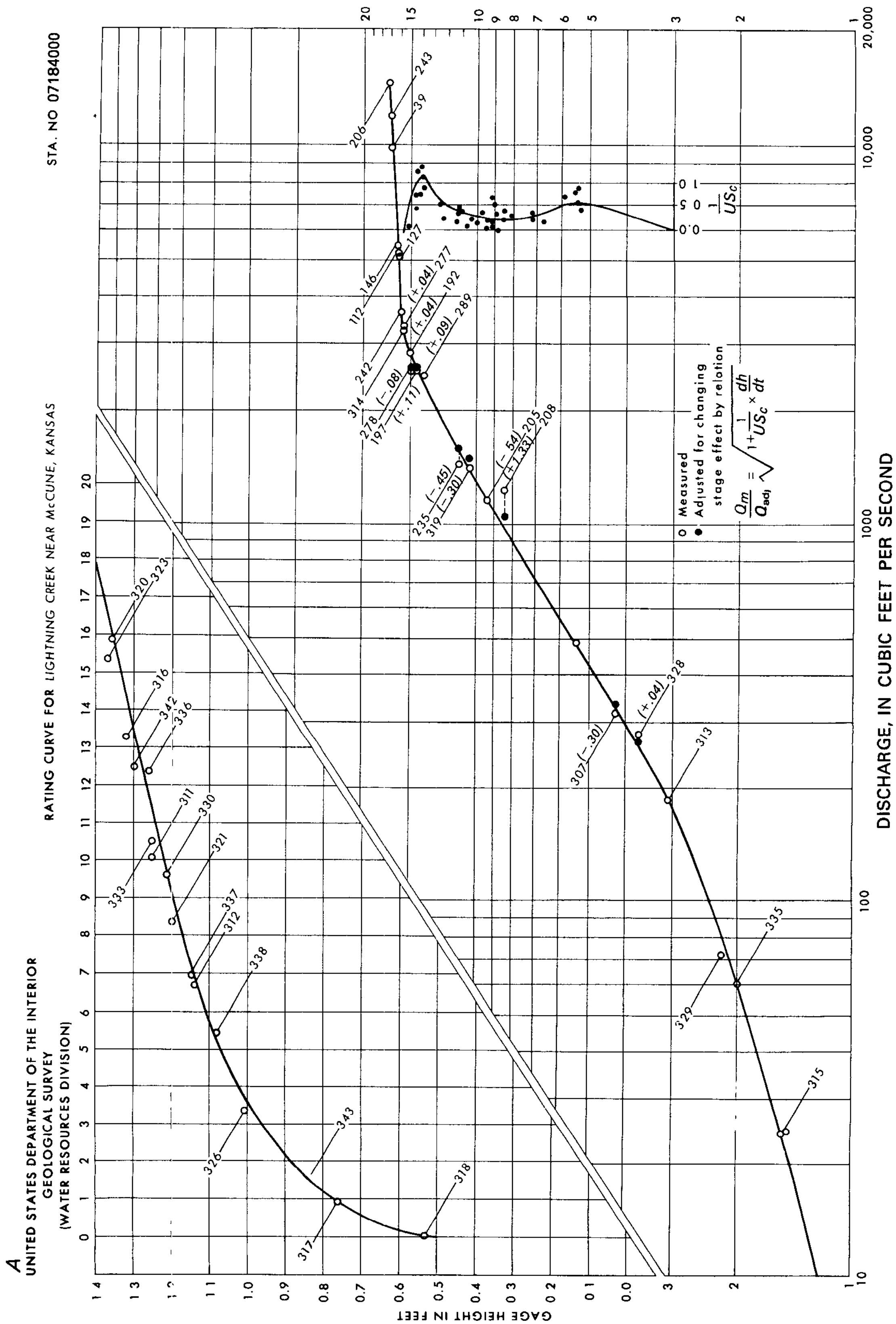


FIGURE 18.—Typical $1/US_c$ discharge rating.

B ANALYSIS PROCEDURE

STEP	OPERATION	INSTRUCTIONS
1	Prepare work sheets and table	Needed: Sheet 1, a log-log rating grid for the constant-stage discharge curve; sheet 2, a rectangular grid for the $1/US_c$ curve; and a computation sheet with columns titled and numbered (1) to (11) as in the example below.
2	Enter data	Fill in (1) to (4) with data from the discharge measurements.
3	Plot Q curve data	Using sheet 1, with an appropriate *Ght* scale offset, plot *Ght* ((2)) vs. Q_m ((3)). Flag each point with J ((4)).
4	Draw first trial	This trial curve should be close to constant-stage measurements, left of rising-stage measurements, and to the right of falling-stage measurements. Skip to step 7 for the first trial computation.
5	Plot Q curve data	On sheet *1, plot *Ght* ((2)) vs. Q_{adj} ((10)).
6	Draw refined Q curve	This discharge curve should average the step 5 points as well as possible.
7	List Q_r values	Fill in * (5) from step 4 curve (first trial), step 6 curve (subsequent trials), or from the curve's descriptors for the final trial.
8	Compute and list Q_m/Q_r	In (6), if J ((4)) is between +0.1 and −0.1 enter a dash. Otherwise, * (6) = (3) ÷ (5)
9	Compute and list $1/US_c$	If a dash is entered in (6), enter a dash in (7). Otherwise, * (7) = ($(3)^2$ − $(5)^2$) ÷ ($(5)^2$ x (4))
10	Plot factor curve data	Plot $1/US_c$ ((7)) vs. *Ght* ((2)) on sheet *2. Use a distinctive symbol for rapid-change points.
11	Draw factor curve	The factor curve should resemble figure 14*B* and be closest to those step 10 points defined by rapid-change measurements. Maximum $1/US_c$ is usually just above bankfull stage. $1/US_c$ is 0 when section control is effective and again when the flood plain contains most of the total discharge.
12	List of curve values of $1/US_c$	Fill in * (8) from the sheet 2 curve values for the early trials. For the final trial, use values interpolated from the curve's descriptors.
13	List computed factor	Fill in *(9) for all measurements, regardless of magnitude of J. (9) = $\sqrt{1 + ((8) \times (4))}$.
14	List Q_{adj}	Fill in * (10) = (3) ÷ (9)
15	Next trial (step 5)	If both the Q and $1/US_c$ curves are unlikely to improve with further trials, proceed to step 16. For an additional trial, return to step 5.
16	Finalize	Prepare descriptors or tables for both curves.
17	Finalize	Recompute * (5) to (10) and compute (11) using the step 16 materials; (11) = 100 x ((10) − (5)) ÷ (5) If (11) values are satisfactory, proceed to step 18. Otherwise, return to step 5.
18	Test	See text and figure 19. If test is satisfactory, proceed to step 19. Otherwise, return to step 5 and adjust curve shapes as necessary.
19	Finalize	Prepare master curve sheet.

*Erase any entries or plotting from previous trials.

C COMPUTATIONS

Lightning Creek near McCune, Kansas

Meas. No.	Ght	Q_m	J	Q_r	Q_m/Q_r	Comp. $1/US_c$	Curve $1/US_c$	Factor	Q_m/Fact = Q_{adj}	%Diff.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
192	15.11	2880	0.04	2780	-	-	0.14	1.00	2872	3.3
205	9.38	1170	-0.54	1230	0.95	0.18	0.22	0.94	1246	1.3
206	17.15	14900	-0.08	14900	1.00	0.00	0.00	1.00	14900	0.0
208	8.44	1240	1.53	1040	1.19	0.28	0.24	1.17	1058	1.7
235	11.09	1450	-0.43	1610	0.90	0.44	0.41	0.91	1597	-0.8
242	15.91	3680	0.05	3770	-	-	0.00	1.00	3680	-2.4
243	16.96	12200	-0.04	12000	-	-	0.00	1.00	12200	1.7
277	15.61	3330	0.04	3280	-	-	0.00	1.00	3330	1.5
278	14.95	2560	-0.08	2680	0.96	1.09	0.40	0.98	2602	-2.9
289	13.82	2500	0.09	2350	1.06	1.46	1.18	1.05	2377	1.2
307	4.24	317	-0.30	344	0.92	0.50	0.57	0.91	348	1.2
319	10.49	1410	-0.30	1470	0.96	0.27	0.27	0.96	1471	0.0
328	3.70	280	0.04	276	-	-	0.51	1.01	277	0.4

SYMBOLS

Comp.	Computed value	Q_m	Measured discharge (ft3/s)
Curve	Value from curve	Q_r	Discharge from rating (ft3/s)
Ght	Gage height (ft)	S_c	Energy slope (ft/ft)
J=**dh/dt*	Rate of change in stage (ft/hr)	U	Velocity of flood wave (ft/s)
Q_{adj}	Adjusted discharge (ft3/s)	%Diff.	Variation of Q_{adj} from Q_r

***J* and *dh/dt* are both conventional symbols for rate of change in stage. *J* is more convenient to use in an equation, especially as part of the numerator or denominator of a fraction.**

FIGURE 18.—Continued.

adjustment has no apparent drawbacks with this rating and probably should be used even if no water sampling is involved.

Slope ratings

Some gaging stations, especially those on large regulated streams, are affected by variable backwater from dams almost all the time. Others, particularly those on flat gradient streams, are subject to occasional periods of backwater from downstream tributaries or from the return of overbank flow into the main channel after floods. Many such gages can be operated as slope stations by using a base gage to measure stage and an auxiliary gage some distance away to measure water-surface fall in the reach. The measured fall is an index of water-surface slope at the base gage.

The location of gages is a factor in determining the reliability of slope ratings, and, where there is a choice, several items should be considered. Both the base gage and the auxiliary gage should be stilling wells, or both should be bubble gages that compensate identically for temperature. The gages preferably should be far enough apart that minimum fall will exceed 0.5 ft, and there should be no significant tributaries or other sources of variable backwater between them. The base gage is best located at the discharge measuring section to eliminate storage adjustments. Where backwater is intermittent, the auxiliary gage should be downstream. This arrangement gives the most sensitive relation between fall and discharge and provides for positive identification of nonbackwater periods. Where backwater is always present or is caused by the return of overbank flow that has about the same magnitude upstream as it does downstream, an upstream auxiliary gage is about as good as one downstream.

Careful attention to the details of field operation (such as precise synchronization of base and auxiliary recorders, close datum control, and avoidance of current-meter measurements at velocities seriously below the limits of accurate meter registration) will improve the reliability of the lower parts of slope ratings.

Techniques that do not involve current meters can be used for low-water extensions of slope ratings at some sites. A power dam close to the gage may be a source of discharge information. Power production records usually include discharge figures, and, if all flow is through the turbines, as it generally is during low-flow periods, the discharge records during steady-flow periods may be used instead of discharge measurements. A dam downstream, where flow is cut off for long periods, may provide a reservoir that can be used as a container for volumetric measurements. The general storage equation (fig. 1) can be used to compute reservoir inflow if bank storage (underground) is not significant. Using records for other stations as a basis for extending a slope rating downward is usually a dubious practice. However, even that procedure may be more accurate than using current-meter measurements whose mean velocities are less than 0.10 ft/s.

Slope ratings fall into two broad categories: (1) constant-fall ratings in which unit fall is a special type and (2) variable-fall ratings. Unit-fall ratings are the simplest and require the fewest discharge measurements for adequate definition. Variable-fall ratings are the most complex, require more adjustments for close calibration to fit the data, and need more dischargemeasurements than the other types. the type of rating applicable to a particular site depends primarily on whether the backwater is intermittent or always present. Constant-fall ratings generally are preferred where backwater is present at all stages at all times, but they can be adapted, somewhat awkwardly, for use with intermittent backwater. Variable-fall ratings, preferable where backwater is intermittent, also can be used for full-time backwater sites but are difficult to define without free-fall discharge measurements.

Unit-fall ratings

A unit-fall rating is the relation between stage and the discharge when the fall in the reach is 1 ft. The rating is developed by plotting each measured discharge divided by the square root of its measured fall against the measurement's base gage height. The rating curve is then fitted to the plotted points. Discharge corresponding to any combination of

A. DAILY DISCHARGE COMPUTATION

May 25

HOUR	GAGE HEIGHT	SHIFT ADJ.	$\frac{dh}{dt}$	$\frac{1}{US_c}$		FACTOR	Q_r	DISCH.
0	3.55		-.10	.25		.98	255	250
1								
2								
3								
4	3.28		-.05	.10		1.00	220	220
5								
6								
7								
8	3.08							196
9								
10								
11								
12	2.95							178
13								
14	2.91							175
15								
16	2.92							176
17								
18	3.60		+1.50	.26		1.18	242	286
19								
20	8.90		+2.30	.22		1.23	1160	1430
21								
22	11.06		+.60	.40		1.11	1660	1840
23								
24	11.92		+.20	.46		1.04	1880	1960
Mean	4.67							520

May 26

HOUR	GAGE HEIGHT	SHIFT ADJ.	$\frac{dh}{dt}$	$\frac{1}{US_c}$		FACTOR	Q_r	DISCH.
0	11.92							1,960
1								
2	12.14		+.04	.52		1.01	1940	1960
3								
4	12.18		+.03	.49		1.01	1950	1970
5								
6	12.25		+.03	.50		1.01	1980	2000 ←
7								
8	12.29	←	0					1990
9								
10	12.23		-.05	.50		.99	1970	1950
11								
12	12.10		-.08	.48		.98	1930	1890
13								
14	11.80		-.15	.45		.96	1850	1780
15								
16	11.37		-.25	.41		.95	1740	1650
17								
18	10.65		-.35	.26		.95	1560	1480
19								
20	9.77		-.50	.26		.93	1340	1250
21								
22	8.65		-.48	.22		.95	1110	1050
23								
24	7.68		-.40	.22		.95	916	870
Mean	11.27							1700

B. HYDROGRAPH

C. LOOP RATING FOR RISE OF MAY 25–27

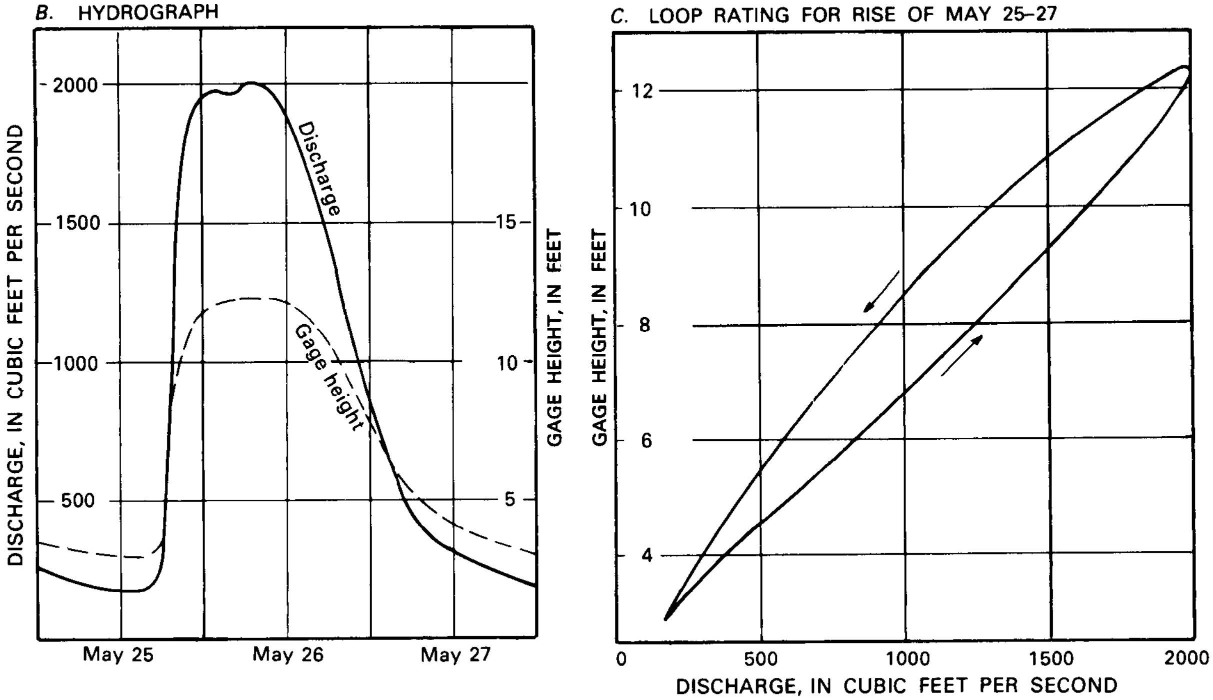


FIGURE 19.—Formats for testing a $1/US_c$ rating.

stage (base gage height) and fall can be computed by multiplying the discharge value corresponding to the stage by the square root of the fall. The rating applies without adjustment when the fall—and its square root—is 1.00. This type of rating usually is satisfactory where backwater is always present, fall is rarely below 0.5 ft, and the datum difference between base and auxiliary gages is known within about 0.05 ft. If these limits are exceeded, the unit-fall rating should be used only in the preliminary analysis for a more complex rating.

Figure 20*A* illustrates a unit-fall rating analysis for a site where backwater from a power dam is high at all times and stages. The same discharge measurement data were used to develop the constant-fall rating shown in figure 21. The measurement percentage differences from both analyses, listed in the last two columns of figure 20*A*, are not significantly different, an indication that the unit-fall rating is about as good as any that can be developed for this station, at least for falls greater than 0.5 ft.

Figure 20*B* illustrates a unit-fall rating analysis for a site where backwater is intermittent during floods and absent at low stages. The discharge measurement data listed also were used to develop the limiting-fall rating in figure 22*A*. The percentage differences in discharge measurements from both analyses, shown in figure 20*B*, are closely comparable. However, a factor other than the fit of the data to the rating must be considered in rating unit falls. The capacity of the channel to carry flow during backwater periods depends on the fall in the reach—the greater the fall, the greater the discharge. The carrying capacity during nonbackwater periods depends only on the geometry and roughness of the controlling reach. Fall in excess of the amount needed to assure the absence of backwater cannot indicate more discharge than the channel's capacity. Constant- or unit-fall ratings lack limiting criteria, and discharge computed by using this kind of rating during a nonbackwater period usually will be greater than the actual discharge. A limiter can be provided by using an auxiliary free-fall rating, a simple rating based only on the nonbackwater discharge measurements. The simple rating is used for a preliminary computation of records. Records for high-water periods when backwater is likely are then computed, by manual methods if only a few days are involved, as figure 20*C* illustrates. The smaller of the two discharge figures for the free-fall rating and the slope rating is accepted as the true value. This combination of free-fall rating and unit-fall auxiliary slope rating would probably be the best rating choice for the site used for the illustration if only a few discharge measurements indicating backwater had been made.

Constant-fall ratings

A constant-fall rating uses two curves: (1) the relation between stage and the discharge when the fall in the reach is some specified value, usually about 1 ft, and (2) a factor curve of fall (F_m) versus discharge ratio (Q_m/Q_r). The symbols used are defined in figure 21*C*. This rating type is similar to a unit-fall rating except that the factor curve replaces the square root relation ($Q_m/Q_r=\sqrt{F_m}$). A unique feature of the constant-fall rating is that the base gages and the auxiliary gages need not be at or adjusted to the same datum. A factor curve showing the relation of gage difference (base *Ght* less auxiliary *Ght*) to discharge ratio (Q_m/Q_r) can be used about as well as the ratio of fall to discharge. Figure 21*A* illustrates a constant-fall rating for a gaging station where backwater from a dam is always present and where slopes are highly variable owing to rapid fluctuation of discharge. The rating analysis computations in figure 21 indicate that instantaneous discharges from the rating are reliable above about 10,000 ft^3/s and satisfactory down to about 5,000 ft^3/s. Daily values are probably reliable at somewhat lower discharges. The factor curve would be close to a square-root relation of factor versus fall if the auxiliary gage datum were raised 0.03 ft. If that datum change is made, the constant-fall rating would be very close to the unit-fall rating in figure 20*A*.

Most constant-fall ratings are developed by drawing a unit-fall rating as a trial curve and using that trial rating to compute a factor (discharge ratio versus fall) curve. The factor curve is then used to improve the rating, which in turn is used to refine the factor curve. The process is continued until consequential improvement stops, usually after about three trials. The analysis can be done by using steps similar to those listed in figure 21*B*. This procedure gives a discharge curve that corresponds to a constant fall of about 1 ft. Some hydrographers prefer a discharge curve whose values approximate actual discharge during floods. Such a curve can sometimes be obtained by using a value closer to the average observed fall as the constant-fall value. If a constant-fall value other than 1.0 is wanted, the figure 21*B* procedure (step 3) provides for the conversion.

Limiting-fall ratings

A gaging station affected by intermittent backwater from tributaries or a dam may be operated for long periods as a simple rating station but needs a slope rating for some or all of the high-water periods. This type of station works best with a limiting-fall rating composed of three parts: (1) a discharge curve that represents a simple rating applicable for nonbackwater conditions and indicates the maximum possible discharge at any stage regardless of fall, (2) a fall curve that varies with stage and indicates the minimum fall in the slope reach under nonbackwater conditions, and (3) a factor curve of the relation Q_m/Q_r (ratio of measured discharge to rating discharge) versus F_m/F_r (ratio of measured fall to rating fall).

Figure 22*A* illustrates a typical limiting-fall rating for a site where backwater is intermittent. The flat-slope channel has a low-water section control, a high-water rating storage loop, and variable backwater from tributaries.

The three-curve rating analysis is much more complex than the two-curve types shown in previous examples. A limiting-fall slope rating has three interrelated component curves (discharge, fall, and factor). When two of the three components corresponding to each discharge measurement are fixed, the magnitude of the third needed to cause a perfect fit for that discharge measurement can be computed. The discharge and factor curves are tentatively drawn and "fixed" as the first step. The value of the "perfect-fit" fall for each discharge measurement is then computed and used as a plotting point to define the fall curve. Each curve is then refined in rotation by fixing the other two curves and using the perfect-fit points defined by the discharge measurements to draw or improve the unfixed or open curve.

Usually, after each curve has been refined about three times in this manner, further improvement is minimal. The computations can be made manually in steps similar to those listed in figure 22*B*, or the trial-and-error work can be facilitated by using an appropriate computer facility and the instructions contained in the WATSTORE User's Guide (Hutchison and others, 1975, 1980).

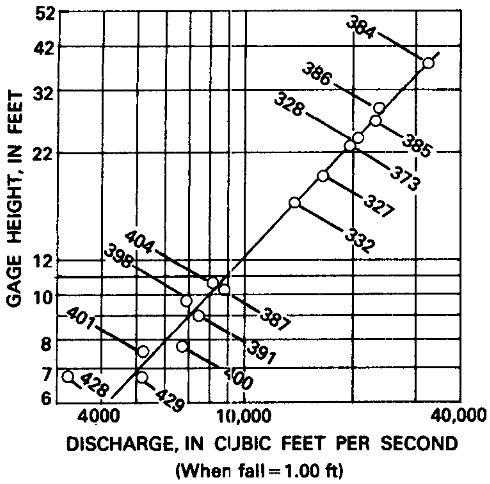
Normal-fall ratings

A normal-fall slope rating is identical to a limiting-fall rating except that the factor curve extends above the coordinates (1,1). Observed fall greater than the normal fall curve value indicates that actual discharge is greater than the discharge curve value instead of equal to it, as it would be for a limiting-fall rating. Normal-fall ratings are used sometimes where high-water measurements fail to indicate a limiting position for the discharge curve. Most such ratings are developed as limiting-fall types below a specified stage and as normal fall above. They also have some application to full-time backwater sites, where the three-component curves provide more opportunity than a two-curve constant-fall rating to achieve agreement between the discharge measurements and the rating. Three-component curves can be a disadvantage, however, because it is possible to warp the rating inadvertently into agreement with faulty data.

The analysis procedure is identical to the limiting-fall method outlined in figure 22*B* except that, in step 1, no dashes are inserted in the computation columns for high-fall measurements, and the discharge curve need not be drawn to the right of the measurement scatter. An example of a normal-fall analysis is not given because of its similarity to the much more common limiting-fall analysis.

Cumberland River at Carthage, Tennessee

A. RATING CURVE

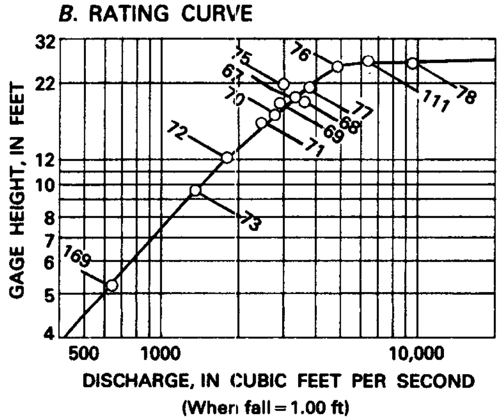


COMPUTATIONS

Meas. No.	Gage Height	F_m	Q_m	$\frac{Q_m}{\sqrt{F_m}}$	Q_r	%Diff. Unit Fall	%Diff. Const. Fall
327	19.38	6.29	41,000	16,300	16,500	- 1.2	- .6
328	23.31	7.16	53,800	20,100	20,400	- 1.5	+ .5
332	16.49	5.24	31,400	13.700	13,900	- 1.4	- .7
373	23.01	7.30	52,700	19,500	19,700	- 1.0	- .5
384	37.92	9.45	99,800	32,500	32,700	- .6	0
385	26.50	6.30	57,900	23,000	22,800	+ .9	+ 1.3
386	28.34	8.70	70,300	23,800	24,400	- 1.6	- 1.2
387	10.30	2.65	14,100	8,660	8,220	+ 5.4	+ .6
391	9.04	2.30	11,200	7,390	7,040	+ 5.0	- 1.0
398	9.72	2.02	10,200	7,180	7,680	- 6.5	-11.5
400	7.74	.67	5,520	6,740	5,810	+16.0	+12.9
401	7.50	.95	5,130	5,260	5,580	- 5.7	- 9.9
404	10.52	3.04	14,500	8,320	8,420	- 1.2	- 5.4
428	6.68	.19	1,410	3,230	4,790	-32.6	-22.5
429	6.60	.20	2,330	5,210	4,710	+10.6	-24.0

Cottonwood River near Florence, Kansas

B. RATING CURVE



COMPUTATIONS

Meas. No.	Gage Height	F_m	Q_m	$\frac{Q_m}{\sqrt{F_m}}$	Q_r	%Diff. Unit Fall	%Diff. Lim. Fall
67	19.54	5.13	7,570	3,340	3,280	+ 1.8	+ 1.4
68	19.38	3.91	6,920	3,500	3,250	+ 7.7	- 1.1
69	18.42	3.28	5,430	3,000	3,050	- 1.6	0
70	17.21	2.84	4,740	2,810	2,800	+ .4	+ 2.3
71	15.96	2.68	4,210	2,570	2,550	+ .8	+ 2.3
72	12.22	1.95	2,490	1,780	1,800	- 1.1	- 1.8
73	9.56	1.59	1,720	1,360	1,360	0	- 2.8
75	21.52	6.67	7,670	2,970	3,700	-19.7	-10.4
76	25.28	8.08	13,800	4,850	4,850	0	+ 5.3
77	21.37	5.15	8,230	3,630	3,670	- 1.1	- 1.1
78	26.50	8.14	27,100	9,500	9,480	+ .2	+ 4.2
111	26.25	8.12	18,500	6,490	6,490	0	0
169	5.20	1.79	839	627	630	- .5	- .7

SYMBOLS

F_m Measured fall (ft)
Q_{adj} Adjusted discharge (ft³/s)
Q_m Measured discharge (ft³/s)
Q_r Discharge from rating (ft³/s)

FIGURE 20.—Typical unit-fall slope ratings.

Ratings for regulating control structures

Dams can be used as gaging-station sites by rating the fixed spillways, gates, turbines, and locks separately. The procedures, explained and illustrated by Collins (1977), have little in common with those described in this manual.

Index-velocity ratings

An index-velocity gaging station generally is used where backwater is variable, particularly from tide, and the water-surface slope is too flat for a slope rating. The equipment consists of a stage recorder and a device that records an indicator of the stream velocity. Stage and index velocity are correlated with discharge in several ways that depend on the type and placement of the equipment. Deflection vanes are used as velocity sensors on most of the older index-velocity stations. Most new installations use electromagnetic meter probes permanently mounted at the index location. An acoustic velocity meter that records the average stream velocity along a line between two underwater transducers mounted diagonally

MERGING OF DISCHARGE VALUES FROM NON-LIMITING SLOPE RATING AND SIMPLE RATING AT END OF BACKWATER PERIOD

HOUR	GAGE HEIGHT	F_m	Q_r	$Q_{adj.}$	Q_r	DISCH.
		Slope Rating			Non-BW Rating	
0	18.61	5.16	3090	7020	7100	7100
1						
2	19.21	5.29	3220	7410	7400	7400
3						
4	19.69	5.36	3320	7690	7640	7640
5						
6	20.05	5.40	3390	7880	7820	7820
7						
8	20.27	5.42	3440	8010	7940	7940
9						
10	20.48	5.42	3480	8100	8040	8040
11						
12	20.55	5.41	3500	8140	8080	8080
13	20.56	5.36	3500	8100	8080	8080
14	20.53	5.30	3490	8030	8060	8030
15						
16	20.43	5.16	3470	7880	8020	7880
17						
18	20.27	5.01	3440	7700	7940	7700
19						
20	20.05	4.83	3390	7050	7820	7050
21						
22	19.71	4.59	3320	7110	7660	7110
23						
24	19.11	4.29	3200	6630	7360	6630
Mean	20.01					7660

FIGURE 20.—Continued.

across the stream from one another can be used, usually at a deep river site that requires instant onsite computation of highly accurate discharge records.

Electromagnetic meters and acoustic meters measure the index velocity directly in feet per second. Deflection-gage readings can be recorded in degrees of rotation but are usually graduated in nonlinear arbitrary units, which complicate the rating analysis considerably. Figure 23 illustrates a relation of index velocity to mean velocity in the cross section. The relation varies considerably with stage. The family of curves shown is typical for magnetic meters or acoustic gages whose sensors are high enough above the streambed to be in a live (stagnant only at zero flow) part of the cross section. Lower sensors would place the index location in a less stable part of the vertical velocity curve, and the family of curves would be less likely to be made up of nearly straight lines. The equivalent curves for a deflection vane would have complex S shapes that are particularly difficult to define.

A curve showing stage versus area represents the total of relatively dead and relatively live parts of a cross section. Cross-section scour or fill in a relatively dead area has little effect on the relation of stage and index velocity to discharge, whereas a similar change within a live area has a large effect on the rating. A change in the total area may or may not indicate a consequential rating change.

An index-velocity rating is composed of from one to three curves. One-curve ratings (stage versus effective area) can be used for most acoustic-velocity-meter installations and for some magnetic-meter stations where the discharge is directly proportional to the index velocity. Two-curve ratings (stage versus coefficient and index velocity versus adjusted discharge) can be used at all index-velocity stations. Three-curve ratings (stage versus coefficient, stage versus area, and index velocity versus adjusted mean velocity), generally more complex and less reliable than two-curve ratings, can be used where there is some special need to derive and maintain a curve showing stage versus total area.

Most of the rating relations can be expressed as equations by using the procedures shown in figure 10. Equations are the only means of entering ratings in some acoustic-meter processors and greatly simplify the use of calculators and computers in all index-velocity rating computations.

Some index-velocity ratings used on canals or estuaries apply to both upstream and downstream flow but most require separate ratings. Vane-gage and magnetic-meter ratings are most reliable in trapezoidal channels where the velocities are reasonably well distributed throughout the cross section and where the velocity sensor is located as high as the stage range allows in a live area free from obstruction. These ratings are least satisfactory where the velocity sensor is isolated from the main channel or wherever there is a combination of a wide channel and low velocities during a period of high winds. Rating problems also can be caused by a sensor that is inaccessible for regular cleaning or that is located where it can be bumped by debris or river traffic. Acoustic velocity meters are less sensitive to these conditions but may malfunction because of unusually high sediment concentration or air entrainment. A channel that is too large or complex

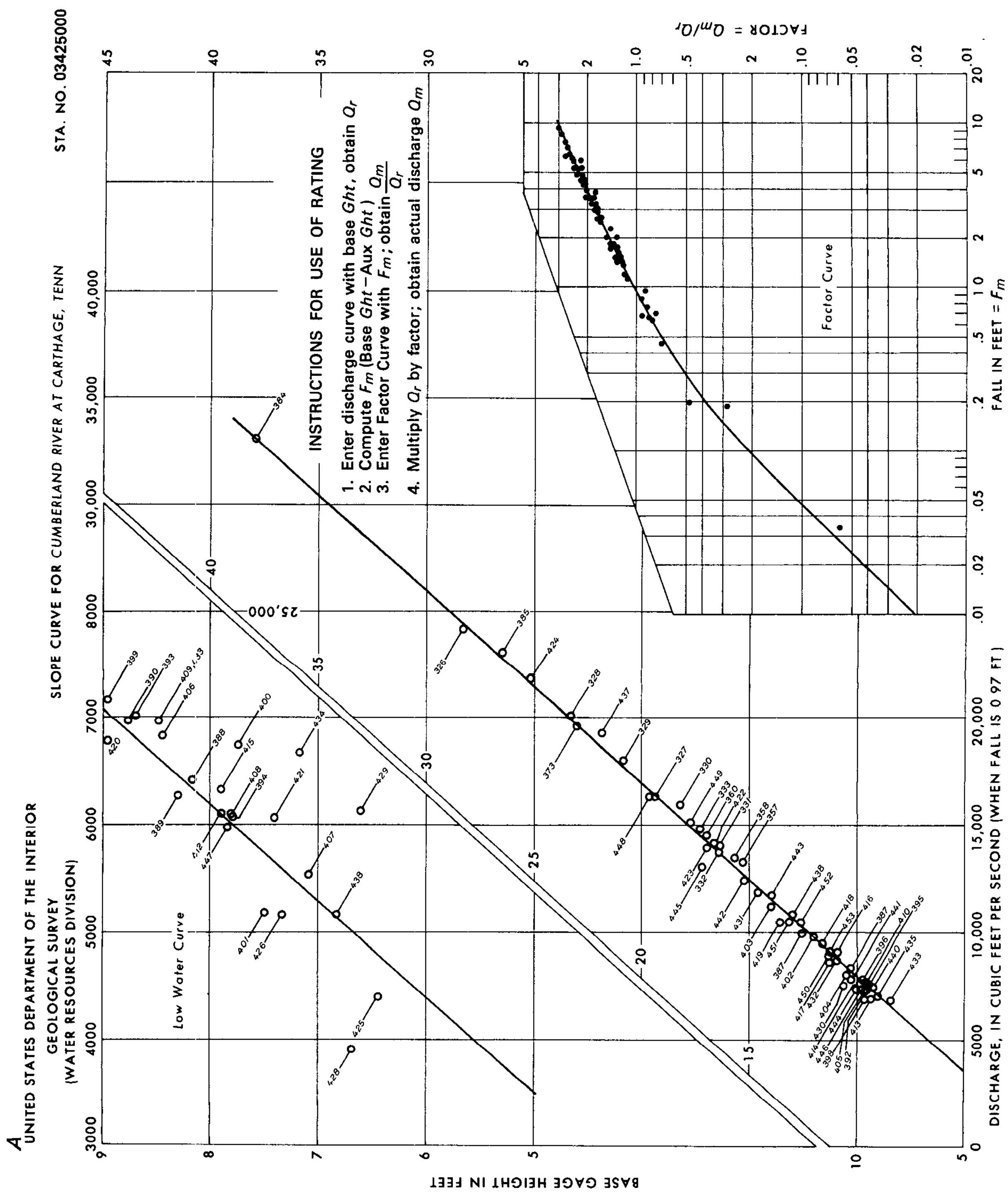


FIGURE 21.—Typical constant-fall slope rating.

B

ANALYSIS PROCEDURE

STEP OPERATION	INSTRUCTIONS
1 Prepare work sheets	Needed: Sheets 1 and 2, log-log rating grids for trial discharge curves; sheet 3, a log-log grid for the factor curve; and a computation sheet with columns titled and numbered ① to ⑩ as in the example below (a column ⑪ is needed for the first trial only).
2 Enter data	Fill in ① to ⑤ with data from the discharge measurements.
3 Compute unit-fall discharge	Fill in ⑪ = ⑤ ÷ $\sqrt{④}$ (for a value of contstant fall, *n*, other than 1 foot, ⑪ = (⑤ ÷ $\sqrt{④}$) ÷ $\sqrt{n}$).
4 Draw preliminary discharge curve	Plot on sheet 1, ⑪ vs. ② . Flag all points whose ④ < 1.0. Draw the curve, using an appropriate *Ght* scale offset, giving the least weight to flagged points.
5 Fill in Q_r	Fill in * ⑥ from the sheet 1 curve (first trial), sheet 2 curve (intermediate trials), or the curve descriptors for the final trial.
6 Draw factor curve	Fill in * ⑦ (*Y*axis) vs. ④ on sheet *3. Draw the curve, giving equal weight to all points. Preferred final curve format is an equation (figure 10*B* or 10*C*).
7 Fill in factor	Fill in * ⑧ from sheet 3 curve or equation.
8 Fill in Q_{adj}	Fill in * ⑨ = ⑤ ÷ ⑧ .
9 Draw trial discharge curve	Plot, with the step 4 *Ght* scale offset, ⑨ vs. ② on sheet 2. Draw the curve, giving equal weight to all but very low velocity discharge measurements (low ④). Preferred final curve format is a set of log descriptors (figure 7).
10 Go to step 5	Repeat steps 5 to 9 about three times or until further improvement is unlikely. Then proceed to step 11.
11 Finalize	Prepare the curves in final format (descriptors, equation, or tables).
12 Finalize	Recompute * ⑥ to ⑨ and compute ⑩ with step 11 materials. ⑩ = 100 (⑨ − ⑥) ÷ ⑥ . If ⑩ values are unsatisfactory, return to step 5 giving special attention to outliers. Otherwise, proceed to step 13.
13 Finalize	Prepare the master curve sheet.

*Erase any entries or plotting from previous trials.

C

COMPUTATIONS
Cumberland River at Carthage, Tennessee

①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪
Meas. No.	Gage Height		F_m	Q_m	Q_r	$\frac{Q_m}{Q_r}$	Factor (Table)	$Q_{adj} = \frac{Q_m}{\text{Factor}}$	%Diff.	1st Trial Q_r $\frac{Q_m}{\sqrt{F_m}} \div \sqrt{n}$
	Base	Aux								
327	19.38	13.09	6.29	41000	16400	2.500	2.508	16300	− .6	16300
328	23.31	16.15	7.16	53800	20000	2.690	2.674	20100	+ .5	20100
332	16.49	11.25	5.24	31400	13800	2.275	2.298	13700	− .7	13700
373	23.01	15.71	7.30	52700	19700	2.675	2.695	19600	− .5	19500
384	37.92	28.47	9.45	99800	33100	2.015	3.018	33100	0	32500
385	26.50	20.20	6.30	57900	22800	2.539	2.510	23100	+ 1.3	23100
386	28.34	19.64	8.70	70300	24500	2.869	2.905	24200	− 1.2	23800
387	10.30	7.65	2.65	14100	8270	1.705	1.695	8320	+ .6	8660
391	9.04	6.74	2.30	11200	7140	1.569	1.585	7070	− 1.0	7390
398	9.72	7.70	2.02	10200	7750	1.316	1.487	6860	−11.5	7180
400	7.74	7.07	.67	5520	5970	.925	.819	6740	+12.9	6740
401	7.50	6.55	.95	5130	5750	.892	.990	5180	− 9.9	5260
404	10.52	7.48	3.04	14500	8470	1.712	1.810	8010	− 5.4	8320

SYMBOLS

F_m Measured fall

Q_{adj} Discharge adjusted to rating fall

Q_m Measured discharge

Q_r Discharge from rating curve

%Diff. Variation of Q_{adj} from Q_r

ⓝ Column number on computation sheet

n Constant fall other than 1.00

FIGURE 21.—Continued.

or whose velocity distribution is too variable to rate with one velocity sensor can be subdivided with a separate sensor and rating for each subarea.

Vane-gage ratings

A vane gage is a mechanical velocity sensor whose components are usually arranged approximately as they are in figure 24*A*. This type of vertical axis vane is deflected by the force of the current acting against the torque from a counterweight. The linkage from the counterweight to the vane varies the resisting torque from zero at zero velocity to a maximum at about 45° deflection. A cam or some other device can vary the torque further at higher deflections. Some vanes have springs rather than counterweights, and others have horizontal axes where the weight of the pendulum vane furnishes the resistance to deflection. The recorder linkage can be arranged so that deflection is recorded in degrees or a multiple of degrees, but counterweight movement is usually recorded. Most velocity-sensor scales are in arbitrary nonlinear units, and the scale often is offset so that zero velocity gives a scale reading of 1, 5, or 10.

The rating analysis method, outlined in detail in figure 24*H*, is a trial-and-error procedure. The relation between vane deflection and discharge is a family of curves, one for each stage, that are parallel to each other on a logarithmic grid. The family of curves is roughly defined by the discharge measurements, as figure 24*C* shows. The best-defined single curve from the family (the 2.5-ft stage curve in fig. 24*C*) is used as the first trial curve for the base-stage rating (fig. 24*D*). The ratio of measured discharge to discharge from the trial base-stage rating (fig. 24*G*) defines a stage-coefficient curve. Each measured discharge is divided by its stage coefficient and used to refine the base-stage rating. The refined rating is then used to improve the stage-coefficient curve and vice versa until, usually after about three trials, further improvement is unlikely.

The base-stage rating curve is best described by logarithmic digital descriptors (fig. 24*E*). The gage-height coefficient curve can be described by an equation using the procedure shown in figure 10*B*. The entire rating is described by the equation in figure 24*F*, which combines the base-stage rating and the stage-coefficient relation.

If subsequent discharge measurements indicate that a rating shift has occurred, the measurements should be used to redefine the base rating curve and to obtain a different set of descriptors. If a temporary condition, such as aqueous growth on the vane, causes the rating to change, shift adjustments varied with time only can be applied to the coefficient a_0 (in this rating, a_0=0.5725). For instance, if a shift to measurement 635 (fig 24*B*) is considered necessary, its amount is ⑥ − ⑦ (see symbols in fig. 24*B*) or 0.86 − 0.93 = −0.07. This shift would modify the rating equation applicable to measurement 635 (G=2.03, V_g=0.60, shift is −0.07, and Q_b=293) to

$$Q_r=293[(0.5725-0.07)+(0.187\times 2.03)-(0.0047\times 2.03^2)]=253\ \text{ft}^3/\text{s}$$

This shifted value changes the percentage difference for measurement 635 from −7.3 to 0, and the daily discharge computation would be changed accordingly.

Few vane-gage ratings are likely to approach the quality of the one illustrated in figure 24. The equipment is a well-designed, well-constructed Keeler deflection meter. The channel is a wooden flume 48 ft wide, and the freshwater site is free from the common, serious problems of channel shifting and heavy aqueous growth on the vane. The good equipment and conditions plus the unusually wide distribution of the discharge measurements result in an exceptionally reliable rating for a vane gage.

Magnetic-meter gage ratings

Electromagnetic meter equipment is usually arranged approximately as figure 25*A* shows. The velocity sensor, the probe of the magnetic meter, is usually attached to the end of a pipe, which generally is part of a frame that permits the probe to extend into an unobstructed area within the live part of the cross section. A typical frame is designed to permit easy removal of the probe for periodic cleaning and to facilitate its replacement in precisely the original location. Minor probe movement or rotation is likely to affect the rating. The rating analysis

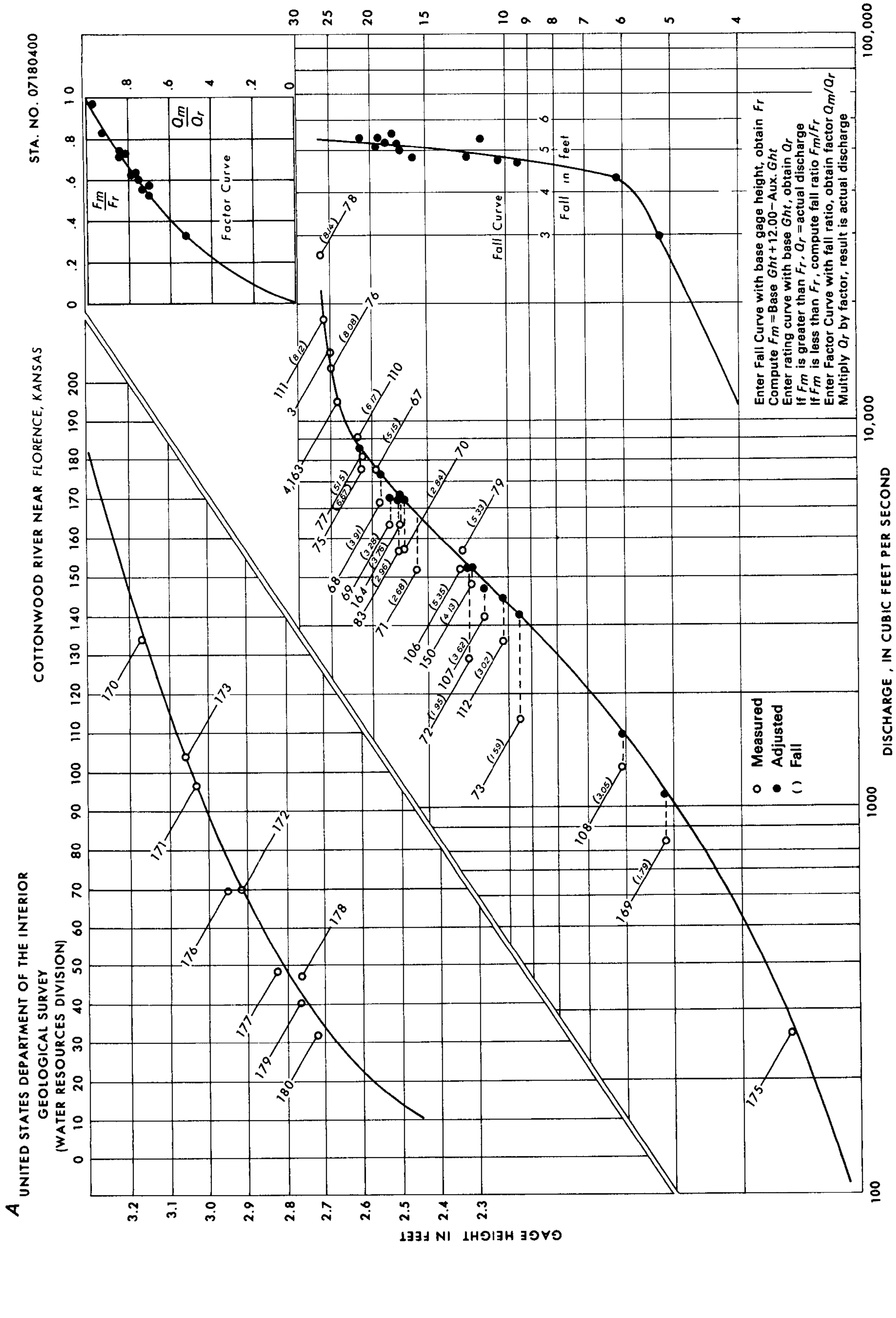


FIGURE 22.—Typical limiting-fall slope rating.

B

ANALYSIS PROCEDURE

STEP	OPERATION	INSTRUCTIONS
1	Prepare work sheets	Needed: Sheets 1 and 2, log-log rating grids for trial discharge curves; sheet 3, a rectangular grid for trial fall curves (*Fm* along the *X* axis); sheet 4, log-log grid for trial factor curves (*Fm/Fr* along the *X* axis); and a computation sheet with the columns titled and numbered from 1 to 14 as in the example (fig. 22*C*).
2	Enter data	Fill in 1 to 5 with data from all discharge measurements made at stages above the low-water section control range. Enter dashes in 7 , 9 , 10 , 13 and 14 for each nonbackwater measurement.
3	Draw preliminary discharge curve	Plot 2 vs. 5 on sheet 1. Choose *Ght* scale offset to straighten the lower end of the curve. Draw the curve close to all nonbackwater measurements and to the right of the backwater measurement scatter.
4	Fill in *Qr*	Fill in 6 from the sheet 1 curve, which can then be discarded.
5	Fill in *Qm/Qr*	Fill in 7 = 5 ÷ 6 .
6	Fill in *Fm/Fr*	For this first approximation, 13 = 7^2 .
7	Fill in F_{adj}	Fill in * 14 = 4 ÷ 13 .
8	Plot fall curve	Plot 14 vs. 2 on sheet *3. Draw the curve, which is usually parabolic. *Fr* is 0 at *GZF* and is usually maximum at the maximum stage. The preferabale final format is a set of descriptors for linear interpolation.
9	Fill in *Fr, Fm/Fr*	Fill in * 8 from the sheet 3 curve or its descriptors and * 9 = 4 ÷ 8 .
10	Plot factor curve	Plot 9 vs. 7 on sheet *4. Draw the curve, which should approximate 9 = 7^2 at its upper end. The preferable final curve format is an equation (figure 10*B* or 10*C*)
11	Fill in factor	Fill in * 10 from the sheet 4 curve or its equation.
12	Fill in Q_{adj}	Fill in * 11 = 5 ÷ 10 .
13	Plot discharge curve	Plot 11 vs. 2 on sheet *2. Draw the curve, giving equal weight to all points except those for very low fall mesurements (low 4). Use step 3 *Ght* scale offset. The preferable final format is a set of log curve descriptors (fig. 7).
14	Fill in *Qr, Qm/Qr*	Fill in * 6 from the sheet 2 curve or its descriptors and * 7 = 5 ÷ 6 .
15	Fill in *Fm/Fr*	Fill in * 13 from the sheet 4 curve or equation. Enter the curve with 7 to obtain 13
16	Go to step 7	Repeat steps 7–15 until improvement stops, then proceed to step 17.
17	Finalize	Prepare all curves in final format (descriptors, equations, or tables).
18	Finalize	Recompute * 6 to 14 from step 17 material. 12 =100(11 − 6)− 6 . If 12 values are unsatisfactory, return to step 7, giving special attention to outliers. Otherwise proceed to step 19.
19	Finalize	Prepare the master curve sheet.

*Erase any entries or plotting from previous trials.

FIGURE 22.—Continued.

procedure, outlined in detail in figure 25*H*, is almost identical to that for a vane gage. However, the direct recording of index velocity in feet per second removes most of the nonlinearity from the relations, reduces the number of trial-and-error steps needed, and makes a reliable rating possible from a limited number of discharge measurements.

The discharge measurements are plotted (fig. 25*C*) in the same manner as those of a vane gage. For a magnetic meter, the family of curves for index velocity versus discharge is likely to be a series of parallel straight lines on a logarithmic grid. One curve from the family is selected as a base curve, and its corresponding stage is the base stage. A stage-coefficient curve (fig. 25*E*) is defined by the ratio of each measured discharge to the discharge from the base-stage rating plotted against stage. The coefficient curve is used to adjust the discharge measurements to the base stage (fig. 25*D*). The base stage rating and the coefficient curve are each used to refine the other until the rating is satisfactory. Both curves can be put into equation form by using the methods outlined in figure 10, and the end product can be a relatively simple equation (fig. 25*G*).

If a temporary condition, such as debris on the probe, causes the rating to shift, adjustments that are varied with time only can be applied to the coefficient a_0 of the stage-coefficient equation (in the rating illustrated, a_0=0.387). For instance, if a shift to measurement 11 was justified, its amount would be 7 − 8 (see symbols in fig. 25*C*) or

C

COMPUTATIONS

Cottonwood River near Florence, Kansas

Meas. No.	Gage Height		F_m	Q_m	Q_r	Q_m/Q_r	F_r	F_m/F_r	Factor from curve	Q_m/Fact = Q_{adj}	% Diff.	F_m/F_r from curve	$\frac{F_m}{F_m/F_r} = F_{adj}$
	Base	Aux.*											
1	2	3	4	5	6	7	8	9	10	11	12	13	14
67	19.54	26.39	5.15	7570	7530	1.01	5.28	0.97	0.99	7680	2.0	1.00	5.15
68	19.38	27.47	3.91	6290	7450	0.84	5.28	0.74	0.85	7410	-0.5	0.73	5.33
69	18.42	27.14	3.28	5430	6980	0.78	5.25	0.63	0.77	7020	0.6	0.63	5.20
70	17.21	26.37	2.84	4740	6400	0.74	5.21	0.55	0.72	6590	3.0	0.58	4.92
71	15.96	25.28	2.68	4210	5840	0.72	5.16	0.52	0.70	6020	3.1	0.55	4.88
72	12.22	22.27	1.95	2490	4210	0.59	5.01	0.39	0.60	4170	-1.0	0.38	5.10
73	9.56	19.97	1.59	1720	3080	0.56	4.77	0.33	0.55	3130	1.6	0.34	4.62
75	21.52	26.85	6.67	7670	8540	--	5.34	--	--	7670	-10.2	--	
76	25.28	29.20	8.08	13800	13400	--	5.40	--	--	13800	3.0	--	
77	21.37	28.22	5.15	8230	8460	0.97	5.33	0.97	0.98	8390	-0.8	0.95	5.42
78	26.50	30.36	8.14	27100	26000	--	5.40	--	--	27100	4.2	--	
79	12.69	19.36	5.33	4720	4420	--	5.04	--	--	4720	6.8	--	
83	17.77	26.81	2.96	4710	6670	0.71	5.22	0.57	0.73	6420	-3.7	0.53	5.61
105	4.96	9.75	7.21	907	939	--	2.29	--	--	907	-3.4	--	
106	12.78	19.43	5.35	4240	4460	--	5.04	--	--	4240	-4.9	--	
107	11.44	19.82	3.62	3170	3880	0.82	4.95	0.73	0.84	3760	-3.1	0.69	5.25
108	6.10	15.05	3.05	1300	1550	0.84	4.20	0.73	0.84	1550	0.0	0.72	4.21
110	21.95	27.78	6.17	9160	8770	--	5.35	--	--	9160	4.4	--	
111	26.25	30.13	8.12	18500	19400	--	5.40	--	--	18500	-4.6	--	
112	10.36	19.34	3.02	2760	3420	0.81	4.85	0.62	0.77	3570	4.4	0.68	4.47
150	12.13	20.00	4.13	3880	4180	0.93	5.01	0.82	0.90	4310	3.1	0.87	4.74
164	17.50	25.74	3.76	5500	6540	0.84	5.22	0.72	0.84	6570	0.5	0.73	5.16
165	5.96	12.37	5.59	1520	1480	--	4.06	--	--	1520	2.7	--	
169	5.20	15.41	1.79	839	1060	0.79	2.73	0.66	0.80	1060	0.0	0.65	2.75

* Datum 12 ft lower than base gage datum

SYMBOLS

F_{adj} Adjusted fall
F_m Measured fall
F_r Fall from rating
Q_{adj} Adjusted discharge
Q_m Measured discharge
Q_r Discharge from rating
%Diff. Variation of Q_{adj} from Q_r
(n) Column number on computation sheet

FIGURE 22.—Continued.

$0.94-0.89=+0.05$. This shift would modify the rating equation (G=3.81, V_g=5.02, shift is +0.05) to

$$Q_r=[5511(5.02-1)^{0.832}][(0.133\times3.81)+(0.387+0.05)]=16{,}500$$

This shifted value would change the percentage difference for measurement 11 from +5.1 percent to 0, and the same degree of adjustment would be applied to the computed daily discharge.

The rating illustrated looks very good, considering that a single sensor was used in a channel more than 400 ft wide where tidal backwater was present. However, this rating gives erratic instantaneous discharge figures when flow is less than about 2,000 ft^3/s and the wind is strong. The faulty record might be eliminated by using additional velocity sensors.

Acoustic-velocity meter gage ratings

The equipment for a typical single-path version of an acoustic-velocity-meter (AVM) gaging station, described in detail by Smith and others (1971), is laid out as figure 26A illustrates. An acoustic signal consisting of a short

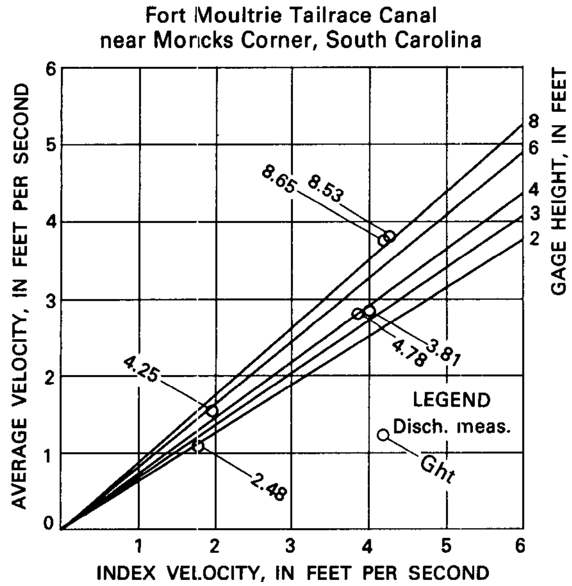


FIGURE 23.—Family of curves relating index velocity to average velocity in the cross section.

burst of energy is transmitted from point A to point B; then, either simultaneously or immediately after, another signal is transmitted from point B back to point A. The time differential between the two transmissions is proportional to the velocity of the water, which has increased the speed of the signal in one direction and decreased it in the other. The "true" velocity of sound in water is computed from the average of the two transmission-reception times. This information, along with the distance A–B and the angle θ, permits computation of the index velocity in feet per second. The index-velocity value is the average velocity of the stream parallel to the banks in the horizontal plane of the diagonal line between A and B. The maximum length of the acoustic path for reliable operation is limited by stream depth and other factors such as maximum sediment concentration and air entrainment.

The type of rating most applicable to an AVM gaging station depends on the channel size and shape and the nature of the flow. A nontidal deep river may need only one acoustic path and may have a relatively simple rating. The rating can be more complex if the acoustic path spans only part of the channel. If the channel contains a stratified mix of saltwater and freshwater at times and has periods of upstream flow, the site may require multiple acoustic paths and a very complex rating. In any event, the rating must be compatible with the program built into the processor by the equipment manufacturer. The two-curve rating analysis, illustrated in figures 24 and 25, can be modified to suit the other types of equipment used at most AVM sites.

The simplest analysis, a one-curve rating, is illustrated in figure 26*A* and can be used only where conditions approach the ideal, as they did at the site used for this illustration. The equipment, a single-path installation, is laid out as figure 26*A* shows. The stream is 80 ft deep at low water, and its stage range is only 15 ft. There is no overbank flow, and reliable discharge measurements are made from a specially designed boat. The acoustic path is located in the upper, relatively straight part of the vertical velocity profile, and the discharge at a given stage is directly proportional to the index velocity.

The one curve used is effective area (measured discharge divided by the index velocity) versus stage. Each measured discharge is divided by its index velocity and plotted against the stage (fig. 26*B*). This relation is fitted to a parabolic curve by using the procedure shown in figure 10*B*. Discharge is computed by using the equation in figure 26*D*. The analysis steps for this type of rating are listed in figure 26*E*, and the computations are tabulated in figure 26*C*. The percentage differences are impressively small.

The rating actually used at The Dalles site is almost the same as the one illustrated except that two curves (stage versus actual area and stage versus stage coefficient) are used. The product of the area and the coefficient is the effective area, which is multiplied by the index velocity to compute the discharge. The rating has not changed during 12 years of AVM operation.

The coefficient a_0 (fig. 26*D*) is $-596{,}500$. This coefficient can be varied if necessary and used as a shift adjustment. For instance, if the variance of measurement 309 (fig. 26*C*) had been due to a channel change and confirmed by subsequent measurements, the rating could have been shifted to fit the measurements by using $-572{,}000$ (C_c) for a_0 in the rating equation.

Rating analysis by computer

Minicomputers and desktop programmable calculators that have adequate storage and peripheral equipment, which may include a printer, a plotter, diskette storage, a CRT viewer, and a digital-tape translator, are used in some field offices to process the daily records locally. The programs that fit the available equipment are complex and often include a discharge rating analysis.

The minicomputer or calculator can be programmed to store all the discharge measurements that were made at a gaging station and to select and plot the relevant ones so that the hydrographer can draw the rating curve and select its descriptors. The computer then tabulates the rating data and computations and prints the rating tables. Table 3 illustrates a computation printed from a typical semiautomatic rating analysis program.

Completely automatic rating analysis using the curve-fitting programs available for each calculator or computer is technically practical but is emphatically discouraged for stage-discharge relations. The programs use a least-squares fitting technique. However, the fitting is done without benefit of human judgment as to the quality of individual measurements, especially outliers, and the hydraulic factors

TABLE 3.

RATING ANALYSIS,PLOT,AND TABULATION

JACK DANIEL SPRING AT LYNCHBERG, TENN.
03580990

MEASUREMENTS USED
ALL AFTER 40170 AND BEFORE 122570

RATING COORDINATES

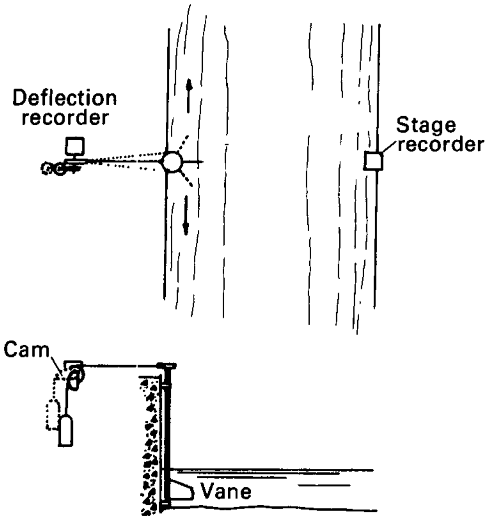
GHT	1.07	1.30	1.40	1.85	1.90
DISC	0.006	0.65	1.4	13.5	15
GHT	2.00	2.30	2.40	2.50	2.70
DISC	17	21.5	23.5	28	41
GHT	2.90	3.00			
DISC	69	84			

GHT SCALE OFFSET=1.04

MST NO	DATE	GHT	DISC	RATING Q	%DIFF	SHIFT
1	42670	2.70	39.5	41.0	-3.8	-0.02
2	42670	2.86	61.9	62.5	-1.0	0.00
3	42670	2.92	67.2	71.8	-6.8	-0.03
4	42670	2.82	60.3	56.4	6.5	0.03
5	51070	1.12	0.049	0.0504	-2.9	0.00
6	51070	1.18	0.168	0.17	-1.2	0.00
7	51070	1.23	0.331	0.329	0.6	0.00
8	51070	1.25	0.403	0.409	-1.5	0.00
9	51070	1.30	0.627	0.65	-3.7	0.00
10	51070	1.34	0.902	0.911	-1.0	0.00
11	122370	1.81	11.3	11.7	-3.5	-0.01
12	122370	1.79	11.4	10.9	4.4	0.01
13	122470	1.58	4.26	4.35	-2.1	0.00
14	122470	1.58	4.44	4.35	2.0	0.00

LAKE WINNIPESAUKEE OUTLET AT LAKEPORT, NEW HAMPSHIRE

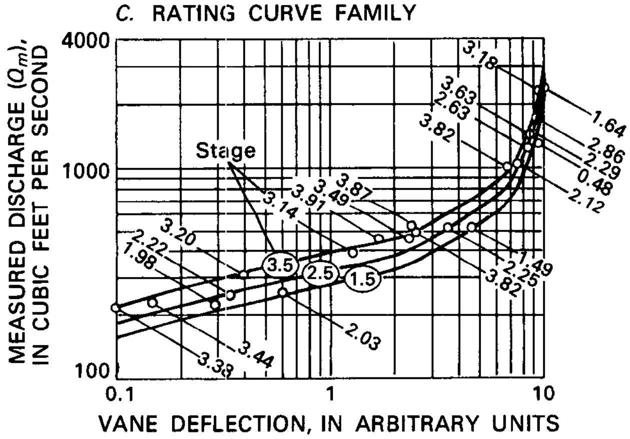
A. TYPICAL DEFLECTION VANE EQUIPMENT ARRANGEMENT



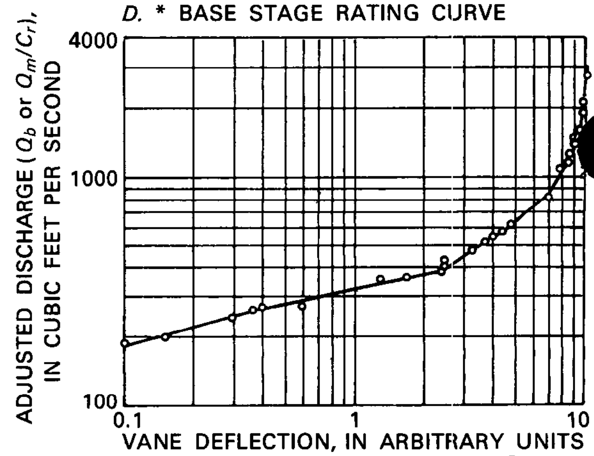
B. COMPUTATIONS

NO (1)	GHT (2)	VG (3)	QM (4)	QB (5)	QM/QB (6)	CP (7)	QM/CP (8)	QR (9)	% (10)
596	2.16	0.40	257	270	0.95	0.96	268	259	-0.8
597	2.21	0.35	249	261	0.95	0.96	256	252	-1.2
598	2.00	4.40	537	576	0.93	0.93	579	534	0.6
599	1.66	9.40	1440	1640	0.88	0.87	1650	1430	0.7
600	3.82	7.00	1010	863	1.17	1.22	829	1050	-3.8
604	2.75	3.30	510	478	1.07	1.05	485	503	1.4
605	3.03	0.40	310	270	1.15	1.10	283	296	4.7
606	1.70	9.00	1340	1430	0.94	0.88	1530	1250	7.2
607	0.48	9.80	1320	2110	0.63	0.66	2000	1400	-5.7
608	1.98	0.30	226	250	0.90	0.92	244	231	-2.2
609	3.82	2.50	495	404	1.23	1.22	406	492	0.6
610	3.63	8.60	1490	1230	1.20	1.19	1240	1460	1.4
611	2.66	9.50	1710	1700	1.01	1.07	1600	1820	-6.0
612	3.18	9.80	2350	2110	1.11	1.12	2100	2360	-0.4
613	3.14	1.30	393	342	1.15	1.11	353	381	3.1
614	2.25	3.70	509	515	0.99	0.97	525	499	2.0
616	2.30	9.00	1360	1430	0.97	0.98	1410	1400	-1.4
617	2.39	3.70	495	515	0.96	0.99	499	511	-3.1
618	3.49	2.40	453	398	1.14	1.17	388	465	-2.6
619	3.20	8.80	1400	1330	1.05	1.12	1250	1490	-6.0
624	2.12	7.80	1040	1040	1.00	0.95	1100	986	5.5
625	1.47	4.70	511	601	0.85	0.84	610	503	1.6
626	2.63	8.60	1230	1230	1.00	1.03	1190	1270	-3.1
628	3.87	2.50	526	404	1.30	1.23	429	495	6.3
629	3.20	3.80	584	524	1.11	1.12	520	588	-0.7
630	2.29	9.20	1480	1530	0.97	0.98	1520	1490	-0.7
631	2.68	8.90	1550	1390	1.12	1.04	1490	1440	7.6
632	3.91	1.70	455	361	1.26	1.23	369	445	2.2
635	2.03	0.60	253	293	0.86	0.93	271	273	-7.2
636	1.64	10.10	2370	2610	0.91	0.87	2730	2260	4.9
637	3.44	0.15	230	208	1.11	1.16	198	241	-4.6
638	3.38	0.10	216	187	1.16	1.15	188	215	0.5
639	3.20	0.40	301	270	1.11	1.12	268	303	-0.7

C. RATING CURVE FAMILY



D. * BASE STAGE RATING CURVE



*Base curve for 2.5 ft stage selected from *C.*

E. DESCRIPTORS FOR BASE RATING

Log scale offset=0

Deflection	Q_b	Deflection	Q_b
0.01	100	5.0	625
.10	187	6.7	800
.40	270	8.6	1230
2.05	375	9.5	1700
2.60	410	10.3	3000

F. RATING EQUATION

$$Q_r = Q_b \times (0.5725 + 0.187G - 0.0047G^2)$$

G. STAGE COEFFICIENT CURVE

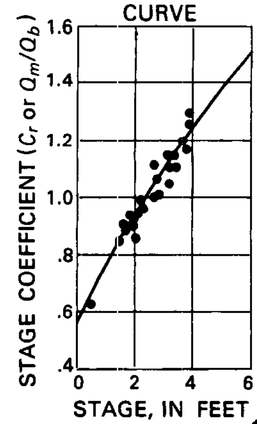


FIGURE 24.—Two-curve index-velocity rating for a vane-gage station.

H. ANALYSIS PROCEDURE

STEP	OPERATION	INSTRUCTIONS
1	Prepare work sheets	Needed: Sheets 1 and 2, log-log grids for rating curves; sheet 3, a rectangular grid for the stage coefficient; and a computation sheet with columns titled and numbered from (1) to (10) as in figure 24*B*. An additional column, unnumbered, may be needed if the deflection scale is offset for negative velocities.
2	Enter data	Fill in (1) to (4) with the data from the discharge measurements.
3	Define family of ratings	Plot V_g ((3)) along *X* axis vs. Q_m ((4)) on sheet 1. Flag each point with its stage (figure 24*C*). Draw a family of curves, based on the plotted points, as completely as the data allow. The curve for each stage should ordinarily be above all points that are flagged with values less than that stage and below the points flagged with higher values. Select the best defined curve from the family as the base rating, and its corresponding stage will be the base stage.
4	First trial curve	Plot the base curve from step 3 on sheet 2 and fill in Q_b ((5)).
5	List Q_m/Q_b	Fill in * (6) = (4) ÷ (5) .
6	Stage coefficient curve	Plot (2) along *X* axis vs. (6) on sheet *3. Draw a curve based on the points. The preferred final format is an equation (figure 10*B*).
7	List C_r	Fill in * (7) from the step 6 curve or equation.
8	List $Q_m/C_r = Q_{adj}$	Fill in * (8) = (4) ÷ (7) .
9	Plot base rating curve	Plot V_g ((3)) along *X* axis vs. Q_{adj} ((8)) on sheet *2. Draw the base stage rating curve based on the points. The preferred final format is a set of logarithmic curve descriptors (figure 7).
10	List Q_b	Fill in * (5) from step 9 curve or descriptors.
11	Go to step 5	Repeat steps 5–10 until further improvement becomes unlikely, then proceed to step 12.
12	Finalize	Prepare descriptors for sheet 2 curve and equation for sheet 3 curve.
13	Recompute final	Recompute * (5) to (8) and compute (9), (10) . (10) = 100 x ((4) − (9)) ÷ (9) using step 12 material. If (10) values are unsatisfactory, return to step 5, giving special attention to outliers. If (10) values are satisfactory, proceed to step 14.
14	Finalize	Prepare the master curve sheet.

*Erase any entries or plotting from previous trials.

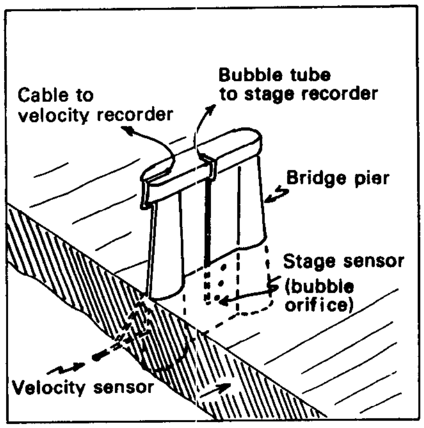
SYMBOLS

$CR = C_r$	Stage coefficient
G	Stage or gage height
NO	Serial number of measurement
$Q_{adj} = QM/CR$	Discharge adjusted to base stage
$QB = Q_b$	Discharge from base stage rating
$QM = Q_m$	Measured discharge
$QR = Q_r$	Discharge computed from rating $= QB \times C_r$
$VG = V_g$	Vane deflection reading
% Diff.	Variation of Q_m from Q_r
(n)	Column number on computation sheet

FIGURE 24.—Continued.

LAKE MOULTRIE TAILRACE CANAL NEAR MONCKS CORNER, SOUTH CAROLINA

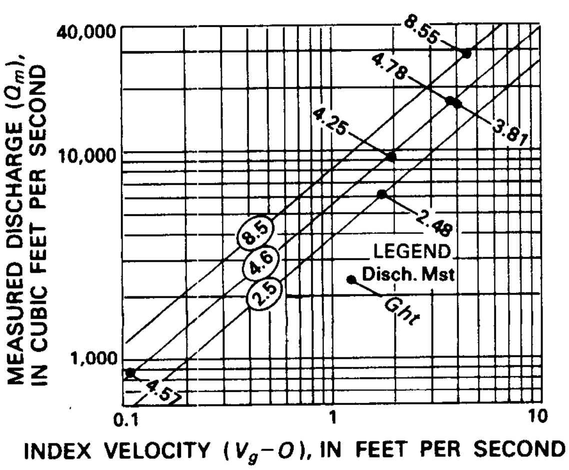
A. TYPICAL MAGNETIC METER EQUIPMENT ARRANGEMENT



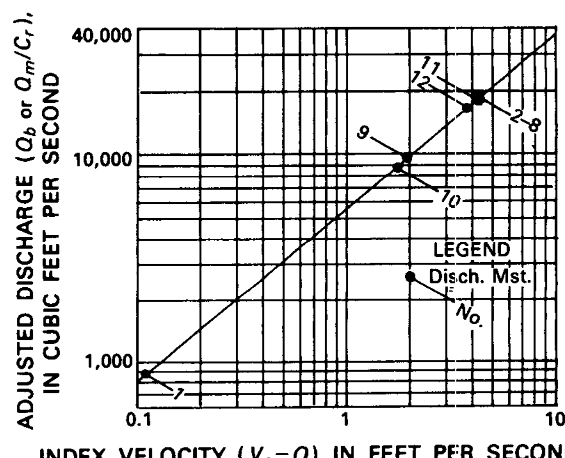
B. COMPUTATIONS

NO ①	GHT ②	VG ③	VG-O ④	QM ⑤	QB ⑥	QM/QB ⑦	CR ⑧	QM/CR ⑨	QR ⑩	% ⑪
1	4.57	0.89	-0.11	-870	-877	0.99	0.99	875	-872	-0.2
2	8.53	5.38	4.38	28300	18900	1.50	1.52	18600	28800	-1.7
3	8.50	5.40	4.40	28800	18900	1.52	1.52	19000	28700	0.3
4	8.50	5.33	4.33	28600	18700	1.53	1.52	18800	28400	0.7
5	8.49	5.29	4.29	28300	18500	1.53	1.52	18700	28000	1.1
6	8.55	5.38	4.38	28300	18900	1.50	1.52	18600	28800	-1.7
7	8.62	5.24	4.24	27900	18400	1.52	1.53	18200	28200	-1.1
8	8.65	5.29	4.29	27900	18500	1.51	1.54	18100	28400	-1.8
9	4.25	2.99	1.98	9140	9730	0.94	0.95	9600	9270	-1.4
10	2.48	2.80	1.80	6190	8990	0.69	0.72	8640	6440	-3.9
11	3.81	5.02	4.02	16500	17600	0.94	0.89	18500	15700	5.1
12	4.78	4.89	3.89	17200	17100	1.01	1.02	16800	17500	-1.7

C. RATING CURVE FAMILY

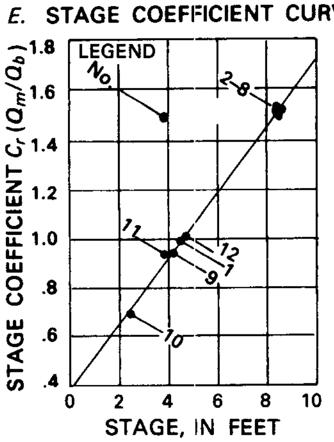


D. BASE STAGE RATING CURVE



*Base curve for 4.6 ft stage selected from (*C.*)

E. STAGE COEFFICIENT CURVE



F. DESCRIPTORS FOR BASE RATING

Log scale offset=0

Index velocity	Q_b
0.10	810
10.00	37,500

G. RATING EQUATION

$$Q_r = Q_b\ (0.133G + 0.387)$$

or

$$Q_r = [5511\ (V_g - 1)^{.832}]\ [0.133G + 0.387]$$

SYMBOLS

$CR = C_r$	Stage coefficient
$GHT = G$	Stage or gage height
NO	Serial number of measurement
O	Magnetic-meter reading at zero velocity
$Q_{adj} = Q_m/C_r$	Discharge adjusted to base stage
$QB = Q_b$	Discharge from base stage rating
$QM = Q_m$	Measured discharge
$QR = Q_r$	Discharge computed from rating
$VG = V_g$	Magnetic-meter reading
% Diff.	Variation of Q_m from Q_r
ⓝ	Column number on computation sheet

FIGURE 25.—Two-curve index-velocity rating for a magnetic-meter station.

H. ANALYSIS PROCEDURE

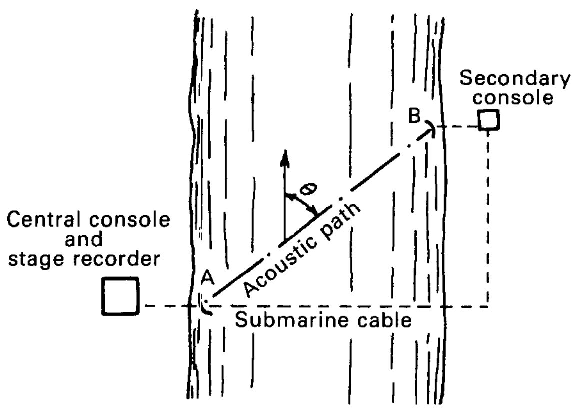
STEP	OPERATION	INSTRUCTIONS
1	Prepare work sheets	Needed: Sheets 1 and 2, log-log grids for rating curves; sheet 3, a rectangular grid for the stage coefficient; and a computation sheet with columns titled and numbered 1 to 11 as in *B*. Columns 3 and 4 are identical if the magnetic-meter scale is zero at zero velocity.
2	Enter data	Fill in 1 to 5 with data from the discharge measurements.
3	Define family of ratings	Plot V_g–0 (4) along *X* axis vs. Qm (5) on sheet 1. Flag each point with its stage (fig. 25*C*). Draw a family of curves, based on the plotted points, as completely as the data allow. The family should be a series of parallel and nearly straight lines (fig. 25*C*). The spread between curves depends largely on the height of the velocity sensor above the streambed. Select the best-defined curve from the family as the base rating, and its corresponding stage will be the base stage.
4	First trial curve	Use the sheet 1 base curve from step 3 as the first trial curve and fill in Qb (6).
5	List $QmQb$	Fill in * 7 = 5 ÷ 6 .
6	Stage coefficient curve	Plot 2 along the *X* axis vs. 7 on sheet *3. Draw a curve based on the points. The preferred final format is an equation (fig. 10*A* or 10*B*).
7	List Cr	Fill in * 8 from the step 6 curve or equation.
8	List $Q_{adj} = QmCr$	Fill in * 9 = 5 − 8 .
9	Plot base rating curve	Plot Vg–0 (4) along the *X* axis vs. Q_{adj} (9) on sheet *2. Draw the base stage rating curve based on the plotted points. The preferred final format is an equation (fig. 10*B* or 10*C*) or a set of logarithmic curve descriptors (fig. 7).
10	List Qb	Fill in * 6 from step 9 curve, equation, or descriptors.
11	Go to step 5	Repeat steps 5–10 until further improvement becomes unlikely, then proceed to step 12.
12	Finalize	Prepare the final equations, descriptors, or tables.
13	Recompute final figures	Recompute * 6 to 9 and compute 10 , 11 . 11 = 100 x (5 − 10)÷10. If the 11 values are unsatisfactory, return to step 5, giving special attention to the outliers. If 11 values are satisfactory, proceed to step 14.
14	Finalize	Prepare the master curve sheet.

*Erase any entries or plotting.

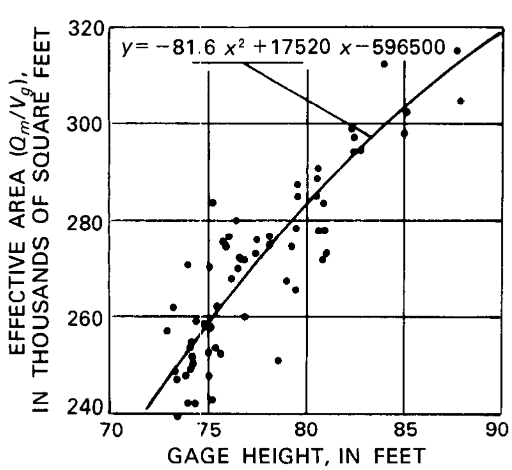
FIGURE 25.—Continued.

COLUMBIA RIVER AT THE DALLES, OREGON

A. TYPICAL ACOUSTIC VELOCITY EQUIPMENT ARRANGEMENT



B. EFFECTIVE AREA CURVE



C. COMPUTATIONS

NO (1)	GHT (2)	VG (3)	QM (4)	QM/VG (5)	CC (6)	CU (7)	QR (8)	% (9)
270	79.54	1.219	347000	284659	-592695	-596500	342000	1.5
271	79.49	1.186	341000	287521	-589606	-596500	333000	2.4
272	82.37	1.510	444000	294039	-595500	-596500	442000	0.5
273	82.28	1.485	443000	298316	-590356	-596500	435000	1.8
274	78.02	1.063	292000	274694	-595575	-596500	291000	0.3
275	78.04	1.071	296000	276377	-593988	-596500	293000	1.0
276	76.38	0.754	211000	279840	-582363	-596500	200000	5.5
277	75.28	0.680	177000	260294	-596252	-596500	177000	0.0
278	75.61	0.694	175000	252161	-606103	-596500	182000	-3.8
279	74.16	0.460	111000	241304	-609279	-596500	117000	-5.1
280	74.04	0.484	123000	254132	-595800	-596500	123000	0.0
281	74.11	0.485	122000	251546	-598766	-596500	123000	-0.8
282	73.82	0.481	130000	270270	-578463	-596500	121000	7.4
283	74.76	0.674	174000	258160	-595643	-596500	173000	0.6
284	74.98	0.649	164000	252696	-602273	-596500	168000	-2.4
285	75.27	0.826	215000	260290	-596204	-596500	215000	0.0
286	75.36	0.816	213000	261029	-595935	-596500	213000	0.0
287	80.77	1.409	399000	283179	-599632	-596500	403000	-1.0
288	80.67	1.366	397000	290629	-591747	-596500	391000	1.5
289	81.00	1.416	393000	277542	-606261	-596500	407000	-3.4
290	81.00	1.422	388000	272855	-610948	-596500	409000	-5.1
293	79.35	1.203	334000	277639	-598850	-596500	337000	-0.9
294	79.43	1.232	327000	265422	-611432	-596500	345000	-5.2
295	79.20	1.195	328000	274476	-601326	-596500	334000	-1.8
297	79.00	1.157	309000	267070	-607810	-596500	322000	-4.0
298	77.35	0.967	264000	273009	-594018	-596500	262000	0.8
299	77.31	0.955	263000	275392	-591439	-596500	258000	1.9
300	75.83	0.769	211000	274382	-585017	-596500	202000	4.5
301	74.17	0.576	144000	250000	-600638	-596500	146000	-1.4
302	74.17	0.576	144000	250000	-600638	-596500	146000	-1.4
303	74.09	0.550	137000	249090	-601114	-596500	140000	-2.1
304	73.41	0.528	131000	248106	-598370	-596500	132000	-0.8
305	73.45	0.548	131000	239051	-607646	-596500	137000	-4.4
306	76.81	0.844	219000	259479	-604882	-596500	226000	-3.1
307	75.05	0.796	215000	270100	-585238	-596500	206000	4.4
308	75.06	0.849	210000	247349	-608042	-596500	220000	-4.5
309	75.08	0.790	224000	283544	-571952	-596500	205000	9.3
310	75.18	0.937	203000	242532	-613490	-596500	217000	-6.5
311	85.13	1.709	517000	302516	-597647	-596500	519000	-0.4
312	85.13	1.713	511000	298307	-601856	-596500	520000	-1.7
313	82.94	1.623	509000	313616	-583119	-596500	486000	4.7
314	84.00	1.634	510000	312117	-583847	-596500	489000	4.3
315	82.41	1.570	466000	296815	-592887	-596500	460000	1.3
316	82.62	1.622	477000	294091	-596471	-596500	477000	0.0
317	76.60	0.970	263000	271134	-592176	-596500	259000	1.5
318	76.56	0.963	260000	269989	-593121	-596500	257000	1.2
319	75.35	0.585	148000	252991	-603921	-596500	152000	-2.6
320	87.82	1.958	617000	315117	-594204	-596500	613000	0.7
321	87.87	1.946	593000	304727	-604753	-596500	609000	-2.6
322	74.29	0.642	166000	258566	-592720	-596500	164000	1.2
323	74.00	0.662	160000	241691	-608024	-596500	168000	-4.8
324	73.89	0.302	74700	247350	-601765	-596500	76300	-2.1
325	73.34	0.596	147000	246644	-599444	-596500	149000	-1.3
326	80.60	1.460	416000	284931	-597140	-596500	417000	-0.2
327	80.71	1.490	414000	277852	-604698	-596500	426000	-2.8
328	73.67	0.588	151000	256802	-586660	-596500	145000	4.1
329	73.14	0.578	151000	261245	-583730	-596500	144000	4.9
330	76.07	1.029	284000	275996	-584632	-596500	272000	4.4
331	76.34	1.034	277000	267891	-594109	-596500	275000	0.7
333	75.10	0.473	122000	257928	-597673	-596500	123000	-0.8
334	80.59	1.089	316000	290174	-591853	-596500	311000	1.6
335	80.53	1.085	313000	288479	-593286	-596500	310000	1.0
336	75.80	0.793	219000	274905	-584340	-596500	208000	4.8

FIGURE 26.—One-curve index-velocity rating for an acoustic-velocity-meter station.

D. RATING EQUATION

$Q_r = V_g(a_2G^2 + a_1G + a_0)$ or

$Q_r = V_g(-81.6G^2 + 17{,}520G - 596{,}500)$

E. ANALYSIS PROCEDURE

STEP	OPERATION	INSTRUCTIONS
1	Prepare work sheets	Needed: a rectangular grid for the effective area curve and a computation sheet with columns titled and numbered (1) to (9) as in *C*. Columns (6) and (7) are unnecessary for sites whose ratings do not shift.
2	Enter data	Fill in (1) to (4) with data from the discharge measurements.
3	Effective area	Fill in * (5) $= Qm \div Vg =$ (4) $\div$ (3) .
4	Plot effective area curve	Plot *Ght* ((2)) vs effective area (5) on rectangular grid. Draw a curve and fit an equation to it (fig. 10*B*). If rating does not shift, skip to step 7.
5	Compute C_c	Fill in (6) = (5) $-a_2$ (2)^2 $-a_1$ (2) .
6	List C_u	Fill in (7) = value of a_0 applicable at time of measurement. a_0 is used as a shift adjustment.
7	List Q_r	Fill in (8) $= Vg$ x (value from step 4 equation).
8	List percentage	Fill in (9) = 100 ((4) − (8)) ÷ (8) .
9	Finalize	Prepare the master curve sheet.

SYMBOLS

a_n	Equation coefficient for a second-degree polynomial
CC	Value of a_0 that makes $Q_m = Q_r$
CU	Value of a_0 applicable at time of measurement. *CU* can be varied and used as a shift adjustment.
$GHT = G$	Gage height
NO	Serial number of measurement
$QM = Q_m$	Measured discharge
$QR = Q_r$	Discharge computed from rating equation
$VG = V_g$	Acoustic-velocity meter reading
% Diff.	Variation of Q_m from Q_r
(n)	Column number on computation sheet

FIGURE 26.—Continued.

that are related to bends and breaks in rating curves. Extrapolation of an automatically fitted curve is particularly unsatisfactory. Fitting an equation to a manually drawn curve by inputting selected points from that curve rather than from the observed data to a fitting program avoids the problem and is encouraged wherever the equation format is needed.

References

Bailey, J. F., and Ray, H. A., 1966, Definition of stage-discharge relation in natural channels by step-backwater analysis: U.S. Geological Survey Water-Supply Paper 1869-A, 23 p.

Brater, E. F., and King, H. W., 1976, Handbook of hydraulics (6th ed., revised): New York, McGraw Hill, 329 p.

Collins, D. L., 1977, Computation of records of streamflow at control structures: U.S. Geological Survey Water-Resources Investigations 77-8, 57 p.

Dawdy, D. R., 1961, Depth-discharge relations of alluvial streams-discontinuous rating curves: U.S. Geological Survey Water-Supply Paper 1498-C, 16 p.

Hutchison, N. E., and others, 1975, WATSTORE user's guide, v.1: U.S. Geological Survey Open-File Report 75-426, 532 p.

———WATSTORE user's guide, v.5: U.S. Geological Survey Open-File Report 77-729-I, 230 p.

Johnson, L. H., 1952, Nomography and empirical equations: New York, John Wiley, 150 p.

Rantz, S. E., and others, 1982, Computation of discharge, v.2: U.S. Geological Survey Water-Supply Paper 2175, 241 p.

Shearman, J. O., 1976, Computer applications for step-backwater and floodway analysis: U.S. Geological Survey Open-File Report 76-499, 103 p.

Simons, D. B., and Richardson, E. V., 1962, The effect of bed roughness on depth-discharge relations in alluvial channels: U.S. Geological Survey Water-Supply Paper 1498-E, 26 p.

Smith, W., Hubbard, L. L., and Laenen, A., 1971, The acoustic streamflow-measuring system on the Columbia River at The Dalles, Oregon: U.S. Geological Survey Open-File Report, 60 p.

INTERNATIONAL SYSTEM OF UNITS (SI) AND INCH-POUND SYSTEM EQUIVALENTS

SI unit		Inch-pound equivalent
Length		
centimeter (cm)	=	0.3937 inch (in)
meter (m)	=	3.281 feet (ft)
kilometer (km)	=	0.6214 mile (mi)
Area		
centimeter^2 (cm^2)	=	0.1550 inch^2 (in^2)
meter^2 (m^2)	=	10.76 feet^2 (ft^2)
kilometer^2 (km^2)	=	0.3861 mile^2 (mi^2)
Volume		
centimeter^3 (cm^3)	=	0.06102 inch^3 (in^3)
meter^3 (m^3)	=	35.31 feet^3 (ft^3)
	=	8.107×10^{-4} acre-foot (acre-ft)
Volume per unit time		
meter^3 per second (m^3/s)	=	35.31 feet^3 per second (ft^3/s)
	=	1.585×10^4 gallons per minute (gal/min)
Mass per unit volume		
kilogram per meter^3 (kg/m^3)	=	0.06243 pound per foot^3 (lb/ft^3)
gram per centimeter^3 (g/cm^3)	=	6.243×10^{-5} pound per foot^3 (lb/ft^3)
Temperature		
degree Celsius (°C)	=	(degree Fahrenheit − 32)/1.8 (°F)